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MAGNETIC LEVITATION SYSTEM CONTROLLED BY SDRE CONTROL

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Abstract: *With the growth of global population, the demand for electrical energy have increased exponentially in the last decades. Therefore, there is a need to search for new ways to harvest energy, even for low power consumption system. Electric energy retrieved from dynamic system have been very important due to the feedback of energy from wasted energy from the system, for example, the kinetic energy provided by vibrations, being possible to use such energy in electric components near these systems. Therefore, this work proposes to control a magnetic levitation system in order to harvest energy from a periodic motion. Depending on the magnetic force, i.e., stiffness of the system, it may become chaotic, which is a difficult way to harvest energy. Thus, the SDRE control strategy will be applied in order to control the chaotic behavior and maintain a good value of energy harvesting. Results showed the SDRE controlled the system and increased significantly the harvested power.*

Key-words: *Magnetic levitation system, nonlinear dynamics, energy harvesting, SDRE control*

1. INTRODUCTION

Nowadays, there are many kind of energy sources that are very promising to be explored. One of them abundantly available in the environment is the kinetic energy provided by vibration of structures (Stephen, 2006). Those explorations are a necessity for, in the last decades, the world demand for electric energy increased through 40%, with fossil fuels usage (Mescia, 2015; Shefiee and Topal, 2009; 2014). The great advantage of the kinetic energy is that it is constantly produced in the environment with unlimited sources, depending on just and external excitation of its means.

In high technological means, there is a foremost necessity of feedback energy from some places, whose energy could be wasted. Magnetic levitation (MagLev) systems have been often used in some applications that offer many advantages related to its use, e.g., no friction and no fatigue of the magnetic effect (Beeby and O'Donnell, 2009).

High-speed bullet trains are one of the most known example that uses the technology of MagLev devices, which reduces the friction between the train and travel rails, making possible the high speed of the train. Due to the train pass, there are many vibrations at these maglev devices, whose vibration could be used to supply the train station light.

Another use of MagLev devices are the wind trees (Nagarkar and Khan, 2013) that are subjected to the magnetic effect, reducing the friction between rolls-device, thus improving the energy harvesting from the wind.

Therefore, this work proposes the control and energy harvesting of a magnetic levitation model, which is based on Mann and Sims (2009), which will be detailed in the next section.

2. MATHEMATICAL MODELLING

The magnetic levitation model considered in this work, illustrated in Fig. 1, consists of a three magnet blocks placed at the top, middle and bottom of the device, where the generalized coordinate x is the displacement of the middle block with mass m. The middle block is positioned between the top and bottom magnets with inverted poles to oscillate when the base of the device is excited.

The oscillation makes possible to variate the magnetic field of the magnets and inducing an electric current in the coils coupled near the structure of the device. Based on the Kirchhoff's law, a RL electric circuit is considered to harvest the induced electric current, consisting of an instantaneous electric current i, the inductance of the coils L, and the internal and external resistances R_{int} and R_{ext}, respectively, and a constant, which represents the electromechanical coupling.

The energy harvesting RL circuit is not illustrated in Fig. 1. It is important to describe that according to an experiment conducted by Mann and Sims (2009), the magnetic force between the top and bottom magnets can be expressed as a power series and compared to a cubic Duffing stiffness, i.e., $kx+k_3x^3$.

The base of the maglev device is excited by an electro-dynamical shaker, which will be considered as an RLC electric circuit. Where R_s is the resistance of the shaker, the inductance is represented by L_s, the capacitance is C_s, with an electric charge q_s. In addition, the output of the electric device is a voltage source of harmonic kind with amplitude e and frequency ω.

The generalized coordinate of the electrical part of the shaker is considered as the charge q_s.

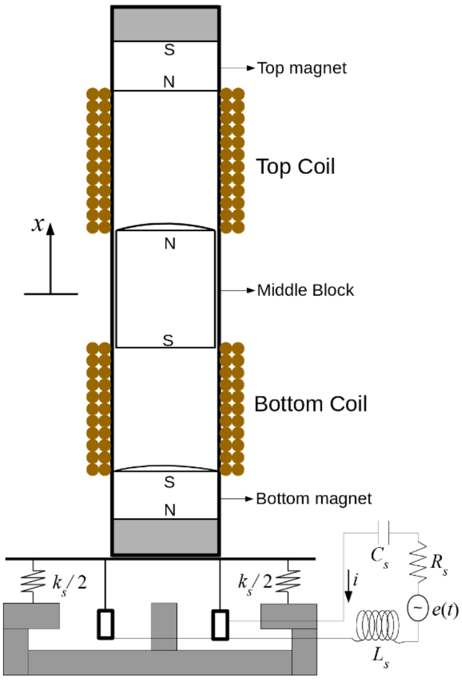


Figure 1. Magnetic levitation system excited by an electro-dynamical shaker

The equations of motion of the system in dimensionless form is given by Eq. (1), where X is the dimensionless displacement of the middle block, Q is the dimensionless charge of the electrical part of the shaker, W_3 is the cubic stiffness of the middle block, α_1 is the damping of the middle block, α_2 is the mechanical coupling, the electrical coupling is α_3, the capacitance of the shaker α_4, a resistive term of the shaker W_4 and E and Θ are the amplitude and frequency of the voltage source, respectively.

$$\begin{aligned} &X'' + \alpha_1 X' + X + W_3 X^3 + \alpha_2 Q' = 0 \\ &Q'' + \alpha_4 Q' + W_4 Q - \alpha_3 X' = E\cos(\Theta\tau) \end{aligned} \quad (1)$$

The energy harvesting of the system is computed using an electric circuit by applying the Kirchhoff's law. According to the Lenz's law, the electric damping is given from the addition of the coil to convert mechanical energy into electricity. Therefore, the dimensionless harvested power of the system is given by Eq. (2).

$$P = (C_e X')^2 \quad (2)$$

where the constant C_e is a relation of the electromechanical coupling with the electric resistances of the circuit.

In the next topic, some numerical simulations involving the governing equations of motion of the system and the harvested power will be presented and discussed.

3. NUMERICAL SIMULATIONS AND DISCUSSIONS

The numerical simulations were performed using the method of Runge-Kutta of 4th order with the parameters of Tab. 1.

Table 1. Parameters of the system

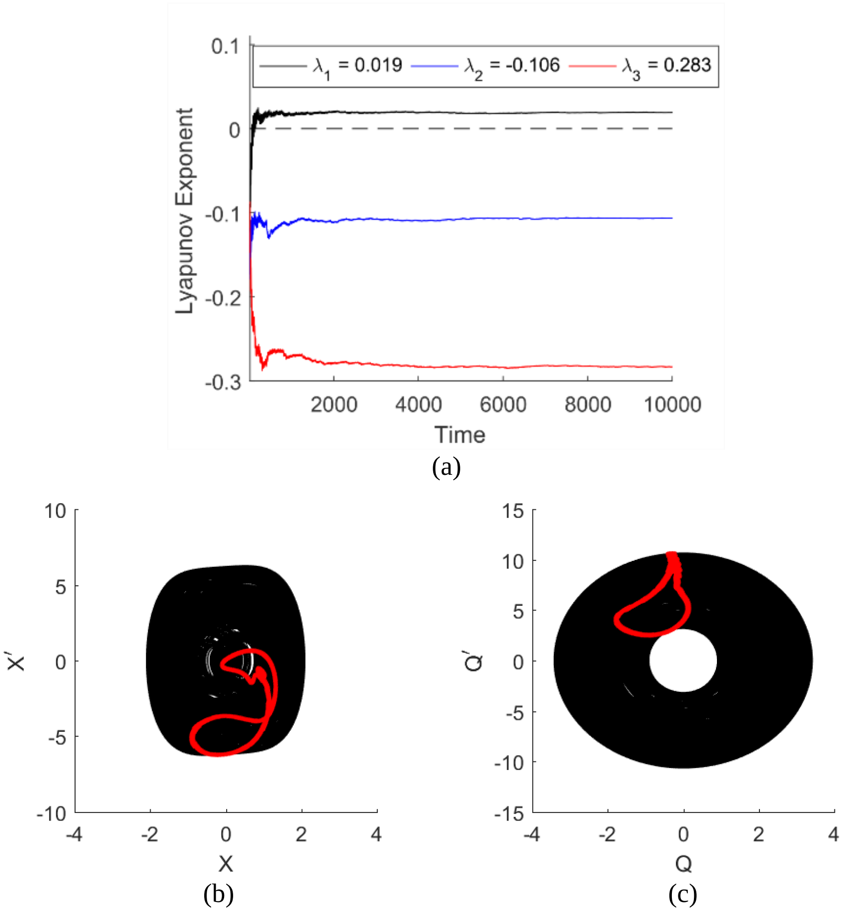
Parameter	Value	Means
α_1	0.62	Damping of the middle block
α_2	0.64944	Mechanical coupling term
α_3	0.64944	Electrical coupling term
α_4	0.1499	Capacitance of the shaker
W_3	4.157	Cubic stiffness of the middle block
W_4	9.108	Resistive term of the shaker
Θ	3.1215	Frequency of the voltage source
E	3.07	Amplitude of the voltage source
C_e	0.1403	Relation of electromechanical coupling with electrical resistance of the shaker

3.1. Dynamical analysis of the system without control

Firstly, the maglev system is analyzed without control, looking for chaotic behaviors and the energy harvesting at such configuration.

Figures 2 show the analysis of the system without any control. Through the Lyapunov exponent in Fig. 2a, the system is chaotic due to, at least, one positive exponent $\lambda_1 = 0.019$. Phase planes and Poincare maps of the middle block and charge of the electric circuit of the shaker, illustrated respectively in Fig. 2b and 2c, show the chaotic period of the system.

In this exactly behavior and parameters, the average harvested power is 0.1874 amount of power.



(a) (b) (c)

Figure 2. (a) Lyapunov exponents; phase plane (in black) and Poincare maps (in red) of (b) middle block, (c) charge of electric circuit

A deep analysis of the stiffness of the oscillator and the resistive term of the shaker is carried out. It provides an overview of energy harvesting with a configuration of both parameters. Figure 3 shows such analysis in the intervals of $1 \leq W_3 \leq 18$ and $5 \leq W_4 \leq 11$. It can be observed two peaks of harvested power, which are near 0.24 amount of power.

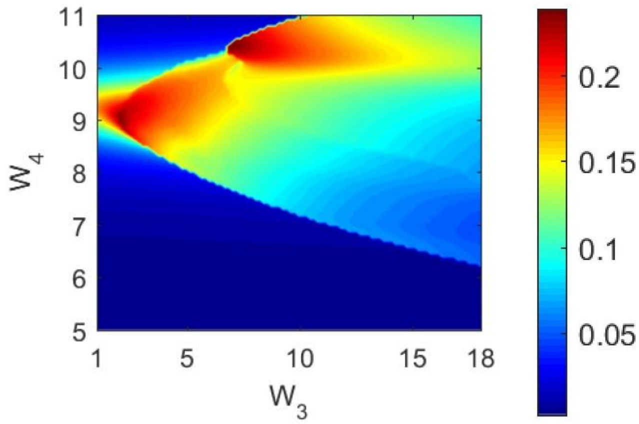


Figure 3. Harvested power of the system varying the cubic stiffness of the mechanical part of the maglev (W_3) and the resistive term of the shaker (W_4)

Next, the SDRE control technique is considered.

3.2. Dynamical analysis of the system using SDRE control

The State-Dependent Riccati Equations (SDRE) control is a consolidated control technique used by many authors whose works involve nonlinear problems. Its great advantage is that its state matrix considers the nonlinearities of the system, which is not necessary to linearize the system (Tusset and Balthazar, 2013; Tusset *et al.*, 2013; 2015; Alves *et al.*, 2017; Balthazar *et al.*, 2014; Lima *et al.*, 2016).

The control will be considered only in the equation of the middle block; thus, the equation of the shaker will be just considered as the excitation of the first one.

Considering the control in the equations of motion, it has

$$\begin{aligned} & X'' + \alpha_1 X' + X + W_3 X^3 + \alpha_2 Q' = U \\ & Q'' + \alpha_4 Q' + W_4 Q - \alpha_3 X' = E\cos(\Theta\tau) \end{aligned} \tag{3}$$

where U is the signal of control which is a feedback control.

The maglev nonlinear system is represented in the matrix form denoted as below:

$$\dot{X} = A(X)X + U \tag{4}$$

where U is the feedback control and is determined, and $A(X)$ is the state matrix.

Considering the dynamic system defined by Eq. (4), it can be parameterized in first order equations and written in the state-dependent coefficient (SDC) form:

$$\begin{aligned} & \dot{X} = A(X)X + BU \\ & Y = C(X)X \end{aligned} \tag{5}$$

where B is the control matrix, and C is the output matrix is the output matrix.

As states feedback and feedforward are adopted to enhance the control performance, the quadratic cost function for the regulator problem is given by:

$$J = \frac{1}{2}\int_{t_0}^{\infty}\left[X^T Q X + U^T R U\right]dt \tag{6}$$

where Q is the semi-positive-definite matrix and R is a positive definite. There are weighting matrices on the outputs and control inputs, respectively. For a point-wise linear fashion, the matrices are assumed as constant coefficients.

Assuming full state feedback, the control law is given by:

$$U = -R^{-1}B^T PX \tag{7}$$

The state-dependent Riccati equation is solved to obtain $P(X)$, whose equation is given by:

$$A^T(X)P(X) + P(X)A(X) - P(X)B(X)R^{-1}B^T P(X) + Q = 0 \tag{8}$$

It is shown in Mracek and Cloutier (1998) that in the neighborhood Ω close to the origin, the SDRE method guarantees a closed-loop solution, local asymptotic stability. In the scalar case, the SDRE method reaches the optimal solution of the feedback regulator problem performance index (16), even when Q and R are functions of X.

The desired orbit of the control is defined as

$$\tilde{X} = 2\sin(\Theta\tau) \tag{9}$$

and the errors of the control are

$$\begin{aligned} e_1 &= X - \tilde{X} \\ e_2 &= X' - \tilde{X}' \end{aligned} \tag{10}$$

Substituting the errors in Eq. (3), it has the system in deviations, which is

$$\frac{de}{dt} = A(e)e + u \tag{11}$$

The feedback control is obtained by

$$u = -k(1,1)e_1 - k(1,2)e_2 \tag{13}$$

Where k is obtained through the command of MATLAB software:

$$k = lqr(A, B, Q, R) \tag{14}$$

Where the matrices are given by

$$A = \begin{bmatrix} 0 & 1 \\ -1 - W_3\left(e_1^2 + 3e_1X + 3Xe_1\right) & \alpha_1 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad Q = 1000\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad R = 1 \tag{15}$$

Therefore, using the control, Fig. 4a shows the new trajectory of the system, which is totally periodic.

Figure 4b shows the time history of the controlled system (in black) and the desired trajectory (in red). It is possible to observe that the controller was efficient to lead the system to the desired orbit keeping the behavior of the system as periodic.

In addition, the average harvested power of the system increased from the chaotic behavior, which was 0.1874, to the amount of 0.3835.

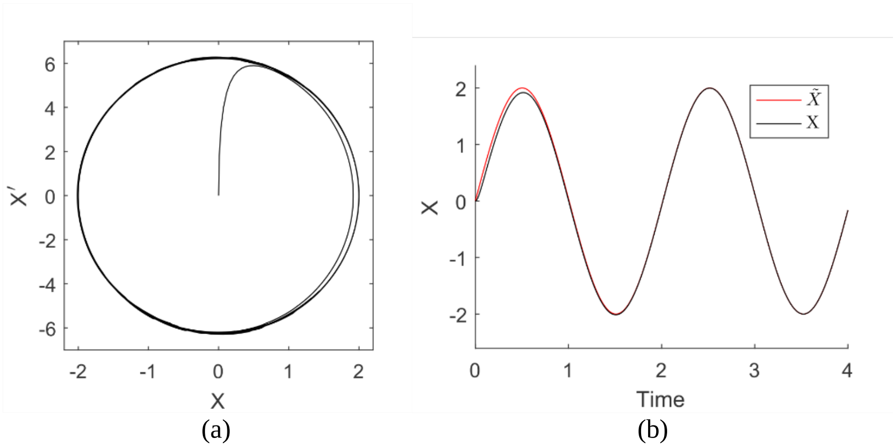


Figure 4. Controlled system; (a) Phase plane; (b) Time history of displacement

The same analysis of Fig. 3 is carried out considering two different ways.

Figure 5a shows the analysis of the harvested power considering the intervals $1 \leq W_3 \leq 18$ and $5 \leq W_4 \leq 11$ with the controlled system. It is observed that the harvested power increased in all over the region of W_4 vs W_3, from near 0.24 to peaks higher than 0.385 amount of power.

However, Fig. 5b shows the analysis of the harvested power considering the intervals of $1 \leq W_3 \leq 18$ and $0.25 \leq \alpha_1 \leq 1.5$. This case uses α_1 and W_3 because both parameters affect directly the state matrix of the controller. It is observed that the harvested power also increased from near 0.24 to peaks of 0.384, in average, amount of power.

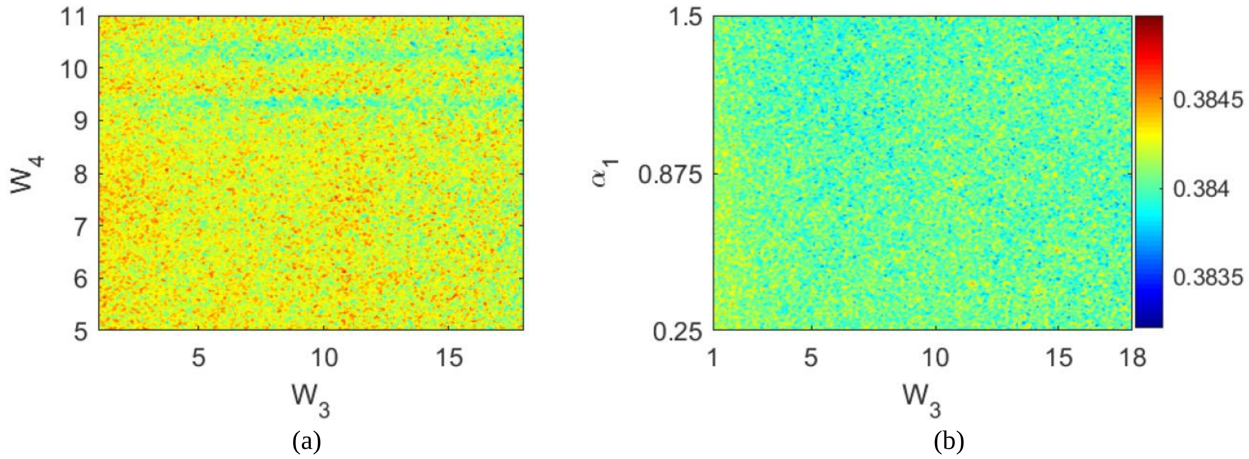


Figure 5. Harvested power of controlled system; (a) W_4 vs W_3; (b) α_1 vs W_3

4. CONCLUSIONS

This work showed the control of a maglev energy harvesting system using the SDRE technique.

The control was efficient to lead the chaotic behavior to periodic one and, in addition, increased significantly the energy harvesting of the system. Considering different parameters to analyze the energy harvesting, two cases were considered: the variation of W_4 vs W_3 and α_1 vs W_3. The first case possesses only W_3 as a parameter in the state matrix of the controller and the second case considers both parameters in the state matrix of the controlled system. The best configuration to obtain the highest amount of power was in the first case, obtaining higher peaks of power. However, the configuration of the second case also increased the harvested power from the uncontrolled system. Moreover, the behavior of the system in all such cases are periodic, due to the fact that the SDRE control is robust.

5. ACKNOWLEDGEMENTS

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7. RESPONSABILITY NOTICE

The authors are the only responsible for the included material in this paper.