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A NUMERICAL METHODOLOGY FOR DESIGNING A VISCOELASTIC VIBRATION NEUTRALIZER WITH TUBULAR GEOMETRY AND CONSTRAINED LAYERS

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Abstract. *Abstract: Vibration neutralizers (also called vibration absorbers) have been widely used in passive vibration control due to their practicality and efficacy. One way of introducing damping into these devices and thus enlarging their range of action is to insert viscoelastic materials in a constrained layer form. However, for flat geometries, among others, vibration control depends on the excitation direction. This leads to the consideration of alternative geometries with a multidirectional energy dissipation character. This paper presents a numerical study in which the efficacy of a viscoelastic vibration neutralizer with tubular geometry and constrained layers is investigated. The neutralizer optimum parameters are found by employing a discrete model, developed by using the finite element method, along with response reanalysis and optimization techniques, in addition to the response surface strategy. In this process, the viscoelastic material is described by a fractional derivative constitutive model. The methodology is designed to work with the frequency response curves of the primary and secondary systems (namely, the system to be controlled and the neutralizer), which makes it possible to exclude the primary system model from the optimization process. For existing systems, those curves can be obtained experimentally. The corresponding plots show that the resulting neutralizer has a distinct and multidirectional action over the frequency band of interest.*

Keywords: *Passive vibration control, Viscoelastic dynamic vibration neutralizers, Finite element method, Response reanalysis.*

1. INTRODUCTION

Continuous advances in engineering have allowed the design of increasingly complex machines and structures, which requires high trust levels for using smaller safety factors and leading, in general, to lighter and slender products. These factors increase susceptibility to dynamic problems requiring advances in vibration control as well.

According to (Rao, 2008), the first task to be tackled in vibration control is to try and change the excitation source, so that it produces a lower level of vibration; but sometimes this is unfeasible or even impossible (i.e. natural excitations). Another option is to reduce the propagation of the excitation from the source to other parts of the system by using vibration isolators. In some cases, it is also possible to perform modifications in mechanical parameters such as mass, stiffness or damping aimed at suitably changing the dynamic characteristics of machines and structures. There is also the approach of introducing an auxiliary system - called ‘vibration neutralizer’ or ‘vibration absorber’ - designed and coupled to the system to be controlled so as to reduce its vibration levels. This is a widely used technique in vibration control due to its expedience, cost, and efficacy.

Vibration neutralizers are secondary mechanical devices designed to reduce vibration levels - for a given frequency or frequency band - in another mechanical system, called ‘primary system’. Undamped vibration neutralizers redistribute the vibration energy, changing the frequency spectrum in such a way as to decrease the response in the frequency of interest. In some cases, only redistributing the vibration energy is not effective, requiring also this energy to be dissipated. This can be achieved by introducing damping into these devices, for instance, by inserting viscoelastic elements. Viscoelastic neutralizers can generate larger reductions in vibratory response, making the devices more efficacious in their task of attenuating vibration levels. The dissipation of vibration energy occurs when the viscoelastic elements deform dynamically.

In the present work, a numerical study on the efficacy of a viscoelastic vibration neutralizer with tubular geometry and curved constrained layers is carried out. The neutralizer optimum parameters are found by employing a discrete model, developed by using the finite element method, along with response reanalysis and optimization techniques, in addition to the response surface strategy. The response reanalysis is particularly useful in describing the effects of the coupling between the primary system and the neutralizer, whereas the response surface strategy is adopted to create, for the optimization process, a continuous objective function from discrete points.

2. CONSTRAINED VISCOELASTIC LAYER DAMPING

This concept has been widely explored in the past five decades (Mead, 1999) and refers to the damping obtained when a viscoelastic material is submitted primarily to shearing deformation when constrained between layers with higher elastic moduli (i.e. metallic layers) (see Fig. 1 for simple unconstrained and constrained configurations). Mead and Markus (1969) presented a fundamental analytical approach of a damped beam with a constrained layer of viscoelastic nature (also known as 'sandwich beam'). The equation of motion was deduced and it was demonstrated that transversal vibration modes must be complex, except for some non-practical cases. It was also shown that orthogonal vibration modes of a sandwich beam become coupled when damping is introduced into the structure.

Usually, the employment of constrained layers is performed by placing a constrained viscoelastic coat over plane beams with a rectangular cross section. Lunden (1979) studied the distribution of the viscoelastic coat over a beam in order to reduce vibration levels, and showed that an optimal distribution leads to the use of 40% less material than a uniform distribution for the same level of reduction.

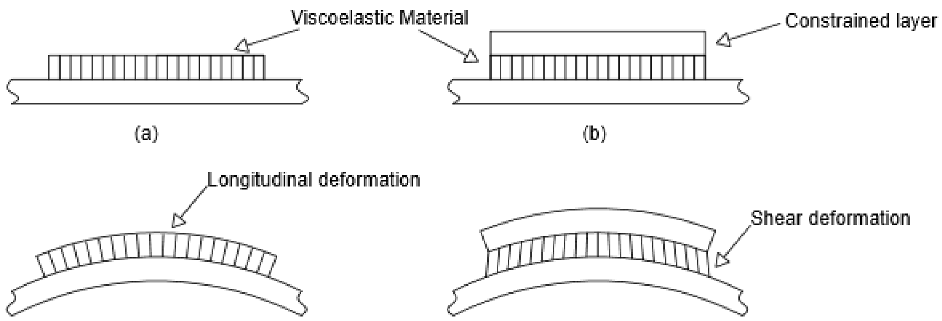


Figure 1. Viscoelastic material layer configuration. (a) unconstrained and (b) constrained layer.

However, the plane configuration can have its energy dissipation potential significantly reduced when the exciting direction is unknown. An alternative is to employ treatments with constrained layers in a cylindrical geometric form. According to Balkema (1994), this format presents some advantages in the above conditions.

It is argued that the use of uniform layers in beams with concentric circular sections does not provide an efficacious damping for flexural vibration, and partitioning the viscoelastic core can lead to better results. Borges *et al.* (2012) used a tubular beam partially covered by constrained layers to reduce vibration levels in oil risers. It is shown that damping introduced by constrained layers depends considerably on the direction of the exciting force. It should be stressed that the analytical approaches quoted so far are based on the theory of plates and shells, applicable to very thin and moderately thin layers.

The vibration theory on cylindrical structures is well developed, and many studies focusing on constrained layer damping have been performed in recent decades. A full review of vibration in cylindrical structures is presented by Hamidzadeh and Jazar (2010), which includes links, beams, shells, thick cylinders, with one or multiple layers, and semi-cylindrical panels. Usually, in these studies, the structural parameters are optimized in order to reduce vibrations in the structures themselves, characterizing a direct structural modification. However, studies about the use of cylindrical vibration neutralizers are scarce in the literature.

3. VIBRATION NEUTRALIZERS

One advantage of using vibration neutralizers is that such devices offer an efficient solution to vibration control without either changing the source of vibration or performing direct structural modifications in the primary system for they act as an auxiliary (secondary) system (Bavastri *et al.*, 2005). The underlying principle is to insert dynamic stiffness into the system to be controlled, mainly for its resonance frequencies, thus reducing significantly the vibration amplitude in those frequencies.

The dynamic stiffness in a point can be computed as the frequency domain ratio between the applied force and the displacement taken at the same point. This definition is exactly the opposite of receptance, a function that is usually obtained experimentally. For vibration neutralizers, the dynamic stiffness is normally determined at its base point, as shown in Fig. 2 for a single-degree-of-freedom device, where m stands for the neutralizer mass, Ω is the angular frequency, $\bar{k}(\Omega)$ is the complex stiffness of the neutralizer 'spring', F is the applied force, and X is the corresponding displacement.

One of the advantages of representing the dynamic stiffness at this specific point is that it is the coupling point between

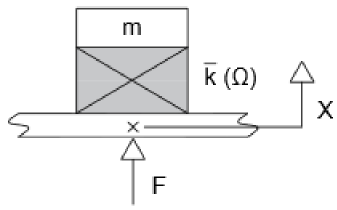


Figure 2. Vibration neutralizer representation.

the neutralizer and the primary system. At this point, the resulting dynamic stiffness is the algebraic sum of the individual dynamic stiffnesses of each system.

4. RESPONSE REANALYSIS AND EQUIVALENT MODELS

The use of dynamic equivalent models provides an alternative to reduce the size of equations in compound systems. Such models are based on the fact that it is not necessary to have a fully representation of the coupled systems - containing all their degrees-of-freedom - to investigate the influence they have on each other. Therefore, compound systems, comprising neutralizers and primary systems, can be described only in terms of primary system coordinates. For this representation, the equivalent dynamic characteristics of the auxiliary system are determined at the coupling point, so that the same dynamic stiffness is obtained (Espindola *et al.*, 2008).

Once a viscoelastic vibration neutralizer is represented by an equivalent model, and its dynamic stiffness at the coupling point is determined, it can be expressed as a particular modification matrix, $\Delta\mathbf{S}(\Omega)$, applied to the global dynamic stiffness matrix of the primary system, $\mathbf{S}(\Omega)$, both of order n. Thus, the dynamic stiffness matrix of the compound system, $\mathbf{S}(\Omega)^*$, can be expressed as (Brandon, 1990; Lopes, 1998)

$$\mathbf{S}(\Omega)^*_{n\text{x}n} = \mathbf{S}(\Omega)_{n\text{x}n} + \Delta\mathbf{S}(\Omega)_{n\text{x}n}. \tag{1}$$

Regarding receptance matrices, the receptance matrix of the primary system, $\mathbf{R}(\Omega)$, is the inverse of the corresponding dynamic stiffness matrix, $\mathbf{S}(\Omega)$, while the receptance matrix of the compound system, $\mathbf{R}(\Omega)^*$, is the the inverse of $\mathbf{S}(\Omega)^*$. Therefore,

$$\mathbf{R}(\Omega)^*_{n\text{x}n} = \left[\mathbf{R}(\Omega)^{-1}_{n\text{x}n} + \Delta\mathbf{S}(\Omega)_{n\text{x}n}\right]^{-1}. \tag{2}$$

For multi-degree-of-freedom systems, in order to obtain a single element of the modified receptance matrix $\mathbf{R}(\Omega)^*$, it would be necessary to know all the elements of the original receptance matrix $\mathbf{R}(\Omega)$, in addition to those of $\mathbf{S}(\Omega)$. However, response reanalysis (Brandon, 1990; Lopes, 1998) shows that if the modification matrix $\Delta\mathbf{S}(\Omega)$ is of sparse nature, the matrices in Eq. (2) can be partitioned by separating the generalized coordinates directly related to the modifications from those not directly related. If the partitions associated to the modifications, of order r, are located at the bottom right of the corresponding full matrices, then

$$\mathbf{R}(\Omega)^*_{r\text{x}r} = \left[\mathbf{R}(\Omega)^{-1}_{r\text{x}r} + \Delta\mathbf{S}(\Omega)_{r\text{x}r}\right]^{-1}. \tag{3}$$

The above equation allows the assessment of the effects of the modifications on specific terms of the receptance matrix. Due to the aforementioned partition, these terms are connected to the generalized coordinates directly affected by the modifications.

5. NUMERICAL MODELS AND PROCEDURES

In order to implement the proposed methodology, a numerical interface in *MATLAB* is created and linked to the finite element software *ANSYS*, allowing to analyze different models, collect and process data, run an optimization algorithm, and search for the optimum design of the neutralizer in focus.

The primary system, or system to be controlled, is required to have multiple resonance frequencies in a wide band to allow checking the wideband control effect. This system is also required to have a minimum weight, so that feasible dimensions can be achieved for the neutralizer, regarding its future manufacturing. To that end, the primary system is defined as a clamped-free beam (Fig. 3), with length, cross section, and material properties as shown in Tab. 1. Because of its simplicity, its numerical model is represented by a beam-type finite element.

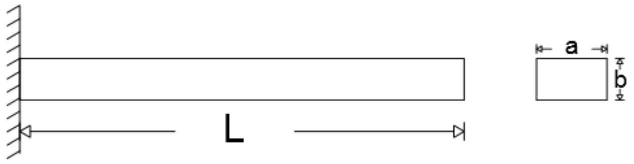


Figure 3. Primary system.

Table 1. Primary system properties.

Properties	Value
L (mm)	1500
A (mm)	50
B (mm)	8
E (GPa)	207
η (-)	0.005

The neutralizer is to be placed at the free end, with its axis parallel to the beam axis and rigidly connected to the primary system at the middle point of the internal cylinder. The frequency band to be controlled is defined between 150 and 713 Hz, comprising the 6^{th}, 7^{th}, 8^{th}, and 9^{th} vibration modes of the beam. This primary system has a connection between the translational $\mathrm{U_y}$ and rotational $\mathrm{rot_z}$ degrees of freedom. When the beam vibrates, the amplitude difference between the vertical displacements of adjacent sections is associated to a relative rotation between them. According to Rodrigues *et al.* (2016), this effect can mislead the response of the compound system while using equivalent models unless the modification matrix account for the related degrees of freedom.

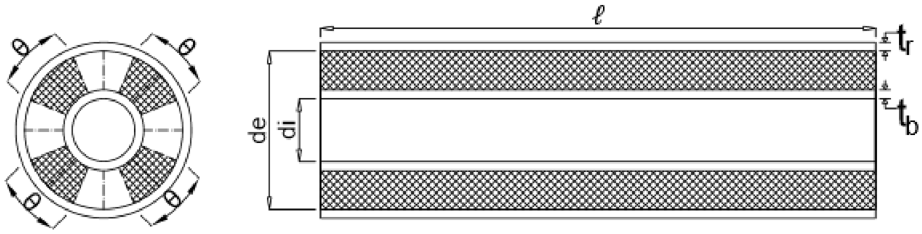


Figure 4. Vibration neutralizer and its design variables for the optimization process.

The vibration neutralizer has its shape as shown in Fig. 4, with a viscoelastic layer placed between two metallic cylinders, which act as constraining layers. Its fixed dimensions are presented in Tab. 2.

Table 2. Neutralizer fixed dimensions.

Properties	Value
$\mathrm{d_i}$ (mm)	12.7
$\mathrm{t_b}$ (mm)	1
$\mathrm{t_r}$ (mm)	1

5.1 Design variables

The design variables consist of parameters of the neutralizer that are controlled by the optimization algorithm in order that the best configuration for the device can be found. They are defined by the user, chosen out of the specific parameters of the neutralizer, to characterize the formulated problem in a proper way.

Among all the possible parameters to describe the neutralizer design, the selected ones are those that have more influence on the dynamic stiffness (length, ℓ; and external diameter, $\mathrm{d_e}$) and damping (viscoelastic layer thickness and angle of coverage, θ). The viscoelastic layer thickness is given by $0.5(\mathrm{d_e} - (\mathrm{d_i} + 2\mathrm{t_b}))$. To ensure a feasible device manufacturing, the following top and bottom limits are established: $0.2 \leq \ell \leq 0.7$ (m) and $15.87 \leq \mathrm{d_e} \leq 57.15$ (mm). As to the coverage angle θ, in order to simplify the finite element modeling, values 22.5°; 45°; 67.5°and 90°can be assumed. That applies for each quadrant in a way that 90°corresponds to a fully covered cross section.

As the neutralizer model is more complex than the beam model, it is represented using both shell elements (internal and external elastic layers) and 3D hexahedral solid elements (viscoelastic layer).

An appropriate constitutive model is crucial for representing the viscoelastic material properties (Nakra, 1998, 2001). A four parameter fractional derivative model is used to describe the viscoelastic properties and dynamic behavior assigned to the viscoelastic core of the neutralizer. The use of fractional calculus to represent the dynamic behavior of viscoelastic materials is an excellent tool for it reduces the number of parameters to be considered (Bagley and Torvik, 1986). The four parameter fractional derivative model used in this analysis is given by:

$$\bar{E}(\Omega, T) = (E_0 + E_\infty\, b_1(i\Omega\alpha_T))/(1 + b_1(i\Omega\alpha_T)^\beta), \tag{4}$$

where the elastic constants E_0 and E_∞ stand for the asymptotic values for the real elastic moduli E_r for very low

($\lim_{x\to 0} E_r(\Omega, T_0) = E_0$) and very high ($\lim_{x\to\infty} E_r(\Omega, T_0) = E_\infty$) frequencies, respectively, while T_0 is the reference temperature. The relaxation moduli b_1 is associated to the horizontal displacement of E_r, regarding the frequency. Parameter β is non-dimensional, $0 \leq \beta \leq 1$, and stands for the fractional derivative order. The shift factor α_T is obtained by the Williams-Landel-Farry model, given by

$$\log \alpha_T = -\theta_1(T - T_0)/(\theta_2 + (T - T_0)). \tag{5}$$

The material parameters for the employed elastomer, a known formulation of butyl rubber, are presented in Tab. 3.

Table 3. Viscoelastic material parameters.

				Shift Factor			
Parameter	β	E_0 (MPa)	E_∞ (MPa)	b_1	T_0 (K)	θ_1	θ_2
Value	0.417	7.21	457	0.0223	243	7.98	81.7

5.2 Objective function

The main aim of the present study is to reduce vibration levels in the primary system. To that end, two different objective functions are defined. The first one uses only the peak amplitude of a given frequency response function (FRF) in the selected frequency band, and is defined by

$$H^{\mathrm{cs}}_{\mathrm{peak}} = max\left(20\log_{10}|H^{\mathrm{cs}}(\Omega)|^{\Omega_b}_{\Omega_a}\right), \tag{6}$$

where H^{cs} is the given FRF of the compound system, Ω_{a} and Ω_{b} are the lower and upper limits of the frequency band, and the $H^{\mathrm{cs}}_{\mathrm{peak}}$ is expressed in decibels. The second one is defined by a mean of the amplitude values of a given FRF in the selected frequency band and is defined by

$$H^{\mathrm{cs}}_{\mathrm{mean}} = \sum_{\Omega=\Omega_a}^{\Omega_b} 20\log_{10}\left[H^{\mathrm{cs}*}(\Omega) * H^{\mathrm{cs}}(\Omega)\right]/N, \tag{7}$$

where $H^{\mathrm{cs}*}(\Omega)$ indicates the complex conjugate of $H^{\mathrm{cs}}(\Omega)$ and N is the number of frequency points considered. Both objective functions are then pondered by the values obtained only from the primary system, before the neutralizer is introduced into the system. In the present study, the amplitude values are related to the vertical displacement at the coupling point.

In addition to the restraints defined by the top and bottom limits that each variable can reach, a penalty function is defined to restrict the total mass of the neutralizer regarding the total mass of the primary system.

To solve the whole optimization problem, the genetic algorithm technique is used. As output, the optimum physical parameters of the neutralizer are obtained.

6. DEVELOPED METHODOLOGY

The developed methodology proposes the use of optimization techniques to identify the optimal parameters of a viscoelastic vibration neutralizer with tubular geometry and curved constrained layers. It aims at controlling the levels of vibration of a certain primary system in a broadband range of frequencies. The coupling between the neutralizer and the primary system is considered to occur at a single point, with two degrees of freedom. This methodology incorporates the following concepts and tools:

- Finite element method (FEM) – to describe the dynamic response of the primary system and the neutralizer, individually;
- Fractional derivative constitutive model – to represent the viscoelastic material properties;
- Equivalent models – to couple the individual models and reduce the compound system order;
- MDOF coupling – to obtain a better representation of the real coupling between both systems;
- Response surface – to represent the objective function through an approximation of the discrete points by a continuum function;
- Genetic algorithms – to search for the minimum of a mixed variable function with possible local minima.

This methodology works as presented in the Fig. 5, where a set of design vectors (points) is firstly created (a). Then this set is evaluated via FEM, defining a set of FRF curves of the NDV (b). Subsequently, the objective function type – either peak value or mean value – and the frequency band to be controlled are defined. Then the algorithm loads the

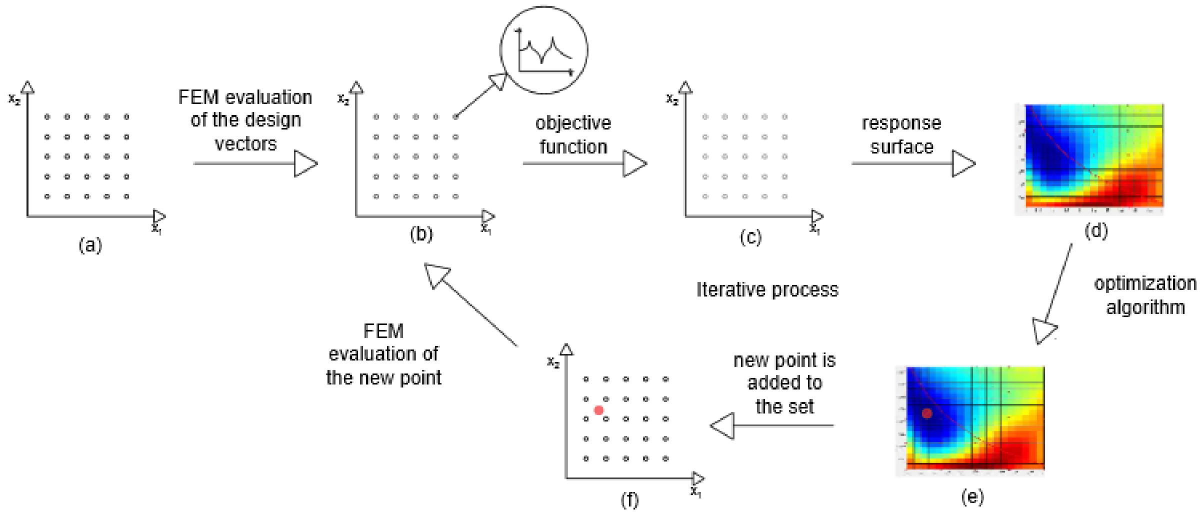


Figure 5. Developed methodology and optimization process.

FRF curves from the primary system, couples them with those from the NDV and evaluates the objective function, with information both from the primary system and the compound system, creating a new set now in the objective function domain (c). The FRF used in the objective function was the receptance curve.

The next step is to create a response surface over these values, that is, to define and adjust an analytical function over these points (d). This function allows carrying out a search using optimization techniques towards the minimum point (e). Then, this new point is also evaluated via FEM and added to its corresponding set (f). The process continues iteratively computing the objective function to this new point, updating the response surface and searching for a new minimum point until a stopping criterion is satisfied.

7. NUMERICAL RESULTS

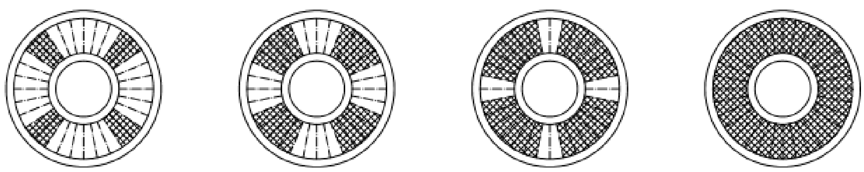
Once the finite element models of both primary system and neutralizer are built and these are coupled with the aid of response analysis, it is possible to perform a search for a minimum inside the response surface using a genetic algorithm technique. This search occurs inside the feasible region of the project with the response surface being updated at each iteration incorporating the previous evaluated solution. The numerical analysis presented below aims at comparing the results obtained through different objective functions to different constrained layer configurations.

The numerically evaluated cases are presented in Tab. 4.

Table 4. Considered cases.

		Constrained layer type	
		Full	**Partial**
Objective function type	**Peak value**	Case 1	Case 3
	Mean value	Case 2	Case 4

The constrained layer configurations are as illustrated in Figs. 6 and 8, and the objective function types are as defined in Eqs. (5) and (6). These cases are considered in order to verify the response due to distinct configurations and the impact the objective function type has in the search for the minimum point for these configurations. The results are presented below.



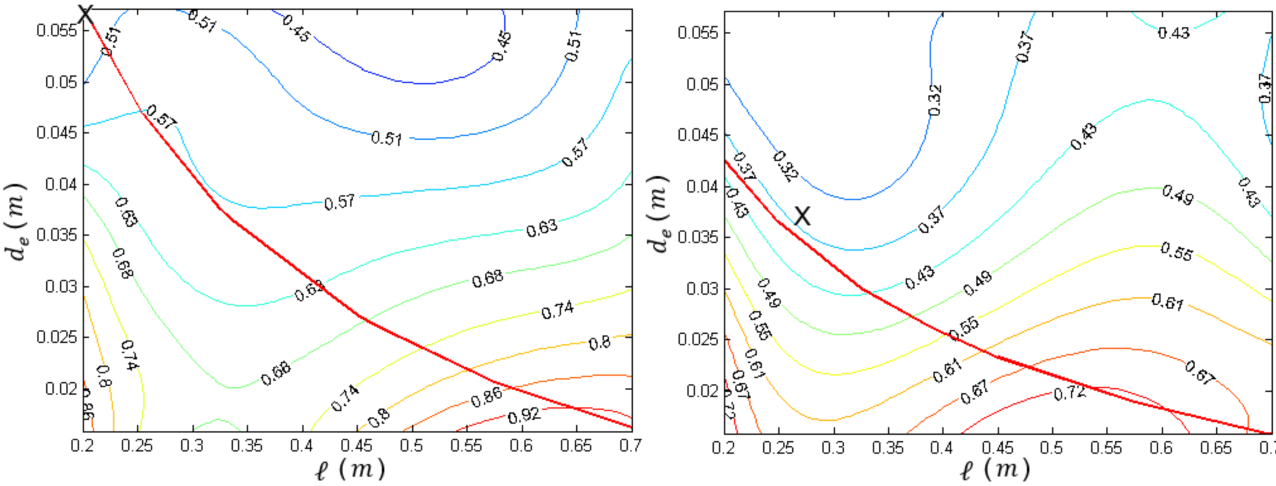
(a) θ = 25% (b) θ = 50% (c) θ = 75% (d) θ = 100%

Figure 6. Developed methodology and optimization process.

7.1 Fully covered constrained layer model

This model consists of two concentric cylinders, connected by viscoelastic segments. Using this geometry, the first and second cases are analyzed. In order to represent a three-dimensional function in a bi-dimensional graphic, fixed values for the viscoelastic layer angle are used and contour lines are created, corresponding to fillings of 25%, 50%, 75%, and 100% of the neutralizer cross section, as represented in Fig. 6.

The optimum design values for cases 1 and 2 are represented by an 'X' in Figs. 7 (a) and (b), respectively.



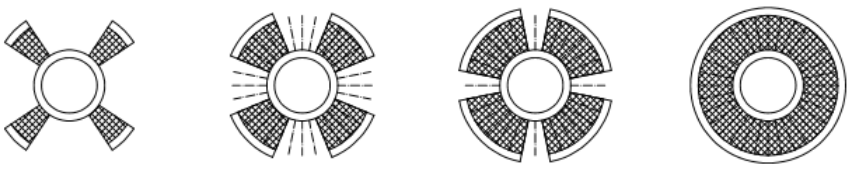
(a) Case 1: response surface contour levels for $\theta = 25\%$. (b) Case 2: response surface contour levels for $\theta = 75\%$.

Figure 7. Optimal design points for fully covered constrained layer configuration.

For Case 1, the point representing the minimum value for the objective function is located at $\theta = 25\%$, $\ell = 0.2$ m and $d_e = 57.15$ mm, and the objective function (f_{obj}) value is 0.465. For Case 2, the optimum point corresponds to $\theta = 75\%$, $\ell = 0.265$ m, $d_e = 37.6$ mm and $f_{obj} = 0.355$.

7.2 Partially covered constrained layer model

This model is characterized by the existence of isolated segments of viscoelastic materials connected only by the internal cylindrical base of the neutralizer (Fig. 8). When compared to the previous one, this model presents a lower structural stiffness besides a certain dynamic independence of these segments. The same assignment of values for the viscoelastic layer coverage angle in the first two cases is repeated here.



(a) θ = 25% (b) θ = 50% (c) θ = 75% (d) θ = 100%

Figure 8. Neutralizer cross section - partially covered viscoelastic layer.

The optimum design values for cases 3 and 4 are presented with an 'X' in Figs. 9 (a) and (b), respectively.

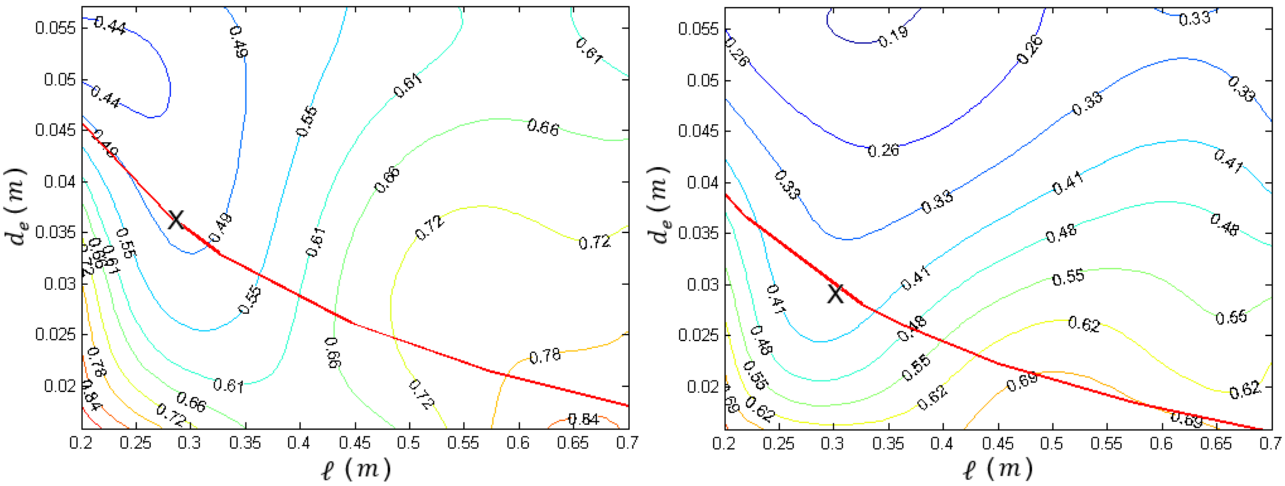
For Case 3, the optimum design point is $\theta = 75\%$, $\ell = 0.281$ m and $d_e = 36.5$ mm, and the f_{obj} value is 0.476. For Case 4, the minimum is at $\theta = 100\%$, $\ell = 0.289$ m, $d_e = 30.5$ mm and $f_{obj} = 0.356$.

The optimal values for Cases 1, 2, 3, and 4 are summarized in Tab. 5 with its respective objective function values.

Given that different functions are used to find the optimum design, it is worth making a comparison among the FRF curves for all the cases. Fig. 10 presents this comparison along with the corresponding primary system FRF curve.

7.3 Evaluation of the multidirectional character of the dynamic stiffness

An analysis of the excitation direction effect is performed. In this case (Fig. 11), the excitation angle φ changes counterclockwise from 0°, when it is parallel to the vertical axis, up to 360°, in order to verify how the dynamic stiffness



(a) Case 3: response surface contour levels for $\theta = 15\%$. (b) Case 4: response surface contour levels for $\theta = 75\%$.

Figure 9. Optimal design points for partially covered constrained layer configuration.

Table 5. Optimal values for the considered cases.

	Variable			
	$\ell\ (m)$	θ	$\mathbf{d_e}\ (m)$	$\mathbf{f_{obj}}$
Case 1	0.2	25%	0.05715	0.465
Case 2	0.265	75%	0.0376	0.355
Case 3	0.281	75%	0.0365	0.476
Case 4	0.289	100%	0.0305	0.356

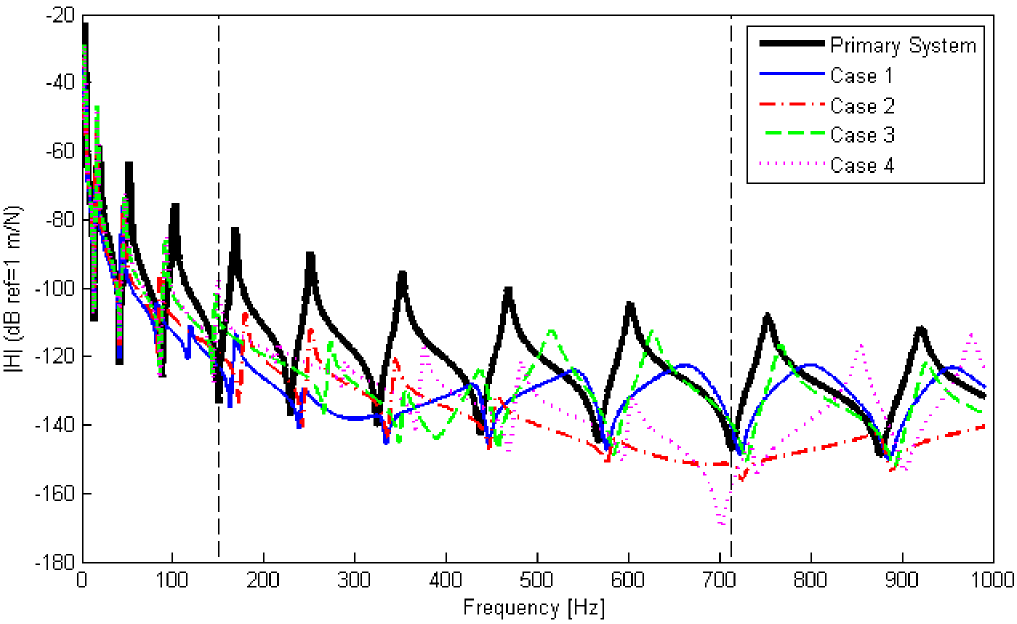


Figure 10. Compound system FRF curves for the considered cases.

and, therefore, the natural frequencies, changes with φ.

Figure 12 compares the changes in the first natural frequency of translation (a) and rotation (b) due to the excitation angle. The circumferential values represent the angle of excitation force and the radial values represent the ratio between the first natural frequency for the given angle and the original value at $\varphi = 0°$.

It is clear that natural frequency depends on the excitation direction, and there is a pattern in the response at each 90° related to rotational symmetry of the cross section. In this case, the influence of the excitation angle is identical for orthogonal excitations, and the maximum difference is for an excitation angle of $45°$. This difference is expected to get lower along with the decrease in the symmetry angle (i.e. using more viscoelastic segments). When the excitation is parallel to the viscoelastic segments, the natural frequency is higher because the transversal stiffness in this direction is higher.

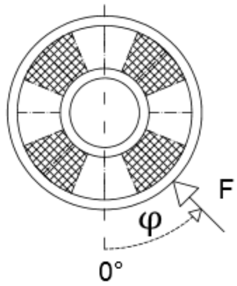
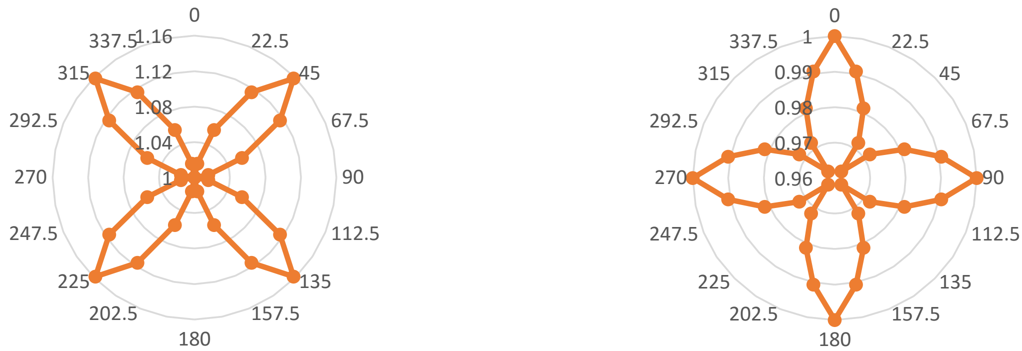


Figure 11. Angle of excitation force at the neutralizer.



(a) Translational mode of vibration.

(b) Rotational mode of vibration.

Figure 12. Changes on the first natural frequency with the excitation angle.

8. CONCLUSIONS

The present work reports a numerical study focusing the efficacy of a viscoelastic vibration neutralizer with tubular geometry and curved constrained layers. A methodology using only FRF curves of the involved systems is developed, and a 2 degrees of freedom coupling is considered between these systems. Such methodology proves to be efficient at carrying out the search for the neutralizer optimal parameters.

The use of a tubular constrained layer geometry proves to be proper for wideband frequency vibration control for the considered primary system. The existence of a rotational symmetry of $90°$ makes it possible to predict similar responses for orthogonal excitations, perpendicular to the neutralizer axis.

The use of response surfaces to create a continuum function is presented as an alternative to reduce the number of evaluations of the compound system by FEM, thereby reducing the time spent in such analyses. The development of a discrete model using the finite element method along with the use of optimization techniques and response surfaces allows the search for the neutralizer optimum parameters to be carried out. In this process, the viscoelastic material is described by a fractional derivative constitutive model.

The methodology is designed to work with FRF curves of the primary and secondary systems (namely, the system to be controlled and the neutralizer), which makes it possible to exclude the primary system model from the optimization process. For existing systems, those curves can be obtained experimentally, which offers an alternative approach to the matter. The corresponding plots show that the resulting neutralizer has a distinct and multi directional action over the frequency band of interest.

9. ACKNOWLEDGMENTS

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