



24th COBEM - 2017



24th ABCM International Congress of Mechanical Engineering
December 3-8, 2017, Curitiba, PR, Brazil

COBEM-2017- 1151

P AND T WAVES HEART MODELING WITH VAN DER POL OSCILLATOR

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Abstract. *The complex functioning of the heart continues to be of great interest in current research that deepens its knowledge. In this work, the alteration of the parameters of a system with coupling of two filtered Van der Pol oscillators was performed, with half-wave rectification and a low-pass filter, similar to that of the Wien Bridge oscillator. The behavior and system responses stability are analyzed through bifurcation diagrams, through the variation of parameters within the range of values of interest. The objective here is being to represent the electrocardiogram variations of lowest magnitude of the P and T waves, through models generated by the oscillator.*

Keywords: *electrocardiogram, Van der Pol oscillator, P and T waves*

1. INTRODUCTION

The extensive reference work on the physiology of Guyton and Hall (2006) reports the basis of heart functioning. The electrocardiogram of a resting individual has an average heart cycles from 0.83 to 1.67 Hz. The spectral characteristics of the normal P and T wave are below 10 to 15Hz. The QRS complex has the highest frequencies, concentrated between 10 and 50Hz.

Ways different modeling electrocardiograms were created, such as use the signal on a three-dimensional phase space trajectory created by McSharry, *et al.* (2003). This model reproduces the QT interval, and amplitude R, with different frequencies. The authors Gidea, *et al.* (2011) built a model with the coupling of sinusoidal and exponential signals, with the variation indexes. Stefanovska (2001) worked in order to generate a physiological synchronization model, emphasizing oscillators with linear coupling. The Van der Pol relaxation oscillators from Van der Pol and Van der Mark (1928), have been analyzed in the modeling of biological systems. The intensity and nature of the synchronization of the coupling of two oscillators were analyzed by Zheng-Ming and Mao-Yuan (2007), was demonstrated that the interference from a third oscillator, as forcing the system, Also in this line Gois and Savi (2009) proposes a mathematical model to describe heart rhythms considering three modified Van der Pol oscillators connected with time delay couplings. On another approach Kaplan, *et al.* (2008) created an oscillator model that coupled two filtered oscillators from Van der Pol, with half-wave rectification and a low-pass filter similar to the Wien Bridge oscillator for simulating the ECG signal. It work Kaplan, *et al.* (2008) was also reference for Das and Maharatna (2013) with fractional oscillators, which alter the QRS complex.

2. DISCUSSIONS

2.1 Heart signal

The electrical signals that stimulate contraction of the cardiac muscle have great variability due to internal and external stimuli for homeostatic control of the body and due to the variation of stimuli of the nervous system connected to it. The real electrocardiogram, Fig. 1, presents the electrical stimuli of the heart, the model helps to quantify each stage, and to identify patterns.

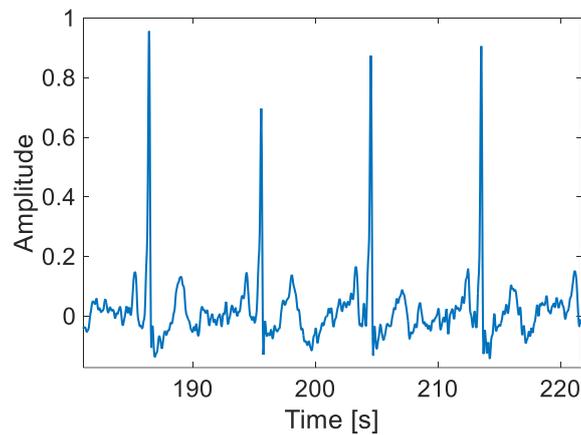


Figure 1 - Electrocardiogram Signal.

2.2 Van der Pol Oscillator

The Van der Pol relaxation oscillator has the mathematical model $\ddot{v} - \varepsilon(1 - v^2)\dot{v} + \omega^2 v = 0$, if ε is positive, the system in small elongation has negative amplitude of the resistance. The behavior is not stable, if $|\varepsilon| \gg |\omega|$ and $|v| \ll 1$ (Van der Pol and Van der Mark, 1928). As an illustration of the behavior of the Vander Pol oscillator, below we show the answer for configuration $\varepsilon = 5.0$, $\omega = 0.5$ at Fig. 2.

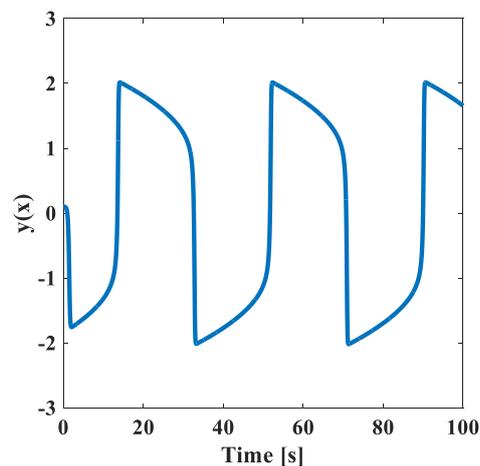


Figure 2 - Oscillator response

2.3 Kaplan Oscillator

This work was based on the oscillator system created by Kaplan, *et al.* (2008). The system was analyzed through bifurcation diagrams looking for settings that get more electrocardiogram detail.

Model with two van der Pol oscillators coupled to, equation for control with time delay of the oscillations and low pass filter similar to the bridge of Wien.

The relaxation oscillator used by Kaplan, *et al.* (2008) as Eq. 1 was simulated with $T = 20$, $\varepsilon = 2$, $\mu = 1$, $\tau = 100$ and $\alpha = 0.05$, this oscillator generates a waveform that characterizes the ventricle stimulus system, as shown in Fig. 3. The signal from the oscillator presents a low frequency spectrum, with dispersion.

$$\begin{cases} \dot{x}_1 = y_1 + \varepsilon(1 - \mu z_1)x_1 \\ \dot{y}_1 = -x_1 + \alpha[x_2(t - \tau) - x_1(t - \tau)] \\ \dot{z}_1 = [((|y_1| - y_1)/2) - z_1]/T \end{cases} \quad [1]$$

$$\begin{cases} \dot{x}_2 = y_2 + \varepsilon(1 - \mu z_2)x_2 \\ \dot{y}_2 = -x_2 + \alpha[x_1(t - \tau) - x_2(t - \tau)] \\ \dot{z}_2 = [((|y_2| - y_2)/2) - z_2]/T \end{cases}$$

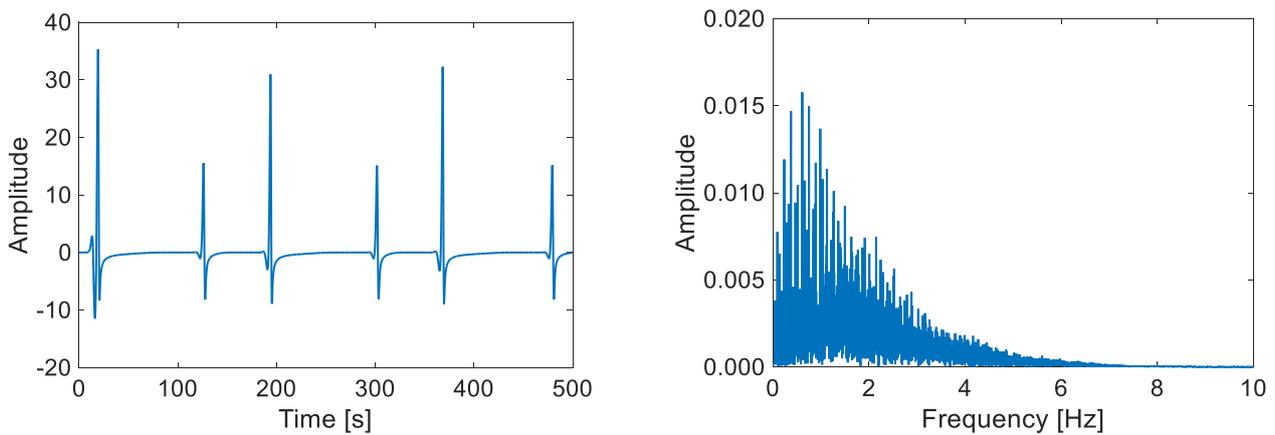


Figure 3 - QRS wave set from Kaplan, *et al.* (2008)

The response of the oscillator system was analyzed by the variation of the coefficient ε in the range that includes the value used by Kaplan, *et al.* (2008). The bifurcation diagram of Fig. 4 shows the wide variety of responses, stationary bands close to zero and 2.5, and others bands of instability in 1.0 to 2.0.

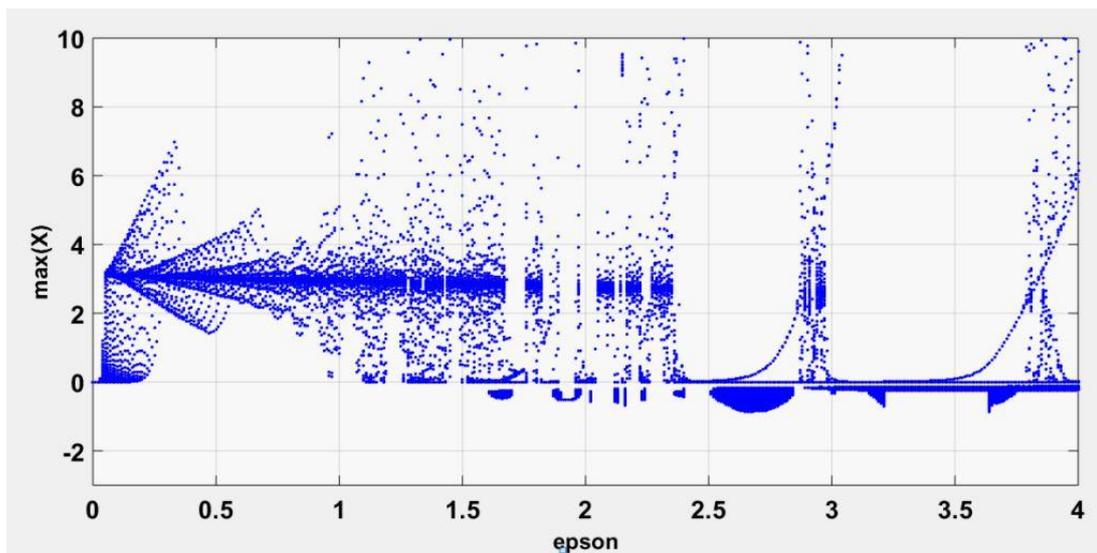


Figure 4 - Bifurcation diagram oscillator.

The Figure 5 shows in detail the response for $\varepsilon = 2$. The oscillator has double points near to zero, once intermediate and double points near thirty, and has stable behavior.

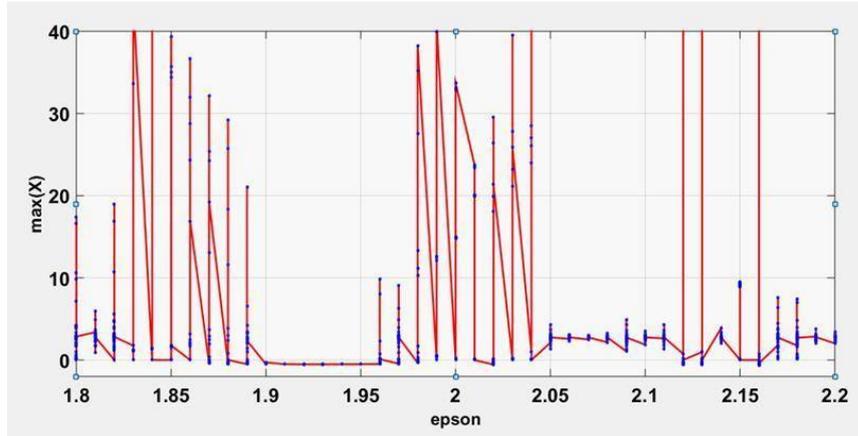


Figure 5 – Detail bifurcation diagram oscillator – ϵ .

The time constant T of the low pass filter affects the oscillator, rectifying the signal according to the equation that sets the delay time τ , participating in waveform modeling. The bifurcation diagram shows the behavior of the system by changing the filter time constant T at Fig. 6. The Figure 7 presents in greater detail the variations in amplitude and regions of instabilities of the system with the variation of T in the Kaplan model.

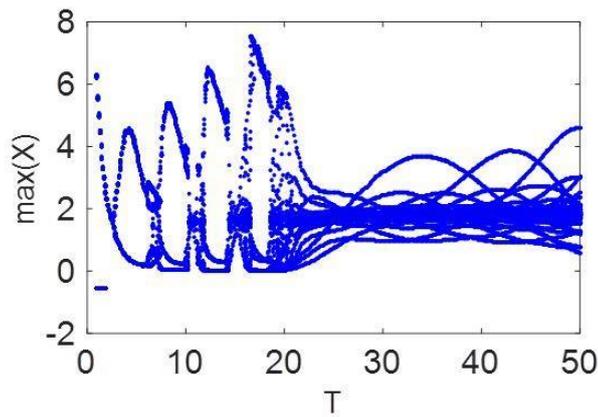


Figure 6 - Bifurcation diagram oscillator.

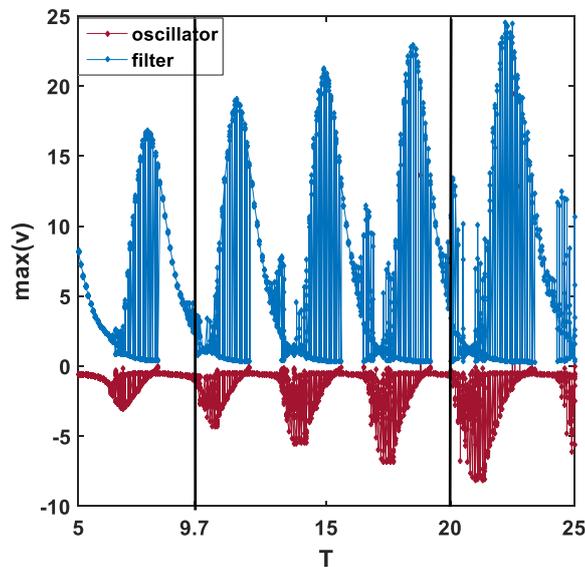


Figure 7 – Detail bifurcation diagram filter and oscillator.

3. IMPLEMENTATIONS

In this work, the γ parameter was included, according Eq. 2, in order to change the amplitude of the wave composition of the oscillator.

$$\begin{cases} \dot{x}_1 = (y_1 + \varepsilon(1 - \mu z_1)x_1)\gamma \\ \dot{y}_1 = -x_1 + \alpha[x_2(t - \tau) - x_1(t - \tau)] \\ \dot{z}_1 = [((|y_1| - y_1)/2) - z_1]/T \end{cases} \quad [2]$$

$$\begin{cases} \dot{x}_2 = (y_2 + \varepsilon(1 - \mu z_2)x_2)\gamma \\ \dot{y}_2 = -x_2 + \alpha[x_1(t - \tau) - x_2(t - \tau)] \\ \dot{z}_2 = [((|y_2| - y_2)/2) - z_2]/T \end{cases}$$

In the bifurcation diagrams of Fig. 8, is observed the restricted stability range for the desired response. Were analyzed two T for composition modeling electrocardiogram. The system was stabilized with parameters $T = 9.7$, $\varepsilon = 0.76$, $\mu = 1$, $\tau = 100$, $\alpha = 0.0189$, and $\gamma = 0.38$, using the initial conditions $[0; 10^{(-6)}; 0; 0; 10^{(-6)}; 0]$. The response from the system of oscillators generates the signal from the QRS wave set, as well as P and T wave, as in Fig. 9(a). Another configuration that managed to stabilize, and suggests the presentation of T, T' and P waves is the alteration of the parameter of filter T for 20, Fig. 9(b).

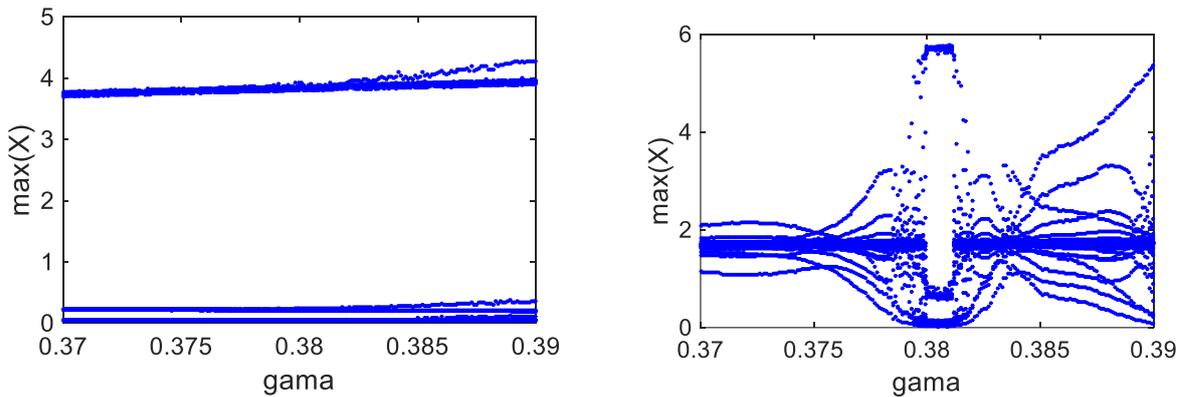


Figure 8 - Bifurcation diagram – γ behavior

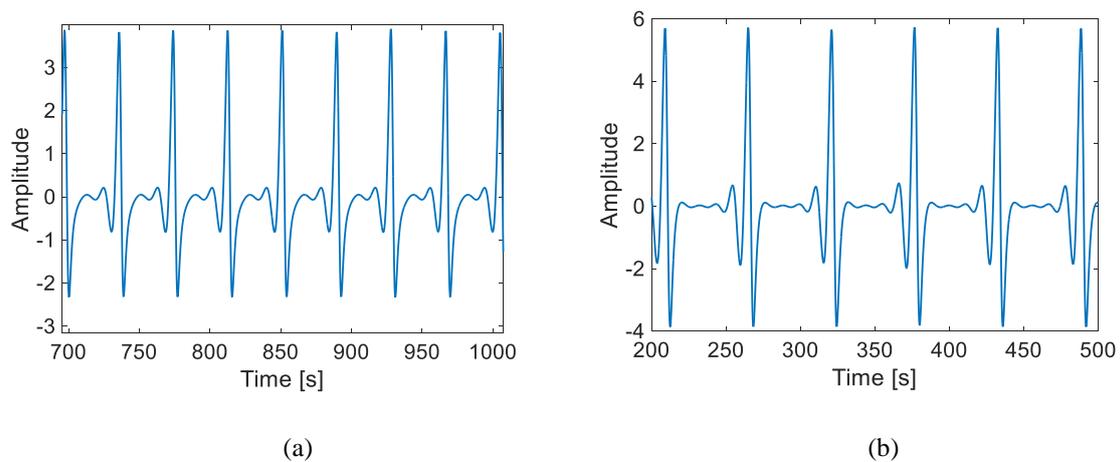


Figure 9 – Modified model responses

The variation of the parameter T, filter time constant promotes great variation in oscillator behavior as can be seen in Fig. 10. Already the alpha α parameter analyzed in the region that was adopted did not present variation in the response observing Fig. 11.

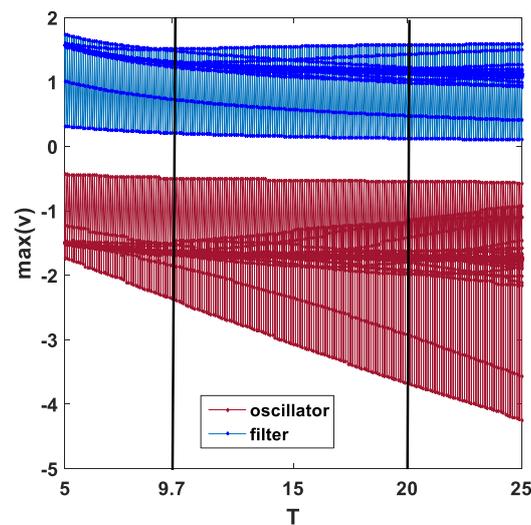


Figure 10 – Modified model responses – T filter

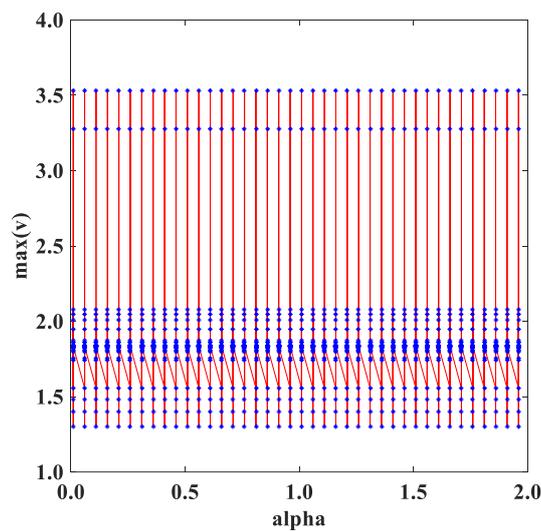


Figure 11 – Modified model responses – α delay

4. CONCLUSIONS

In this work, we analyzed the bifurcation diagram of relevant parameters of a model with two Van der Pol oscillators coupled to, control with time delay of the oscillations and low-pass filter similar to the Wien Bridge. After analyzing the parameters of the oscillator were altered in order to generate a signal closer to the complete electrocardiogram. The resulting signal, besides the complex QRS of the electrocardiogram, allowed us to draw the P and T waves. This work brought a small advance in the modeling of heart signals.

5. ACKNOWLEDGEMENTS

The authors would like to thank CAPES and CNPq for financial support.

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