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DIMENSIONLESS FACTORS TO DESIGN HYDROPNEUMATIC SUSPENSION SYSTEMS

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Abstract. *The degree of stability of classical hydropneumatic suspension systems is directly affected by variations of sprung mass. Robustness of the degree of stability can be achieved by CRONE (french acronym for Fractional Order Robust Control) suspension, in which the damping coefficient becomes independent of variations of sprung mass. This article proposes dimensionless factors to develop a design methodology to recreate the physical parameters of CRONE and classical suspensions, considering significant changes of sprung mass and cutoff frequency. This methodology makes the procedure straightforward either for new users or for those unfamiliar to fractional order control theory or hydropneumatic suspensions. Furthermore, the theoretical expressions of the design factors are related with the design parameters. Numerical simulations confirm the equivalence of the suspensions designed through the regular process or based on the proposed dimensionless factors*

Keywords: *hydropneumatic suspension, CRONE suspension, dimensionless factor*

1. INTRODUCTION

There is a wide variety of application areas for hydropneumatic suspension, for example automotive, seismic protection of structures and heave compensation in offshore drilling. Each application has its own requirements and therefore different hydropneumatic suspension systems have been developed to achieve these requirements (**?**).

The requirement of robustness of damping coefficient for load variation is very important in automotive field. The classical hydropneumtic suspension does not achieve this requirement, because variation of the sprung mass M modifies the stiffness k of hydropneumatic suspensions and the damping coefficient (the stiffness according to mass variation is determined by Eq. (**??**) with the nominal sprung mass M_{nom} and nominal stiffness k_{nom}). On the other hand, the CRONE suspension presents robustness of stability degree versus load variation (**?**).

$$k = \left(\frac{M}{M_{nom}}\right)^2 k_{nom} \tag{1}$$

Figure **??** presents the suspension as a naturally closed-loop system, in which the plant corresponds to a double integrator and the controller $C(s)$ is related to the suspension impedance. Therefore, the suspension design may be addressed as a control problem in which the transmittance transfer function is defined by Eq. (**??**), where M is the sprung mass, Z_1 and Z_2 are respectively the sprung mass and input movement displacements with respect to their equilibrium points. The expression of the controller depends on the hydropneumatic structure.

$$\frac{Z_1(s)}{Z_0(s)} = \frac{C(s)}{Ms^2 + C(s)} \tag{2}$$

The design of CRONE suspension is based on fractional order automatic control theory, which includes mainly three steps: design a fractional controller; approximation by a rational controller and computation of the system physical

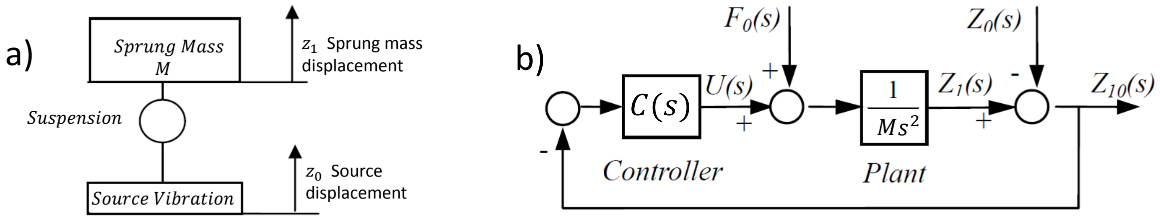

Figure 1. Hydropneumatic suspensions. (a) One-degree-of-fredom model. (b) One-degree-of-fredom model block diagram.

parameters. The last step may be achieved with two hydropneumatic structures: a parallel arrangement of dissipative and capacitive components, namely R and C, in series (RC cells), or a gamma arrangement of R and C elements (**?**). Both arrangements are shown in Fig. **??**. The capacitances are obtained with hydraulic accumulators of oil and gas, which is separated by an impermeable diaphragm. The dissipative components are obtained by hydraulic dampers.

The design of CRONE suspension requires the knowledge of CRONE control, fractional to rational approximation and the relation between the physical parameters and the rational controller. To simplify the process, a methodology to re-design a previous CRONE and classical suspensions with different cutoff frequency and sprung mass is proposed. This methodology is based on dimensionless factors, which have a constant value for each damping coefficient.

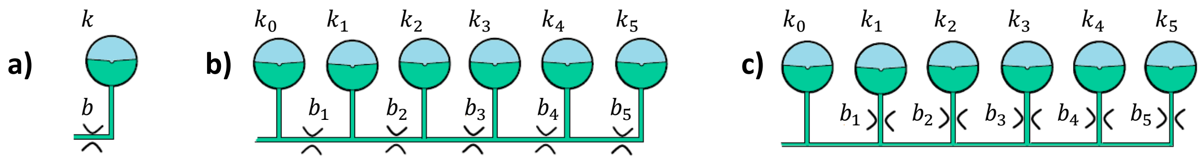

Figure 2. Hydropneumatic suspensions. (a) Classical. (b) CRONE with gamma arrangement. (c) CRONE with parallel arrangement of RC cells

This article is divided into 4 parts. In Section **??** , the factor expressions are shown, the methodology to design the suspension is described and the theoretical factors are found in function of the design parameters for classical suspension. Section **??** shows the theoretical factor for CRONE suspension with arrangement of RC cells. Section **??** presents the design of three suspensions using the methodology proposed here. The three of suspension systems are: classical and CRONE with parallel arrangement of RC series cells and gamma arrangement (see Fig. **??**). The frequency response of suspension designed with factor are presented, and the factor and physical parameters are summarized in tables. Finally, in Section 6. conclusions are discussed.

2. FACTOR

The factors present here are important because they are used to propose a methodology for designing hydropneumatic suspensions. These factors are stiffness factor f_k and the viscous friction coefficient factor f_b, which depend on viscous friction coefficient b (independent of sprung mass value), cutoff frequency ω_{co} and stiffness k_o in the nominal sprung mass M_o. The factors are constant for each value of damping coefficient, as it is shown by their theoretical expressions shows in Subsesction **??**.

$$f_k = \frac{(\omega_{co})^2}{k_o} M_o \tag{3}$$

and

$$f_b = \frac{\omega_{co}}{b} M_o. \tag{4}$$

2.1 Methodology for designing hydropneumatic suspensions using dimensionless factors

The factors allow to develop a simple methodology for designing hydropneumatic suspensions with the desired frequency response. This methodology is applied to three kinds of hydropneumatic suspensions: classical suspension, CRONE suspension with arrangement of parallel RC in series cells and gamma arrangement.

The methodology is formed by three parts, which are summarized in Fig. **??**. Firstly, identify the parameters of a previously designed or existing suspension (nominal sprung mass M_o, nominal stiffness k_o, viscous friction coefficient b and cutoff frequency for nominal parameters ω_{co}). Secondly, calculate the factors with Eq. (**??**) and (**??**). Finally, use these factors, the desired cutoff frequency $\bar{\omega}_{co}$ and the desired sprung mass $\bar{M}_o$ to obtain the physical parameter of the desired suspension ($\bar{k}_o$ and $\bar{b}$) with the following equations:

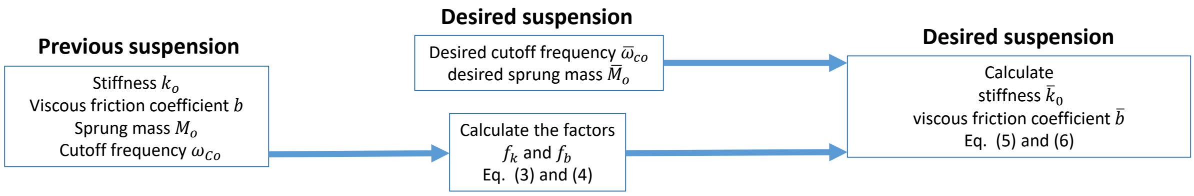

Figure 3. Steps to project hydropneumatic suspension using dimensionless factors

$$\bar{k}_o = \frac{(\bar{\omega}_{co})^2}{f_k}\bar{M}_o \tag{5}$$

and

$$\bar{b} = \frac{\bar{\omega}_{co}}{f_b}\bar{M}_o. \tag{6}$$

In the case of CRONE suspension that has more than one stiffness and one viscous friction coefficient, the methodology have to be re-applied for each parameter. This methodology is a simple way to design a suspension with a desired frequency response, but it has a limitation, the factors have different values for each desired damping coefficient.

2.2 Theoretical factor expression for classical hydropneumatic suspension

To begin the proof, the expression of $C(s)$ for classical suspension is presented (**?**).

$$C(s) = bs + k. \tag{7}$$

The above expression of $C(s)$ is introduced in Eq. (**??**) to obtain the transmittance of classical hydropneumatic suspension system

$$\frac{Z_1(s)}{Z_0(s)} = \frac{(\frac{b}{M}s + \frac{k}{M})}{(s^2 + \frac{b}{M}s + \frac{k}{M})} = \frac{2\zeta\omega_n s + \omega_n^2}{(s^2 + 2\zeta\omega_n s + \omega_n^2)}. \tag{8}$$

The natural frequency ω_n and the damping coefficient ζ are the performance parameters. The stiffness k and viscous friction coefficient b are related with the performance parameter and the sprung mass by the following expressions:

$$b = 2\zeta M\omega_n, \quad \text{and} \quad k = \omega_n^2 M. \tag{9}$$

A dimensionless factor $l(\zeta)$ was defined in (**?**), which relates the cutoff frequency with the natural frequency and this is a function of the damping coefficient value. This factor $l(\zeta)$ has a constant value for each damping coefficient. Thus,

$$\omega_n = l(\zeta)\omega_c. \tag{10}$$

Substituting the above expression of ω_n in Eq. (**??**) and arranging to have the same right side of Eq. (**??**) and (**??**) yields:

$$\frac{1}{2\zeta l(\zeta)} = \frac{\omega_c}{b}M \quad \text{and} \quad \frac{1}{l(\zeta)^2} = \frac{\omega_c^2}{k}M. \tag{11}$$

Thus, the theoretical factors are:

$$f_{b-theo} = \frac{1}{2\zeta l(\zeta)} \tag{12}$$

and

$$f_{k-theo} = \frac{1}{l(\zeta)^2}. \tag{13}$$

The theoretical viscous friction coefficient factor f_{b-theo} has a direct relation with the damping coefficient ζ. The theoretical stiffness factor f_{k-theo} is determined by the relation between cutoff and natural frequencies $l(\zeta)$. These expressions shows that the factors are independent of cutoff frequency and sprung mass, but they are dependent of damping coefficient. A similar proof for CRONE suspension with parallel arrangement of RC cells is developed in the next section.

3. THEORETICAL FACTORS FOR PARALLEL ARRANGEMENT OF RC CELLS

The design of CRONE suspension with parallel arrangement of RC series cells is briefly explained, because it is required some knowledge and expressions of this process to show how the factors are obtained in function of the designed parameters. After that, the factor for the parameter k_0 is presented and finally the other stiffness and viscous friction coefficient factors are obtained in similar way.

3.1 Basic about CRONE suspension

The CRONE controller is C_f, where C_0 is the static gain, n presents the fractional order of the controller and the frequencies ω_h and ω_l stablish the frequency interval in which the system has the fractional behaviour.

$$C_f(s) = C_o \left(\frac{1 + \frac{s}{\omega_b}}{1 + \frac{s}{\omega_h}} \right)^n \tag{14}$$

The frequencies ω_h and ω_b are defined in (**?**), which have relation with the the crossover frequency ω_u, maximum and minimum sprung masses (M_{max} and M_{min}). The parameter A is generally chosen with an order of magnitude of 10.

$$\omega_h = A\omega_u, \qquad \omega_b = \omega_u \left(\frac{M_{min}}{M_{max}} \right)^2 \frac{1}{A} \tag{15}$$

It is necessary to synthesize a rational approximation C_R of fractional controller C_f, this approximation is given by N poles ω_i and zeros ω_i'.

$$C_R(s) = C_o \prod_{i=1}^{N} \frac{1 + \frac{s}{\omega_i'}}{1 + \frac{s}{\omega_i}} \tag{16}$$

Recursive coefficients α and η are used to obtain the relations between the zeros ω_i' and poles ω_i with respect to the parameter of fractional controller as introduced in (**?**)

$$\alpha\eta = \left(\frac{\omega_h}{\omega_b} \right)^{\frac{1}{N}}, \qquad \alpha = (\alpha\eta)^n, \qquad \eta = (\alpha\eta)^{1-n} \tag{17}$$

$$\frac{\omega_i}{\omega_i'} = \alpha, \qquad \frac{\omega_{i+1}'}{\omega_i} = \eta \tag{18}$$

$$\omega_l' = \omega_b \sqrt{\eta} (\eta\alpha)^{l-1}, \qquad \omega_l = \omega_b \sqrt{\eta} \alpha (\eta\alpha)^{l-1} \tag{19}$$

The expression of ω_h/ω_b is obtained with Eq. (**??**) and is replaced in Eq. (**??**), this allows to re-write the recursive coefficients α and η in function of the value A, the relation between maximum and minimum masses M_{max}/M_{min}. It is important to highlight that the recursive coefficients are independent of crossover frequency and the sprung mass. They have the same value for the same mass relation M_{max}/M_{min} and any crossover frequency value.

$$\alpha = \left(A\frac{M_{max}}{M_{min}}\right)^{\frac{2n}{N}}; \qquad \eta = \left(A\frac{M_{max}}{M_{min}}\right)^{\frac{2(1-n)}{N}}; \tag{20}$$

The N poles and N zeros of rational controller are linked to $2N+1$ physical parameters (N viscous friction coefficient and $N+1$ stiffness). These relations are established in (**?**).

$$A_i = \frac{1}{C_o}\left(\prod_{N}^{l=1}\frac{\omega_l'}{\omega_l}\right)\left(\frac{\prod_{l=l}^{N}(\omega_l-\omega_i')}{\prod_{l=1,l\neq i}^{N}(\omega_l'-\omega_i)'}\right) \tag{21}$$

$$k_0 = C_o\prod_{l=1}^{N}\frac{\omega_l}{\omega_l'}, \quad b_i = \frac{1}{A_i}, \quad k_i = \omega' b_i \tag{22}$$

3.2 Theoretical factor for stiffness k_0

The proof idea of the theoretical dimensionless factor for CRONE suspension is to transform the above equation of k_0 in the following equation. Where the factor f_{k_0-theo} is independent of cutoff frequency and sprung mass.

$$k_0 = M_o\omega_c^2\frac{1}{f_{k_0-theo}} \tag{23}$$

Equation (**??**) is composed by two parts, the parameter C_o and the $\prod_{l=1}^{N}\frac{\omega_l}{\omega_l'}$. The parameter C_o must be ensure that the gain of open loop in crossover frequency ω_u is equal to 1. The open loop $B(s)$ is:

$$B(s) = C_o\left(\frac{1+\frac{s}{\omega_b}}{1+\frac{s}{\omega_h}}\right)^n\frac{1}{Ms^2} \tag{24}$$

The requirement $\|B(s=j\omega_u)\| = 1$ is obtained with

$$C_o = K_x M\omega_u^2; \qquad K_x = \left\|\left(\frac{1+A\left(\frac{M_{max}}{M_{min}}\right)^2 j}{1+\frac{j}{A}}\right)^{-n}\right\| \tag{25}$$

Where k_x is a function of relation M_{max}/M_{min} and A. The crossover frequency could be expressed in function of cutoff frequency, using a factor l_c, this is similar with the factor $l(\zeta)$ presented in (**?**). Figure **??** shows that there is a similar factor l_c between the cutoff and crossover frequency for CRONE suspensions.

$$\omega_u = \omega_c l_c(\zeta) \tag{26}$$

Now, the product is expressed in function of the recursive parameter α with Eq. (**??**).

$$\prod_{l=1}^{N}\frac{\omega_l}{\omega_l'} = \alpha^N \tag{27}$$

The Equations (**??**) (**??**) and (**??**) are used to obtain the following expression of k_0.

$$k_0 = M\omega_c^2 l_c(\zeta)^2 K_x\alpha^N \tag{28}$$

Comparing with Eq. (**??**) is observed that the theoretical factor f_{k_0-theo} is:

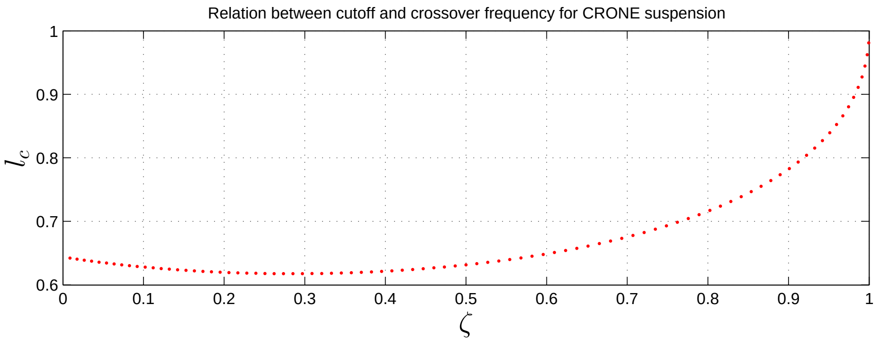

Figure 4. Factor l_c in function of damping coefficient ζ

$$f_{k_0-theo} = \frac{1}{l_c(\zeta)^2 K_x \alpha^N} \tag{29}$$

This factor is a function of $l_c(\zeta)$, K_x and α. These parameters are independent of cutoff frequency and sprung mass. This factor is valid when the design requires the same damping coefficient, less than or equal interval mass and the same number of cells. If the desired interval mass is less than the the interval of suspension used to calculate the factors, the suspension designed with this factors will have the desired performance.

3.3 Theoretical factor for stiffness and viscous friction coefficient

In the same way, the factor expressions are obtained for the other parameters. The product of Eq. (**??**) is modified using Eq. (**??**) and (**??**).

$$A_i = \frac{1}{M\omega_c^2 l_c(\zeta)^2 K_x \alpha^N}\left(\sqrt{\eta}\omega_b \frac{\prod_{l=l}^{N} \quad (\alpha(\eta\alpha)^{l-1} - (\eta\alpha)^{i-1})}{\prod_{l=1,l\neq i}^{N} ((\eta\alpha)^{l-1} - (\eta\alpha)^{i-1})}\right) \tag{30}$$

The frequency ω_b (Eq. (**??**)) is re-written in function of cutoff frequency with Eq. (**??**).

$$\omega_b = \omega_c l_c(\zeta)\left(\frac{M_{min}}{M_{max}}\right)^2 \frac{1}{A}$$

The expression of ω_b are replaced in Eq. (**??**) and with Eq. (**??**) is obtained the parameter b.

$$b_i = M\omega_c l_c(\zeta) K_x \alpha^N A \left(\frac{M_{max}}{M_{min}}\right)^2 \left(\frac{1}{\sqrt{\eta}} \frac{\prod_{l=1,l\neq i}^{N} ((\eta\alpha)^{l-1} - (\eta\alpha)^{i-1})}{\prod_{l=l}^{N} \quad (\alpha(\eta\alpha)^{l-1} - (\eta\alpha)^{i-1})}\right) \tag{31}$$

Comparing with Eq. (**??**) is determined the factor f_{b_0}.

$$f_{b_i-theo} = \frac{\sqrt{\eta}}{l_c(\zeta) K_x \alpha^N A}\left(\frac{M_{min}}{M_{max}}\right)^2 \left(\frac{\prod_{l=l}^{N} \quad (\alpha(\eta\alpha)^{l-1} - (\eta\alpha)^{i-1})}{\prod_{l=1,l\neq i}^{N} ((\eta\alpha)^{l-1} - (\eta\alpha)^{i-1})}\right) \tag{32}$$

The expression of theoretical factor f_{k_i-theo} is found in the same way.

$$f_{k_i-theo} = \frac{1}{(\eta\alpha)^{i-1} l_c(\zeta)^2 K_x \alpha^N}\left(\frac{\prod_{l=l}^{N} \quad (\alpha(\eta\alpha)^{l-1} - (\eta\alpha)^{i-1})}{\prod_{l=1,l\neq i}^{N} ((\eta\alpha)^{l-1} - (\eta\alpha)^{i-1})}\right) \tag{33}$$

4. APPLICATIONS

The methodology for designing hydropneumatic suspension using the factors is applied to design three different suspension with different requirements (cutoff frequency and sprung mass). Their frequency responses are plotted to show that the system has the desired response. The physical parameters and the factors are summarized in tables.

4.1 Case of a classical hydropneumatic suspension

A classical suspension system is used as a passive heave compensator in (?), their physical and performance parameters are shown in left side of Tab. ??. The factors f_k and f_b are calculated with Eq. (??) and (??), see center of Tab. ??.

Table 1. Parameter of classical hydropneumatic suspension

Design in (?)				Factors		Design with factors			
M_o $(tonnes)$	ω_{co} Hz	k_o (kN/m)	b (kNs/m)	f_k	f_b	$\bar{M}_o$ $(tonnes)$	$\bar{\omega}_c$ Hz	$\bar{k}_o$ (kN/m)	$\bar{b}$ (kNs/m)
350	0.056	17.2	25.7	2.50	4.77	0.4	13.5	1154	7.119

The **desired classical suspension** must have the same damping coefficient of the above suspension, but with different cutoff frequency $\bar{\omega}_{co} = 13.5Hz$ and different sprung mass $\bar{M}_o = 400kg$. The cutoff frequency gain was chosen as $-3dB$. This value may be different or even the resonance frequency, but the factors must be adjusted for the design of the new suspension.

The stiffness $\bar{k}_o$ and viscous friction coefficient $\bar{b}$ are calculated with Eq. (??) and (??) to obtain the desired performance. The parameter of the desired suspension are summarized in the right side part of Tab. ??. The suspension frequency responses are obtained with Eq. (??) and are shown in Fig. ??. The suspension designed with the factor has the desired cutoff frequency and damping coefficient for the required sprung mass.

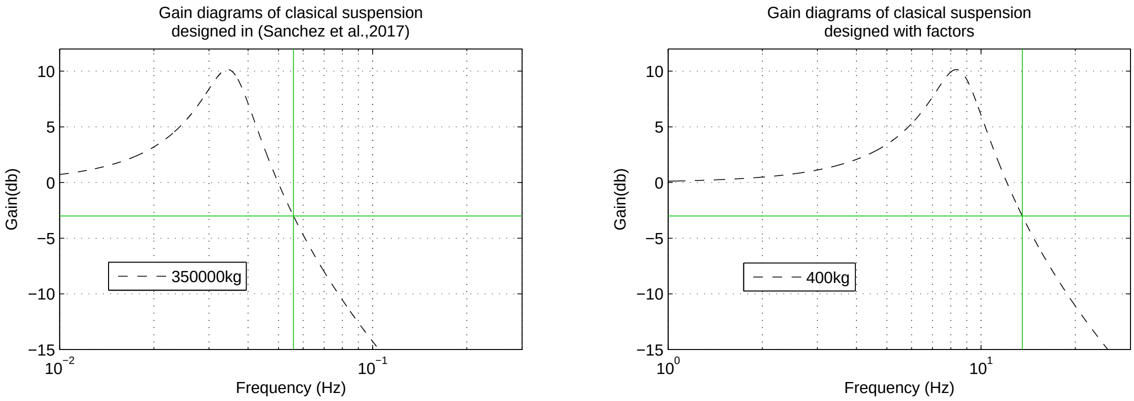

Figure 5. Frequency response of classical suspension. (a) Suspension designed in (?). (b) Suspension designed with factors.

The parameters ζ and l, used to calculate the theoretical factors with Eq. (??) and (??), are obtained from (?) and presented in Tab. ??. The relative errors with respect to the factors computed with Eq.(??) and (??) (see Tab. ??) are smaller than 0.5%. This means that the theoretical factor expressions really explain the fact that the factors are constant for each damping coefficient. It is important to highlight that the parameters ζ and l are not used in the design of this suspension, these parameters were exclusively used to calculate the theoretical factors.

Table 2. Theoretical Factors for classical suspension

f_{k-theo}	f_{b-theo}	ζ	$l(\zeta)$
2.51	4.77	0.166	0.631

4.2 Case of CRONE suspension systems

Two structures of hydropneumatic suspension to achieved the fractional behaviour are considered in (?), a parallel arrangement of RC cells, and a gamma arrangement. New structures are designed with the factors methodology proposed in this paper. The frequency responses with the maximum and minimum masses are added to show the robustness of the CRONE suspension against load variation.

4.2.1 Parallel Arrangement of RC cells

A CRONE suspension with a parallel arrangement of RC cells has been designed in (**?**). Its requirements are: the robustness of damping coefficient for mass variation from $75kg$ to $150kg$, such that the maximum gain of frequency response is $3.34dB$; and an open-loop crossover frequency of $0.96Hz$ which yields to a cutoff frequency of $1.5Hz$. The stiffness and viscous friction coefficients for minimum mass obtained from (**?**) are presented in Tab. **??**.

The factors, also shown in Tab. **??**, are calculated with Eq. (**??**) and (**??**) for each stiffness and viscous friction coefficient.

Table 3. Parameters of parallel arrangement of RC cells

	Design in (**?**)		Factors		Design with factors	
i	$k_{io}\ (N/m)$	$b_i\ (Ns/m)$	f_{k_i}	f_{b_i}	$\bar{k}_{io}\ (mN/m)$	$\bar{b}_i\ (Ns/m)$
0	10468		0.6430		7.8	
1	578	4111	11.64	0.1728	0.4	0.289
2	1722	3144	3.9086	0.2260	1.3	0.223
3	3681	1724	1.8285	0.4121	2.7	0.121
4	7649	919	0.8799	0.7731	5.7	0.065
5	17749	547	0.3792	1.2989	13.2	0.039

The **desired CRONE suspension with the parallel arrangement of RC cells** has the following requirements, mass variation between $0.5kg$ and $1kg$, cutoff frequency of $0.016Hz$ and maximum gain of $3.34dB$. The required suspension has the same proportional interval of sprung mass and the same maximum gain of the suspension designed in (**?**). Therefore, the factors may be used to design the new suspension.

The desired minimum mass and the desired cutoff frequency are used to calculate the physical parameter ($\bar{k}_o$ and $\bar{b}$) of the desired CRONE suspension, which are presented in Tab. **??** and are calculated with Eq. (**??**) and (**??**).

The frequency responses of both CRONE suspensions with parallel arrangement of RC cells are shown in Fig. **??**. These diagrams (see **?** for more information) are plotted using Eq. (**??**) and

$$C(s) = \frac{1}{\frac{1}{k_0} + \sum_{i=1}^{N} \frac{1}{b_i s + k_i}}. \quad (34)$$

The frequency response for the maximum mass is obtained in the same way as for minimum mass, but the stiffness is modified with Eq. (**??**) (**?**). On the other hand, the viscous friction coefficient values of b_i are not affected by mass changes and are maintained constant.

Figure **??**.a shows the frequency response of the CRONE suspension designed in (**?**), this suspension has a robust damping coefficient to mass changes. The suspension designed with the factors has the desired frequency response for the nominal mass, and it also presents robustness to mass changes, because the frequency response for maximum and minimum masses are almost equal (Fig. **??**.b).

Theoretical factors have also been computed for the parallel arrangement of RC cells (see subsection 4.3), and they are equal to the values based on physical parameters presented in Tab. **??**.

4.2.2 Gamma arrangement

The factors also work with CRONE suspension based on gamma arrangement. The design process is similar to the design of parallel arrangement of RC cells, the only difference lies in the controller expression $C(s)$, which is described by Eq. (**??**). The gamma arrangement designed in (**?**) has the same performance of parallel arrangement of RC cells.

$$C(s) = \frac{1}{\frac{1}{k_0} + \frac{1}{b_1 s + \frac{1}{\frac{1}{k_1} + \frac{1}{b_2 s + \frac{1}{\frac{1}{k_2} + \dots + \frac{\dots}{\dots + \frac{1}{b_5 s + k_5}}}}}}} \quad (35)$$

The **desired CRONE suspension with gamma arrangement** has a mass variation between $400kg$ and $800kg$, a cutoff frequency of $13.53Hz$ and maximum gain of $3.34dB$. Table **??** has the most relevant parameters of the arrangement gamma use in (**?**), the dimensionless factors, and the parameter of arrangement gamma designs with the factors. Figure **??** shows that the suspension has the desired performance with the robustness for load variation.

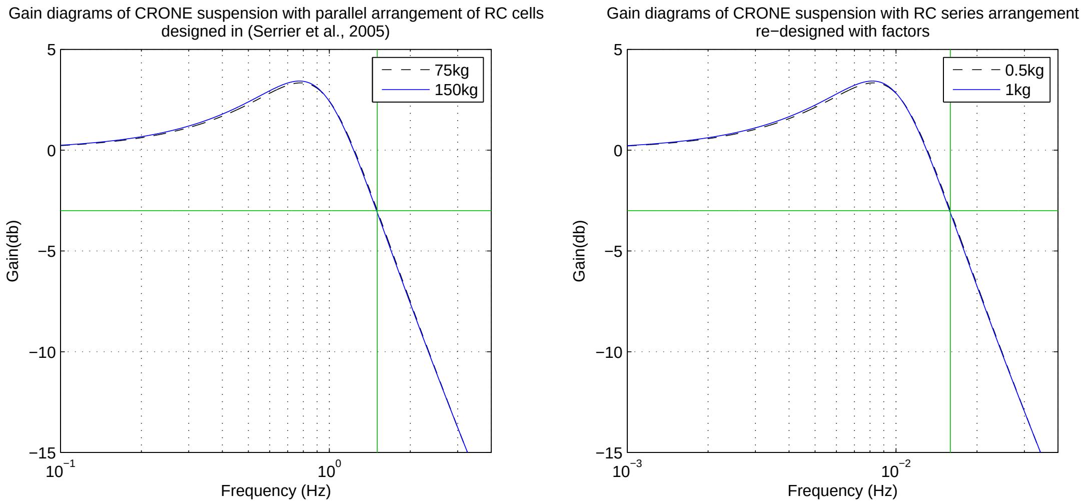

Figure 6. CRONE suspension with parallel arrangement of RC cells. (a) Design in (**?**). (b) Design with factors

Table 4. Parrameters of gamma arrangement

i	Design in (**?**)		Factors		Design with factors	
	k_{io} (N/m)	b_i (Ns/m)	f_{ki}	f_{bi}	$\bar{k}_{io}$ (kN/m)	$\bar{b}_i$ (kNs/m)
0	10625		0.632		4570	
1	4231	244.5	1.588	2.903	1820	11.71
2	2595	371	2.589	1.913	1116	17.77
3	1521	703	4.417	1.010	654	33.67
4	1109	1648	6.058	0.431	477	78.94
5	1694	6737	3.966	0.105	729	322.6

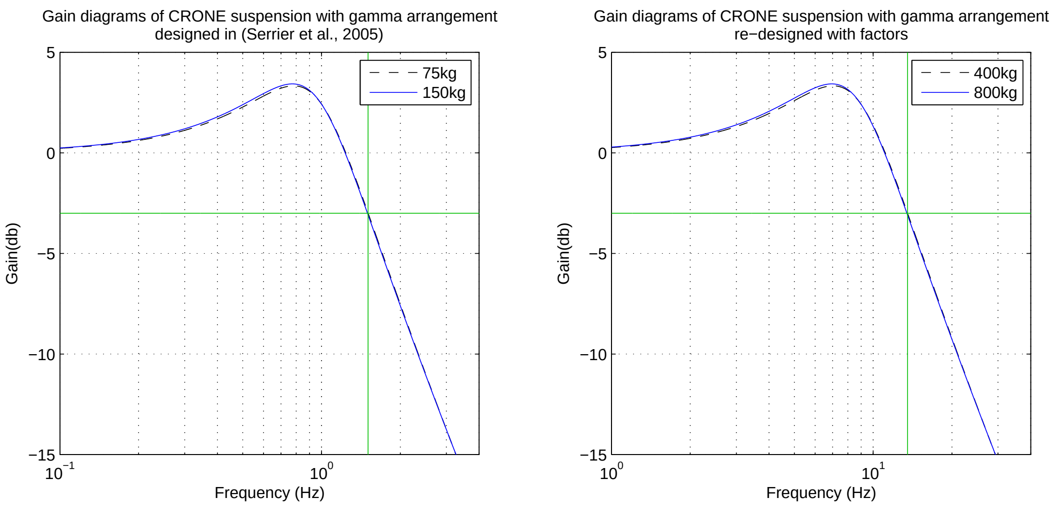

Figure 7. CRONE suspension with gamma arrangement. (a) Designed in (**?**). (b) Designed with factors

4.3 Calculating theoretical factors for parallel arrangement of RC cells

The theoretical factors are calculated with the parameters values taken from (**?**) ($n = 0.5$, $N = 5$, $\alpha = \eta = 1.975$). The parameter $l_c(\zeta)$ is proposed in this article and has a value of 0.6317, that it is found with Fig. **??**. The parameter are calculated with Eq. (**??**), (**??**) and (**??**) and are shown in the following Table.

The theoretical factors of this arrangement have the same values of the factors presented in Tab. **??**. In this way, it is shows that the factors are constant for the same required coefficient damping and the same relation between maximum and minimum masses.

Table 5. Theoretical factors for parallel arrangement of RC cells

i	0	1	2	3	4	5
f_{k-theo}	11.6548	3.9110	1.8290	0.8803	0.3794	0.6432
f_{b-theo}		0.1728	0.2260	0.4121	0.7732	1.2989

The factors proof of gamma arrangement is not developed in this article, its main difficulty lay in finding the expressions to relate directly the physical factors with the zeros and poles of rational controller. The procedure to find physical parameter of gamma arrangement involves an step to arrange the rational controller in similar way of Eq. (**??**), this is obtained with help of Maple (**?**).

5. ACKNOWLEDGEMENTS

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6. CONCLUSIONS

Dimensionless factor for the design of hydropneumatic suspensions are presented in this paper. A simple methodology based on an existing suspension allows the computation of dimensionless factors for the suspension stiffness and viscous friction coefficients. It has been shown how these factors are used in order to design new suspensions with the same damping coefficient. The proposed methodology allows the re-design of hydropneumatic suspension based on very simple operations, such that it does not require knowledge of automatic control techniques, usually employed to mitigate vibrations.

Three applications have been described including, a classical hydropneumatic suspension and two CRONE suspensions (parallel arrangement of RC cells in series and gamma arrangement). It is important to remark that the robustness of the CRONE suspension is maintained through the design of new suspensions using the proposed methodology.

Theoretical factors have also been deduced which confirms the use of the proposed methodology.

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