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VIBRATION CONTROL OF SMART STRUCTURES WITH A FUZZY SLIDING MODE CONTROL SCHEME

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Abstract. *Smart structures and systems have the main purpose to mimic living organisms, which are essentially characterized by an autoregulatory behavior. Therefore, this kind of structure has adaptive characteristics which are capable of changing its geometry or physical properties with the purpose of performing a specific task. In this work, a sliding mode controller with a proportional fuzzy inference is applied for active vibration control in a shape memory von Mises truss. In order to obtain a simpler controller, a polynomial model is used in the control law, while a more sophisticated version, which presents close agreement with experimental data, is applied to describe the shape memory alloy behavior of the structural elements. This system has a rich dynamic response and can easily reach a chaotic behavior even at moderate loads and frequencies. Therefore, this approach has the advantage of not only obtaining a simpler control law, but also allows its robustness to be evidenced. Numerical simulations are carried out in order to demonstrate the control system performance.*

Keywords: *Sliding mode control, fuzzy logic, smart structures, von Mises truss, shape memory alloy (SMA).*

1. INTRODUCTION

The term smart structure has been used to identify structures that are made or have smart materials, either embedded or in surface layer, and can perform sensing, control and actuation function, it is a primitive analogue of a biological body. Despite the smart materials, they can be equipped with sensors and actuators with the purpose of performing particular tasks. For these structures, greater attention is given to shape memory alloys (SMAs), which are used in situations where high force, large strain, and low frequency structural control are needed. These alloys have the ability to return to a previous shape or dimension when subjected to an appropriate thermomechanical procedure, which is characterized by complete strain recovery accompanied by large hysteresis loop in a loading-unloading cycle (Otsuka and Ren, 1999). This property make them very appealing for applications in civil engineering structures (Cismaşiu and dos Santos, 2008). The mechanism behind SMAs remarkable behavior is related to the martensitic phase transformations that can be induced by stress and/or temperature.

The remarkable properties of SMAs are attracting much technological interest and motivating different applications in several fields of sciences and engineering. Specifically, SMAs have been used in the actuators and micromanipulators manufacture, in the biomedical field, in aerospace technology, among others (Mohd Jani *et al.*, 2014; Lagoudas, 2008). SMA actuators are relatively lightweight, easy to manufacture and able of producing high forces or displacements driven by thermal loads. In the biomedical field, these alloys are mainly used in the fabrication of small surgical devices and implants, due to its excellent biocompatibility (Konh *et al.*, 2015; Machado and Savi, 2003). In aerospace technology, SMAs has been used in self-erectable structures and mechanisms of stabilization (Chau *et al.*, 2006; Loughlan *et al.*, 2002). In addition, they are used in the manufacture of micromanipulators and robotic actuators to mimic the muscles movement (Paiva and Savi, 2006), and as actuators for vibration and buckling control of flexible structures (Amarante dos Santos, 2016).

It is a typical feature of structural analysis theory that its fundamental characteristics can be analyzed using very simple models, with one or two degrees of freedom (DOFs). Archetypal models are usually employed in order to investigate the general aspects of the structural dynamics, providing a global comprehension of the system behavior. The von Mises truss, also known as two-bar truss, is one of these archetypal models used to analyze the stability aspects of structures (Bazant and Cedolin, 2010). One of the remarkable feature of this structure is the snap-through behavior where, for a given load level, two displacement configurations are possible. Snap-through behavior is a classical geometrical nonlinearity. This structure represents one of the most popular system related to stability analysis, defining some of the most important characteristics of framed structures as well as of flat arches and of many other physical phenomena associated with bifurcation buckling (Savi, 2015).

The stability and control of SMA von Mises truss has been studied by some authors. Savi and Nogueira (2010) and Savi *et al.* (2002a) discussed the main aspects of its dynamical behavior by considering two different constitutive models to describe the thermomechanical behavior of SMAs. The constitutive model stated by Savi and Nogueira (2010) presents close agreement with experimental data, while Savi *et al.* (2002a) presented a simpler one, based in a polynomial constitutive model. Bessa *et al.* (2013) and de Paula *et al.* (2014) presented strategies to control SMA two-bar truss based on the sliding mode control technique and time-delayed feedback method, respectively, which proved to be effective.

In this work a smooth sliding controller with a fuzzy inference system is applied to control the vibration of a SMA von Mises truss. In order to simplify the controller design, a polynomial model (Savi *et al.*, 2002a) is used in the development of the control law, while a more sophisticated one (Savi and Nogueira, 2010) is assumed to simulate the thermomechanical behavior of the structure elements, which presents close agreement with experimental data. Considering that the sliding mode control technique requires a model to estimate the plant dynamics to be controlled, and that the sophisticated model given its complexity would not be the most appropriate, the polynomial model will be the one employed in the control law. This approach has the advantage of not only obtaining a simpler control law, but also allows its robustness to be evidenced.

In this context, the control scheme is mainly based on the sliding mode control (SMC) methodology, however, it is reinforced with a fuzzy inference system to deal with system modeling inaccuracies and external disturbances. The adoption of a robust control law, as used in the present work, allows simple constitutive models, such as the polynomial equation considered, can be considered to control systems with more complex models. After developing the control technique, numerical simulations are carried out to prove its robustness and effectiveness in the control of SMA structures even in the presence of uncertainties in the system. It is important to highlight that SMA properties are being used to achieve others goals than control. This situation is common in distinct applications which include aerospace systems as self-erectable structures (Bessa *et al.*, 2013).

2. DYNAMIC MODEL

In this paper the structure analyzed was the von Mises truss, this structure is basically composed of two identical bars, which have the same length L, same cross-section A_s and horizontal projection B, and form the same angle φ with its supports plane, as shown in Fig. 1. The bars are free to rotate in their supports and are connected at the central junction of the structure.

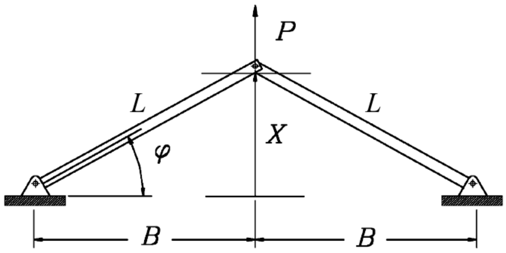


Figure 1. Schematic representation of a von Mises truss.

In the present analysis, we consider that the mass of the structure, m, is concentrated at the central junction, which can only move vertically, and the critical Euler load of both bars is assumed to be sufficiently large that buckling will not occur here. Under these assumptions, only symmetric motions will be considered in the structure, it is usually treated as a single DOF system, therefore its dynamic behavior is expressed through the following equation of motion:

$$-2F\sin\varphi - c\dot{X} + P = m\ddot{X} \quad (1)$$

where F is the force in each bar, c is an equivalent coefficient damping, P is an external disturbance and X represents the vertical and symmetric displacement of the central junction, as shown in Fig. 1. The force F is related to the mechanical behavior of the structure material, which in this work will be SMA.

There are many different works dedicated to describe the thermomechanical behavior of shape-memory alloys, however, this is not a well-established topic. In order to explore all potentialities of SMAs, there is an increasing interest on the development of mathematical models capable to describe the main behaviors of these alloys (Paiva and Savi, 2006).

Some authors proposed polynomial models to describe the thermomechanical behavior of SMA bars (Falk, 1980, 1983; Müller and Xu, 1991), the great advantage of this model is its simplicity. Nevertheless, the model provides a good description of the system dynamics, but can not present some important phenomena, such as pseudoelasticity, shape memory effect and the material hysteresis. On the other hand, there are some works which present models that are able to reproduce the main phenomena of this material, by using internal variables related to SMA phase transformation, that should obey internal constraints (Savi *et al.*, 2002b; Baeta-Neves *et al.*, 2004; Savi and Paiva, 2005; Paiva *et al.*, 2005). These contributions were based mainly on Fremond (1987) and Fremond and Miyazaki (1996) models, which present a three-dimensional model considering three volumetric fractions associated with austenite and detwinned martensite variants.

In order to present the constitutive equations to describe the thermomechanical behavior of each bar in the SMA structure, let us consider ε as strain, σ as normal stress, T as temperature and three more state variables associated with the volume fraction of each macroscopic phase, ρ_1 is associated with tensile detwinned martensite $(M+)$, ρ_2 is related to compressive detwinned martensite $(M-)$ and ρ_3 represents austenite (A). A fourth phase that is related to twinned martensite ρ_4 (M) is also considered, which can be obtained from the following phase coexistence condition:

$$\rho_4 = 1 - (\rho_1 + \rho_2 + \rho_3) \tag{2}$$

According to Savi and Nogueira (2010), with this assumption it is possible to obtain a complete set of constitutive equations that describes the thermomechanical behavior of SMAs as follows:

$$\sigma = E\varepsilon + [\alpha + E\alpha_h]\,(\rho_2 - \rho_1) - \kappa\,(T - T_0) \tag{3}$$

$$\dot{\rho}_1 = \frac{1}{\bar{\eta}_1}\big\{\alpha\varepsilon + \sigma_\Lambda + [2\alpha_h\alpha + E\alpha_h^2](\rho_2 - \rho_1) + \alpha_h[E\varepsilon - \kappa(T - T_0)] - \partial_1 J_\pi\big\} + \partial_1 J_\chi \tag{4}$$

$$\dot{\rho}_2 = \frac{1}{\bar{\eta}_2}\big\{-\alpha\varepsilon + \sigma_\Lambda - [2\alpha_h\alpha + E\alpha_h^2](\rho_2 - \rho_1) - \alpha_h[E\varepsilon - \kappa(T - T_0)] - \partial_2 J_\pi\big\} + \partial_2 J_\chi \tag{5}$$

$$\begin{aligned}\dot{\rho}_3 =& \frac{1}{\bar{\eta}_3}\left\{\frac{1}{2}(E_M - E_A)[\varepsilon + \alpha_h(\rho_2 - \rho_1)]^2 + (\kappa_A - \kappa_M)(T - T_0)[\varepsilon + \alpha_h(\rho_2 - \rho_1)] + \sigma_{\Lambda_3} - \partial_3 J_\pi\right\} + \\ &+ \partial_3 J_\chi\end{aligned} \tag{6}$$

where α_h is related to horizontal width of the material hysteresis loop on the stress-strain diagram, while α controls its vertical length, E is the elastic modulus and κ is related to the thermal expansion coefficient of the material, which are given by:

$$\begin{aligned} E &= E_M + \rho_3(E_A - E_M) \\ \kappa &= \kappa_M + \rho_3(\kappa_A - \kappa_M) \end{aligned} \tag{7}$$

The subscripts 'A' and 'M' present in the Eq. (6) and (7) refers to the austenitic and martensite phases, respectively. The terms $\partial_i J_\pi$ $(i = 1, 2, 3)$ are sub-differentials of the indicator function $J\pi$ with respect to ρ_i (Rockafellar, 1970). The indicator function $J_\pi = J_\pi(\rho_1, \rho_2, \rho_3)$ is related to a convex set π, which provides the internal constraints related to the phases' coexistence. With respect to evolution equations of volume fractions, the parameters $\bar{\eta}_i$ $(i = 1, 2, 3)$ are associated with internal dissipation in phase transformations. To study different characteristics of phase transformations, it is possible to consider different values for $\bar{\eta}_i$ as $\bar{\eta}_i^L$ and $\bar{\eta}_i^U$ during loading and unloading process, respectively. Moreover $\partial_i J_\chi$ $(i = 1, 2, 3)$ are sub-differentials of the indicator function J_χ with respect to $\dot{\rho}_i$ (Rockafellar, 1970). This indicator function is associated with the convex set χ, which establishes conditions for the correct description of internal subloops due to incomplete phase transformations and also avoids phase transformations $M+ \to M$ or $M- \to M$, for instance.

Concerning the parameters' definition, linear temperature dependent relations are adopted for σ_Λ and σ_{Λ_3}, which are related to stress level for phase transformation, as follows:

$$\sigma_\Lambda = \begin{cases} -\bar{L}_0 + \dfrac{\bar{L}}{T_M}(T - T_M) & \text{if} \quad T > T_M \\ -\bar{L}_0 & \text{if} \quad T \leq T_M \end{cases} \tag{8}$$

$$\sigma_{\Lambda_3} = \begin{cases} -\bar{L}_0^A + \dfrac{\bar{L}^A}{T_M}(T - T_M) & \text{if} \quad T > T_M \\ -\bar{L}_0^A & \text{if} \quad T \leq T_M \end{cases} \tag{9}$$

where T_M is the temperature below which the martensitic phase becomes stable, $\bar{L}_0$, $\bar{L}$, $\bar{L}_0^A$ and $\bar{L}^A$ are parameters related to critical stress for phase transformation. Now, the following strain definition is considered:

$$\varepsilon = \frac{L}{L_0} - 1 = \frac{\cos\varphi_0}{\cos\varphi} - 1 \tag{10}$$

with L_0 and φ_0 representing the nominal values of L and φ, respectively. The temperature T_0, present in Eq. (3-6), is the temperature when $\varepsilon = 0$.

From the stress definition $F = \sigma A_s$ and the Eq. (3) and (10), the equation of motion of the structure can be rewrite as:

$$m\ddot{X} + c\dot{X} + 2A_{st}\frac{X}{(X^2+B^2)^{1/2}}\left\{E\left[\frac{(X^2+B^2)^{1/2}}{L_0} - 1\right] + [\alpha + E\alpha_h](\rho_2 - \rho_1) - \kappa(T - T_0)\right\} = P(t) \tag{11}$$

Considering a external periodical disturbance $P(t) = P_0\,\mathrm{sen}(\omega t)$ in the central junction, the Eq. (11) may be written in non-dimensional form as:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \gamma\,\mathrm{sen}(\Omega\tau) - \bar{\xi}x_2 - \zeta_E\left[1 - \frac{1}{(x_1^2+\beta^2)^{1/2}}\right]x_1 - [(\bar{\alpha} + \zeta_E\alpha_h)(\rho_2 - \rho_1) - \bar{\kappa}\zeta_\kappa(\theta - \theta_0)]\frac{x_1}{(x_1^2+\beta^2)^{1/2}} \end{aligned} \tag{12}$$

where

$$\begin{aligned} &x_1 = \frac{X}{L_0}, \quad \beta = \frac{B}{L_0}, \quad \theta = \frac{T}{T_M}, \quad \tau = \omega_0 t, \quad \Omega = \frac{\omega}{\omega_0}, \quad \zeta_E = \frac{E}{E_M}, \quad \zeta_\kappa = \frac{\kappa}{\kappa_M}, \\ &\bar{\alpha} = \frac{\alpha}{E_M}, \quad \dot{x}_1 = \frac{dx_1}{d\tau}, \quad \gamma = \frac{P_0}{mL_0\omega_0^2}, \quad \omega_0^2 = \frac{2E_M A_{st}}{mL_0}, \quad \bar{\kappa} = \frac{\kappa_M T_M}{E_M} \quad \text{e} \quad \bar{\xi} = \frac{c}{m\omega_0} \end{aligned} \tag{13}$$

3. CONTROL DESIGN

Now a smooth sliding mode controller with uncertainties compensation is applied to the SMA von Mises truss, to ensure that the state vector $\mathbf{x} = [x_1 \quad x_2]$ reaches the desired state vector $\mathbf{x}_d = [x_{1d} \quad x_{2d}]$ and the stabilization error $\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x}_d = [\tilde{x} \quad \dot{\tilde{x}}] \to \mathbf{0}$ when $\tau \to \infty$, even in the presence of an external disturbance. To ensure the uncertainties compensation and a convergence of the system to an error close to zero, a fuzzy function will be determined on the basis of the stabilization error (fuzzy P).

The control of SMA actuators and structures requires a robust control technique that can deal with uncertainties and nonlinear behaviors, which are the main characteristics of these materials. In the control of SMA actuators, some authors have applied only the sliding mode control methodology (Ianagui and Tannuri, 2015; Lee *et al.*, 2013; Romano and Tannuri, 2009), while other ones have added to it other techniques to improve their performance, such as networks neural (Song *et al.*, 2003) and fuzzy logic (Nakshatharan *et al.*, 2015), mainly to compensate uncertainties in the mathematical models of SMAs. For the control of SMA structures, the sliding mode control technique is also effective, especially when it has a uncertainties compensation system (Bessa *et al.*, 2013).

In order to ensure the vibration control of the structure, a linear actuator is supposed to be installed vertically at the central junction, as illustrated in Fig. 2. The combination of linear actuators with shape memory elements enables the development of variable geometry trusses that also have the ability of self-attenuate their vibration levels.

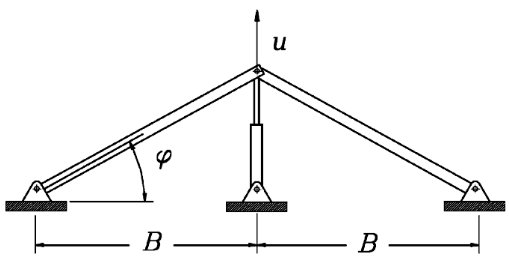


Figure 2. Schematic representation of an actuated von Mises truss.

On this basis, the related control variable u must be added to the Eq. (12), which for control purposes could be simply rewritten as:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= d + f + u \end{aligned} \tag{14}$$

where u is the control action, $d = \gamma\sin(\Omega\tau)$ is the external disturbance, assumed to be unknown, and f is the system dynamics. Regarding the development of the control law, the following assumptions must be made:

Assumption 1 *The state vector* $\mathbf{x}$ *is available.*

Assumption 2 *The system dynamics* $f(\mathbf{x})$ *is unknown but bounded by a known function of* $\mathbf{x}$*, i.e.* $|\hat{f}(\mathbf{x}) - f(\mathbf{x})| \leq \mathcal{F}(\mathbf{x})$*, where* $\hat{f}$ *is an estimate of* f*.*

Assumption 3 *The external disturbance* d *is unknown but bounded by a known function* $\mathcal{D}$*, i.e.* $|d| \leq \mathcal{D}$*.*

The design of a sliding mode controller provides a systematic approach to problems which it is necessary to maintain stability and performance even in the presence of uncertainties. According to Slotine and Li (1991), considering the following systems:

$$x^{(n)} = f(\mathbf{x}, t) + b(\mathbf{x}, t)u + d(t) \tag{15}$$

the switching variable is given by:

$$s(\mathbf{x}, t) = \left(\frac{\partial}{\partial t} + \lambda\right)^{n-1} \tilde{x} \tag{16}$$

where x is the interest output, u is the control input, $f(\mathbf{x}, t)$ and $b(\mathbf{x}, t)$ are nonlinear functions which represent the system dynamics and control action gain respectively, d is an external disturbance, λ is a positive constant and $\tilde{x}$ is the tracking error associated with the state variable x. Thus the sliding mode controller for the SMA von Mises truss can be defined with the combination of an equivalent control law $u = -\hat{f} - \hat{d} - \lambda\dot{\tilde{x}}$ and another term, $K\text{sat}(s/\phi)$, to confer robustness to the system, as follows:

$$u = -\hat{f} - \hat{d} - \lambda\dot{\tilde{x}} - K\text{sat}(s/\phi) \tag{17}$$

where $\hat{d}$ is an estimate of d, K is a positive gain, ϕ is a strictly positive constant that represents the boundary layer thickness and sat(s/ϕ) is defined as

$$\text{sat}(s/\phi) = \begin{cases} \text{sgn}(s/\phi) & \text{if} \quad |s/\phi| \geq 1 \\ s/\phi & \text{if} \quad |s/\phi| < 1 \end{cases} \tag{18}$$

Considering Assumptions 2 and 3, the robustness of the adopted smooth sliding mode controller against parametric uncertainties, modeling inaccuracies and external disturbances is assured by defining the gain K according to (Slotine and Li, 1991):

$$K \geq \eta + \mathcal{F} + \mathcal{D} + |\hat{d}| \tag{19}$$

where η is a strictly positive constant related to the time required to reach the sliding surface.

Since the controller requires a model to estimate the dynamics of the plant to be controlled, and given the mathematical model complexity of the structure, a polynomial model will be employed in the control law, Eq. (17). This approach has the advantage of not only obtaining a simpler control law, but also allows its robustness to be evidenced. Thus, the estimate of f will be given from the von Mises truss dynamics with a polynomial model to simulate the SMA thermomechanical behavior (Savi *et al.*, 2002a), which is given as follows:

$$\begin{aligned} \hat{f} = & -\xi x_2 + x_1\big\{-[(\theta-1) - 3\alpha_2 + 5\alpha_3] + [(\theta-1) - \alpha_2 + \alpha_3](x_1^2 + \beta^2)^{-1/2} + \\ & -[3\alpha_2 - 10\alpha_3](x_1^2+\beta^2)^{1/2} + [\alpha_2 - 10\alpha_3](x_1^2+\beta^2) + 5\alpha_3(x_1^2+\beta^2)^{3/2} + \\ & -\alpha_3(x_1^2+\beta^2)^2\big\} \end{aligned} \tag{20}$$

where ξ is a non-dimensional viscous damping coefficient. The dissipation due to hysteretic effect on the system model may be considered by assuming an equivalent viscous damping related to this parameter. Moreover, α_2 and α_3 are non-dimensional material constants, which are given by:

$$\alpha_2 = \frac{a_2}{a_1 T_M} \quad \text{and} \quad \alpha_3 = \frac{a_3}{a_1 T_M} \tag{21}$$

where a_2 and a_3 are related to the material constants.

In order to obtain a good approximation to the disturbance, $\hat{d}$ will be computed directly by a fuzzy algorithm. The fuzzy logic were established by Zadeh (1965), through the introduction of fuzzy sets. Due to its simplicity and computational efficiency (Jang *et al.*, 1997), the adopted fuzzy inference system is the zero-order TSK (Takagi-Sugeno-Kang), whose rules can be stated in a linguistic manner as follows:

If $\tilde{x}$ *is* $\tilde{X}_r$ *then* $\hat{d}_r = \hat{D}_r$*;* *r = 1, 2,..., N* where $\tilde{X}_r$ are fuzzy sets related to the stabilization error $\tilde{x}$, whose membership functions could be properly chosen, and $\hat{D}_r$ is the output value of each fuzzy rule r. Considering that each rule defines a numerical value as output $\hat{d}_r$, the final output $\hat{d}$ can be computed by a weighted average:

$$\hat{d}(\tilde{x}) = \frac{\sum_{r=1}^{r=N} \mu_r \cdot D_r}{\sum_{r=1}^{r=N} \mu_r} \tag{22}$$

or

$$\hat{d}(\tilde{x}) = \hat{\mathbf{D}}^{\mathrm{T}}\mathbf{\Psi}(\tilde{x}) \tag{23}$$

where $\hat{\mathbf{D}} = [\hat{D}_1, \hat{D}_2, ..., \hat{D}_N]^{\mathrm{T}}$ is the vector containing the attributed values $\hat{D}_r$ to each rule r, $\mathbf{\Psi}(\tilde{x}) = [\psi_1(\tilde{x}), \psi_2(\tilde{x}),$ ..., $\psi_n(\tilde{x})]$ is the vector with components $\psi_r(\tilde{x}) = \mu_r / \sum_{r=1}^{r=N} \mu_r$, and μ_r is the firing strength of each rule.

A detailed discussion on the boundedness of all closed-loop signals and the convergence properties of the fuzzy sliding mode control of n^{th}-order uncertain nonlinear systems is presented in Bessa *et al.* (2010).

4. NUMERICAL SIMULATIONS

Numerical simulations are now in focus exploring the controller capability to perform vibration reduction in a smart structure. The operator split technique (Ortiz *et al.*, 1983), the orthogonal projection algorithm (Savi *et al.*, 2002b) and the classical fourth order Runge−Kutta method was developed to deal with nonlinearities in the formulation. Orthogonal projections ensure that volume fractions of the macroscopic phases obey the imposed constraints. In all simulations, we have used the material properties presented in Tab. 1. These values were chosen in order to represent a typical SMA behavior at 373 K as shown in the stress-strain diagram of Fig. 3, which was obtained from Eq. (3).

Table 1. SMA constitutives parameters considered in numerical simulations.

E_A (GPa)	E_M (GPa)	α (MPa)	α_h
54	54	150	0,052
L (MPa)	L_0 (MPa)	L_0^A (MPa)	L^A (MPa)
41,5	0,15	0,63	185
κ_A (MPaK^{-1})	κ_M (MPaK^{-1})	T_M (K)	T_A (K)
0,74	0,17	291,4	307,7
	$\bar{\eta}^C$ (MPa s)	$\bar{\eta}^D$ (MPa s)	
	10	27	

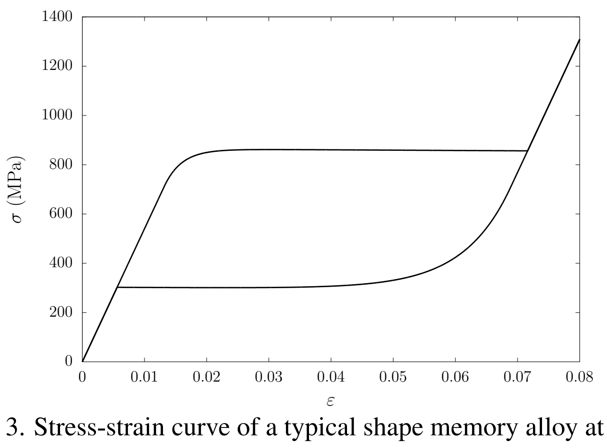
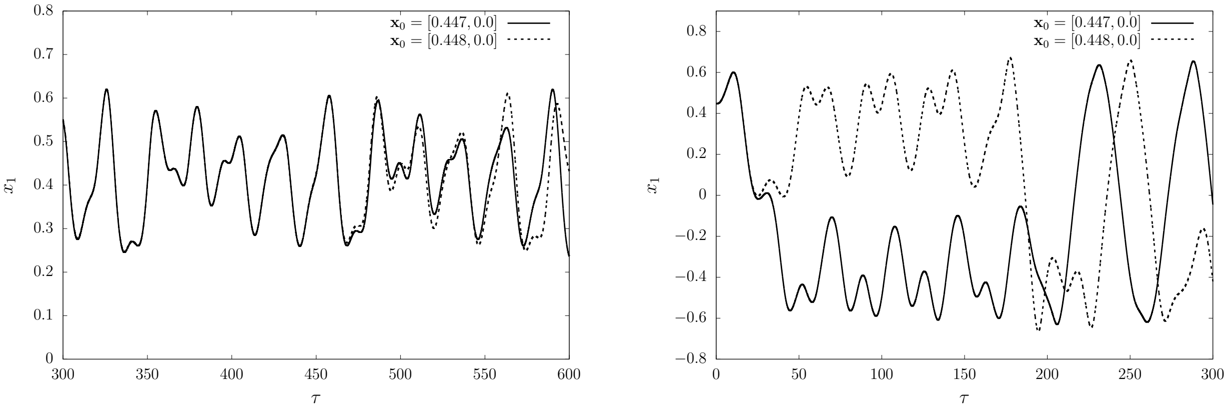


Figure 3. Stress-strain curve of a typical shape memory alloy at 373 K.

Savi and Nogueira (2010) studied the chaotic behavior of this SMA structure considering bifurcation diagrams under the slow quasi-static increase of disturbance parameters. Bifurcations diagrams were analyzed driving frequency Ω with fixed amplitudes γ and driving amplitudes γ with fixed frequencies Ω. Both diagrams shown regions with cloud of points which are related to chaotic motions and also regions with discrete numbers of points that are associated with periodic motions.

In order to present the chaotic motion of the uncontrolled structure, we simulate its behavior using specific values for Ω and γ according to Savi and Nogueira (2010) that present chaotic motion. From the data in the Table 1, the parameters



(a) Response for $\gamma = 0.008$ and $\Omega = 0.475$. (b) Response for $\gamma = 0.01$ and $\Omega = 0.3347$.

Figure 4. Uncontrolled response of the system.

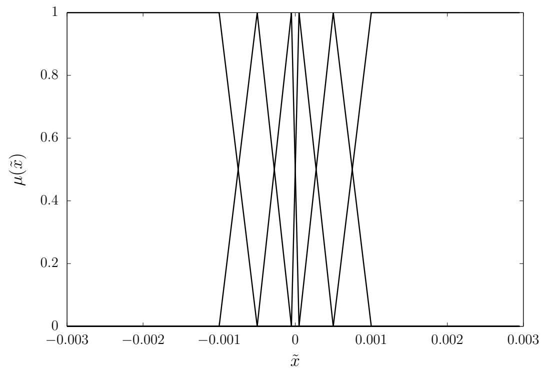


Figure 5. Adopted fuzzy membership functions.

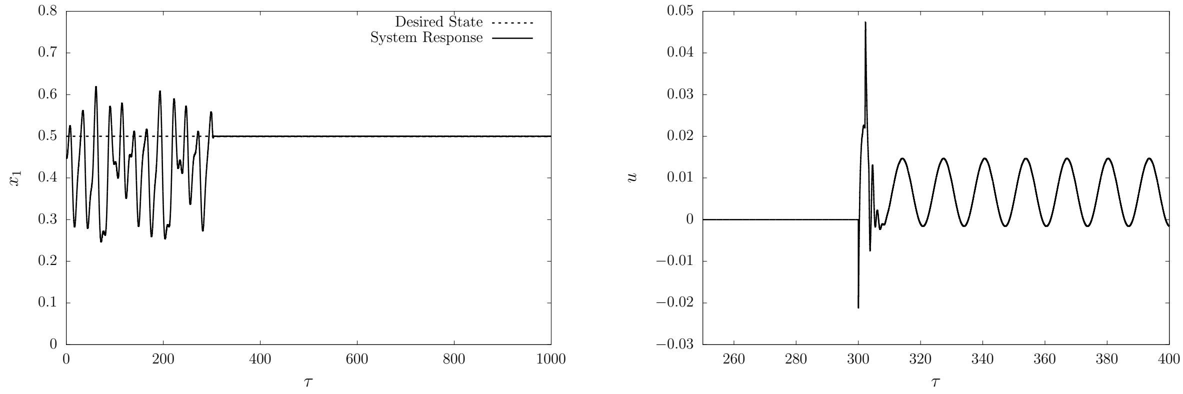
defined in Eq. (12) assume the values: $\mathbf{x}(0) = [0.447\,, 0.0]$ and $\mathbf{x}(0) = [0.448\,, 0.0]$, $\beta = 0.894$ and $\bar{\xi} = 0$. The chaotic response x_1 related to τ is shown in Fig. 4 for different values of Ω and γ. This analysis is done by considering the high temperature ($T = 373$ K) behavior that is related to the pseudoelastic effect, where the austenitic phase is stable in the stress-free state. Time steps are chosen according to $\Delta\tau = \pi/1000\Omega$.

From the Figure 4 we can see the rich dynamic behavior of the shape memory two-bar truss. The chaos phenomenon is associated with the unpredictability of the system response with small variations of its initial conditions, however small are these variations (Khalil, 2002). The system responses are similar until a certain period of time and soon after begin to diverge from each other up to the point where the curves have completely different behaviors. The SMA two-bars truss is very sensitive to its parameters and can present totally different behaviors, as we can see in the Fig. 4a, that the response starts to become unpredictable around $\tau = 500$ and does not present the snap-through behavior, while the response of the Fig. 4b starts to be unpredictable in $\tau = 25$ and presents the snap-through behavior.

The controller capabilities are now investigated. Sampling rates of $200\Omega/\pi$ for control system and $1000\Omega/\pi$ for dynamical model are assumed. In order to demonstrate that the adopted control scheme can deal with both modeling inaccuracies and external disturbances, a simplified model to simulate the SMA behavior (Savi *et al.*, 2002a) was considered in the control law. Moreover, the external disturbance is treated as an unknown periodic excitation. Under this assumption, $\gamma \sin \Omega\tau$ is not taken into account, it will be estimated by the fuzzy inference system. On this basis, according to Savi *et al.* (2002a) the values of the constants a_1, a_2 and a_3 from Eq. (21) are chosen in order to match with the hysteresis loop of the SMA stress-strain diagram. For the proposed controller, the constants assumed the following values: $a_1 = 566.62$ (MPa/K), $a_2 = 2.09 \times 10^7$ (MPa) and $a_3 = 2.69 \times 10^9$ (MPa). Besides, the following parameters were adopted in the control: $\mathcal{F} = 0.05$; $\mathcal{D} = 0.05$; $\phi = 0.1$; $\lambda = 0.6$ and $\eta = 0.03$. Concerning the fuzzy system, trapezoidal (at the borders) and triangular (in the middle) membership functions are adopted for $\tilde{X}_r$, with the central values defined respectively as $\mathbf{c}_{\tilde{X}} = \{-\Phi/\lambda;\ -0{,}08\Phi/\lambda;\ -0{,}03\Phi/\lambda;\ 0{,}03\Phi/\lambda;\ 0{,}08\Phi/\lambda;\ \Phi/\lambda\} \times 10^{-1}$, as shown in Fig. (5), with the output value of each rule: $\hat{\mathbf{D}} = \{3{,}0;\ 2{,}0;\ 1{,}0;\ -0{,}5;\ -1{,}0;\ -1{,}5\} \times 10^{-2}$. The universe of discourse was based on the residual error presented by the smooth sliding mode control technique (Bessa *et al.*, 2009).

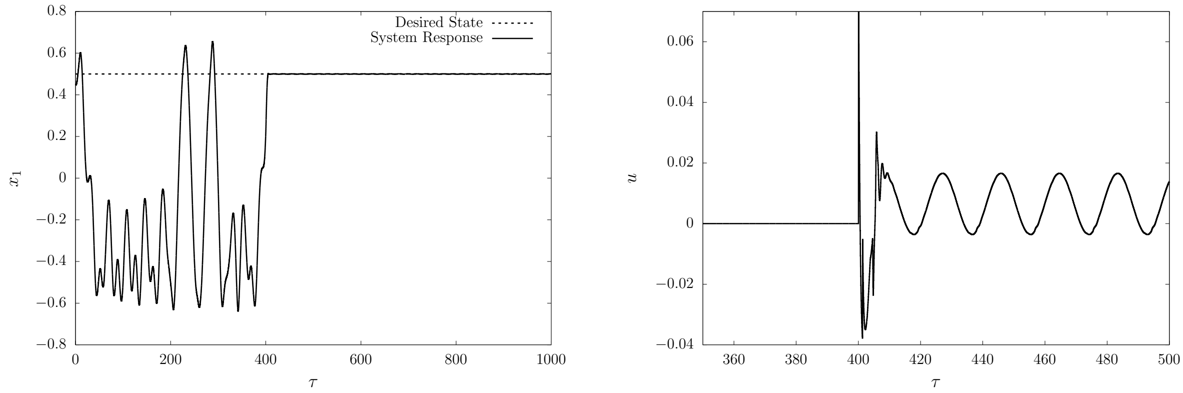
In the numerical simulations we considered the initial state $\mathbf{x}(0) = [0.447\ \ 0.0]$ with the desired state $\mathbf{x}_d = [0.50\ \ 0.0]$. The hysteresis effect of the system in Eq. (20) was considered by an equivalent viscous damping $\xi = 0.002$. The dissipative term ξ of the polynomial model was determined by the logarithm decrement method applied to the system in the free vibration case.

The stabilization of the state vector in the neighborhood of one equilibrium point of the shape memory two-bar truss is carried out. This approach shows that the adopted control scheme can significantly reduce the vibration level and also avoid the undesired chaotic and snap-through behaviors, as presented in Fig. 6a and 7a. Note that the chaotic behavior with large amplitudes of the uncontrolled response is replaced by a regular behavior with small amplitudes around the desired state when the controller is activated.



(a) System Response x_1. (b) Control action u.

Figure 6. Controller performance for $\gamma = 0.008$ and $\Omega = 0.475$.



(a) System Response x_1. (b) Control action u.

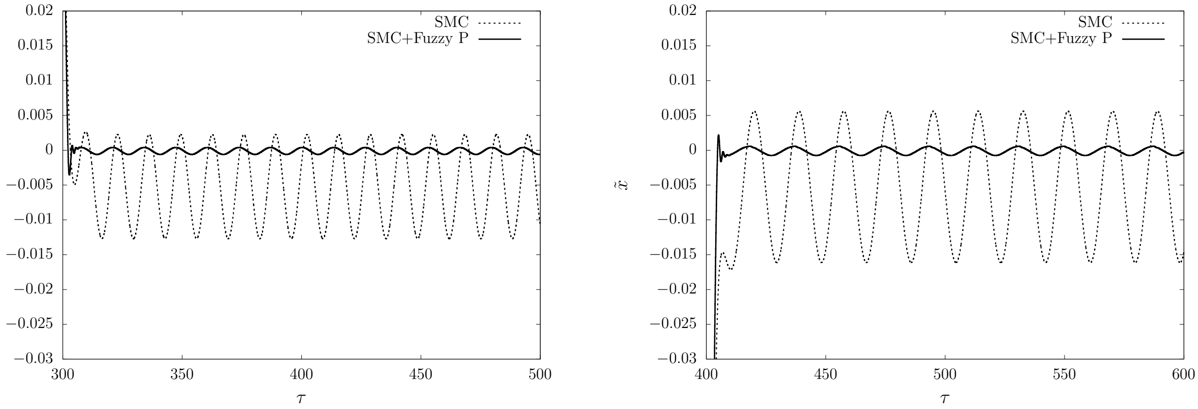
Figure 7. Controller performance for $\gamma = 0.01$ and $\Omega = 0.3347$.

The robustness of the designed controller can also be seen that even in the presence of modeling inaccuracies, which were associated with the simplified model used in the control law and the external disturbance, it is able to provide the desired stabilization with a small error associated. In the Figure 6b and 7b we can see the moment that the fuzzy compensation therm is taken into account, through the small variations that occur in the control effort when it is activated.

Through the comparative analysis shown in Fig. 8, the improved performance of the sliding mode control with fuzzy compensation (SMC + Fuzzy P) over the uncompensated control (SMC) can be clearly checked. By considering simulation purposes, the control with fuzzy compensation can be easily converted to the classical SMC by setting the estimated disturbance to zero, $\hat{d} = 0$.

5. CONCLUDING REMARKS

In this paper, a fuzzy sliding mode controller was considered for vibration attenuation in a smart structure. A constitutive model with internal constraints was assumed to describe the thermomechanical behavior of the structure elements while a simplified one was considered in the control law. The controller applied presented a great contribution, since it allows that structures with complex behaviors, which present a mathematical formulation of hard analytical approach, can be controlled from a simpler model. As observed in the numerical simulations, the incorporation of a fuzzy algorithm in the control law made a better stabilization of the system in the desired state. Undesirable behaviors such as snap-through and chaotic responses are avoided by the controlled structure. It should be highlighted once again that the controller robustness to modeling inaccuracies and external disturbances is an important issue that allows the use of simple constitutive models for control purposes.



(a) Stabilization error for $\gamma = 0.008$ and $\Omega = 0.475$. (b) Stabilization error for $\gamma = 0.01$ and $\Omega = 0.3347$.

Figure 8. Comparative analysis of the stabilization error with and without fuzzy compensation.

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