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A SIMPLIFIED MODEL FOR THE AERODYNAMIC ANALYSIS OF A SMALL PROPELLER

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Abstract. *The flow through the rotor of a propeller is complex due to the rotor wake interaction. Different methods with more or less simplifications are used to represent the flow field and its interaction with the rotor and how these simplifications can affect the precision of the predicted results and computation time. These methods include momentum theory, blade element theory, lifting line theory, finite volume methods, panel methods, boundary element methods and CFD analysis. Most of these methods consume memory and computational time. The present proposes using a robust and simple method to calculate the aerodynamic characteristics of the propeller with acceptable precision and small computational time and small CPU time of around 0.7s. In this study both the momentum theory and blade element theory are used in home built FORTRAN calculation program. The predicted results are compared with those obtained from the panel method and available experimental results showing acceptable agreement. Further calculations were realized to investigate the effects of chord distribution and local pitch angle on the blade loading, on the torque and the thrust coefficients of the propeller.*

Keywords: *small propeller, panel method, momentum theory, blade element theory, blade aerodynamics*

1. INTRODUCTION

Small and medium size rotors are used in many recent applications as propellers and wind turbines. Momentum theory applied to rotors and blade element theory were widely used for light loaded blades. Theodorsen (1948) developed the propeller theory with ideal load distribution from the dynamics of the wake vortex sheet.

The flow over the rotors is very complex due to the circular movement of the blades and the strong interaction with the wake (Palmiter and Katz, 2010). For this reason, the precise calculation of the aerodynamic behavior of the rotor depends on the correct modeling of the rotor wake, whose complex structure limits pure analytical methods and hence the numerical methods are inevitably necessary (Dumitrescu and Cardos, 1998).

There are two main approaches to model the rotor wake. The first is to assume a prescribed wake where the wake is known a priori and hence the velocity field. The second method is based upon assuming a free-wake and needs a tremendous computing time.

Dumitrescu and Cardos (1998) used a lifting line method to replace the wind turbine blades with the trailing vortices shed along the turbine blade. The model is nonlinear and was solved iteratively. The performance parameters were calculated by the Biot-Savart law and the Kutta-Joukowski theorem.

Palmiter and Katz (2010) used a tridimensional potential flow based panel code to model the flow over rotating propeller blades. They modified an existing panel code, and studied the wind turbine and propeller flows and validated their predictions with available results.

The present study is focused on developing and validating a calculation routine for small propellers and wind turbines that has simple formulation, acceptable precision and low computing time. The routine based on the Momentum theory and Blade element theory is used to calculate the performance characteristics of the propeller using the geometry of the rotor and the aerodynamic characteristics of the blade sections along the rotor radius. The numerical predictions were validated against experimental and numerical results showing acceptable agreement. The calculation procedure was used to investigate the effects of the chord and pitch angle distributions on the blade loading and on the performance characteristics of the propeller.

2. COMPUTATIONAL PROCEDURE

The propeller develops an axial force called thrust T at an advance V for a rotational n due to a torque Q . In this manner the propeller efficiency is $\eta = (TV)/(2\pi nQ)$.

The aerodynamic characteristics are expressed by no dimensional coefficients which depend on the Reynolds and Mach numbers and the advance ratio $J = V/(nD)$. They are thrust coefficient $k_T = T/(\rho n^2 D^4)$, torque coefficient $k_Q = Q/(\rho n^2 D^5)$ and the power coefficient $k_P = 2\pi k_Q$ that can be obtained experimentally or by analytically theory.

The calculation routine to predict the general performance of the propeller associates the momentum theory due to Rankine and Froude (Wald, 2006) with Glauert blade element theory (Glauert, 1926). The advance velocity V is corrected by the inflow factor a , as $V_0 = V(1+a)$. The velocity component in the plane of rotation V_w can be calculated from $V_w = 2\pi n(1-b)r$, where b is the swirl factor which accounts for the effects of the wake vortex system.

Applying the principle of conservation of linear and angular momentum to the flow in an infinitesimal radial ring and also from the blade element theory one can obtain

$$a = (1+a) \frac{\sigma}{4 \sin^2 \phi} (C_\ell \cos \phi - C_d \sin \phi) \quad (1)$$

$$b = (1+a) \frac{\sigma}{4 \sin^2 \phi} (C_\ell \sin \phi + C_d \cos \phi) \frac{J}{2\pi(r/D)} \quad (2)$$

where $\sigma = (Bc)/(2\pi r)$ is the solidity and the angle $\phi = \tan^{-1}[J(1+a)/(2\pi(r/D)(1-b))]$. The two dimensional lift coefficient C_ℓ and drag coefficient C_d can be obtained from the aerodynamic characteristics of the airfoil for the angle of attack $\alpha_c = \theta_c - \phi$; where θ_c is the pitch angle of the blade section. The pitch angle of the blade is defined at $r = 0.75R$, where R is the radius of the blade tip, $R = D/2$.

Equations (1) and (2) can be used to calculate the factors a and b iteratively.

The thrust loading coefficient δk_T of the blade element δr at r is given by

$$\delta k_T = \frac{\frac{dT}{dr}}{B\rho n^2 D^4} \delta r = \frac{1}{2} c \left[\frac{J(1+a)}{D \sin \phi} \right]^2 (C_\ell \cos \phi - C_d \sin \phi) \delta r \quad (3)$$

While de torque loading coefficient of the blade δk_Q is given by

$$\delta k_Q = \frac{\frac{dQ}{dr}}{B\rho n^2 D^5} \delta r = \frac{1}{2} c \left[\frac{J(1+a)}{D \sin \phi} \right]^2 (C_\ell \sin \phi + C_d \cos \phi) \frac{r}{D} \delta r \quad (4)$$

The coefficients of thrust k_T and torque k_Q of the propeller are obtained by integrating Eq. (3) and Eq. (4).

Hence the performance characteristics of the propeller can be presented in terms of k_T , k_Q , k_P and η in terms of the advance ratio J . The detailed calculation procedure can be found in Ismail and Rosolen (2017).

The computer used in the calculation is a notebook, Intel® core™ i7-4510U CPU of 2.6 GHZ, RAM memory of 8 GB. The CPU time varies approximately in the range of 0.6 to 0.7 sec.

3. RESULTS AND DISCUSSION

The method adopted here can be used to pre dimension a light loaded propeller and determine its aerodynamic characteristics. First the method is validated by comparing the present predictions with experimental results and other results obtained from using the panel method. Then, the calculation procedure was used to investigate the effects of the chord and pitch angle distributions on the blade loading, on the torque and thrust coefficients of the propeller.

3.1 Validation

The validity of this method and its viability in calculating the propeller performance and/or its pre dimensioning is established by comparing the numerical predictions from the present method with experimental results (Hartman and Biermann, 1938) and numerical predictions based on the panel method (Palmiter and Katz, 2010).

The available experimental results are for the propeller Clark Y 5868-9, with airfoil Clark Y, diameter 3.048 m (10 ft), two blades and for two blade pitch angles of 25° and 35° (Hartman and Biermann, 1938). Recent numerical results for the same propeller were realized by Palmiter and Katz (2010), they used three dimensional panel method.

Figure 1 shows a comparison of coefficient of thrust k_T predicted from the present method compared with the experimental results and with the numerical predictions calculated by the panel method (Palmiter and Katz, 2010). As can be seen the agreement is good for the case of pitch angle of 25°. When the pitch angle is increased to 35°, there is noticeable divergence between the present predictions and the experimental results for low advance ratios due to possible flow separation (Palmiter and Katz, 2010), which is not accounted for in the both numerical methods. Palmiter and Katz (2010) justified the low values calculated by the panel method due to considering rigid wake to reduce the computational time.

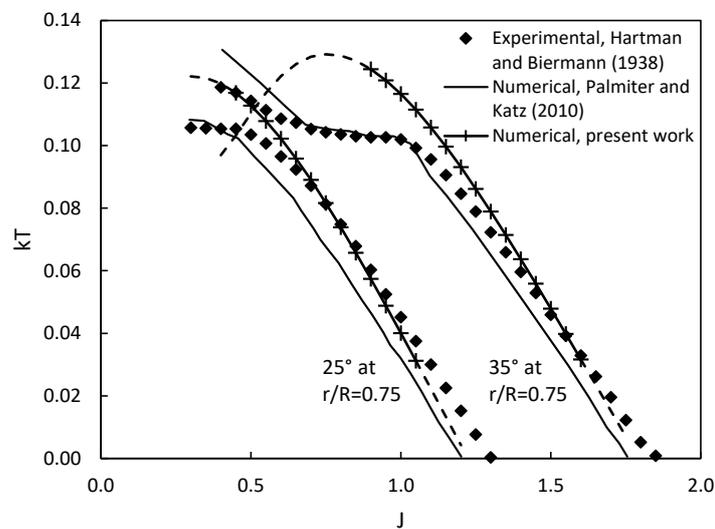


Figure 1. Variation of the thrust coefficient with the advance ratio for propeller Clark Y 5868-9

Figure 2 shows a comparison between the present predicted values of coefficient of power k_p , the experimental results and the numerical results due to Palmiter and Katz (2010). It is possible to verify that the results from the present method are closer to the experimental results. As can be seen the agreement between the present predictions and experiments are good except for values of J below 1.0 for the case of pitch angle of 35°. In this region, both numerical predictions indicate noticeable divergence in comparison with experiments.

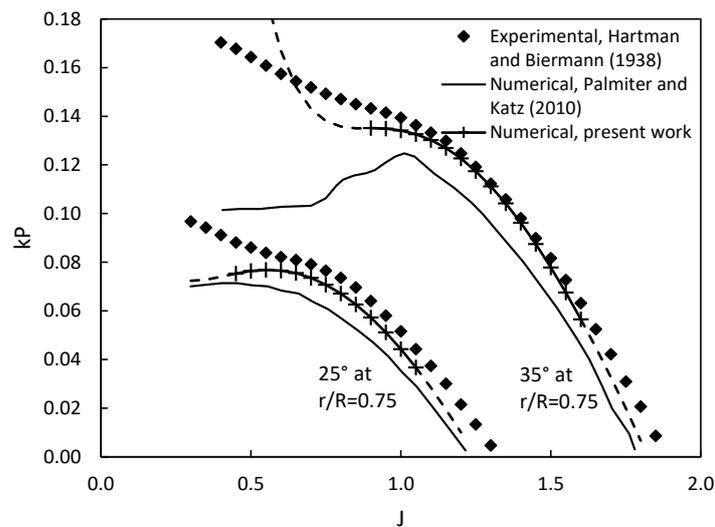


Figure 2. Variation of the power coefficient with the advance ratio for propeller Clark Y 5868-9

Figure 3 shows the efficiency of the propeller predicted from the present method compared with experimental results and numerical ones due to Palmiter and Katz (2010). As can be seen the agreement is good with the results of panel method. However, both numerical methods overestimate the efficiency.

It is interesting to observe that around the advance ratio for best efficiency, that is, around $J = 0.95$ ($\eta = 0.91$) for the case of pitch angle of 25° and around $J = 1.4$ ($\eta = 0.93$) for the pitch angle of 35° , the present predicted results are good for both coefficients k_T and k_P as can be verified from Fig. 1 and Fig. 2. These results are significant for a simple rapid method in comparison with other elaborate methods.

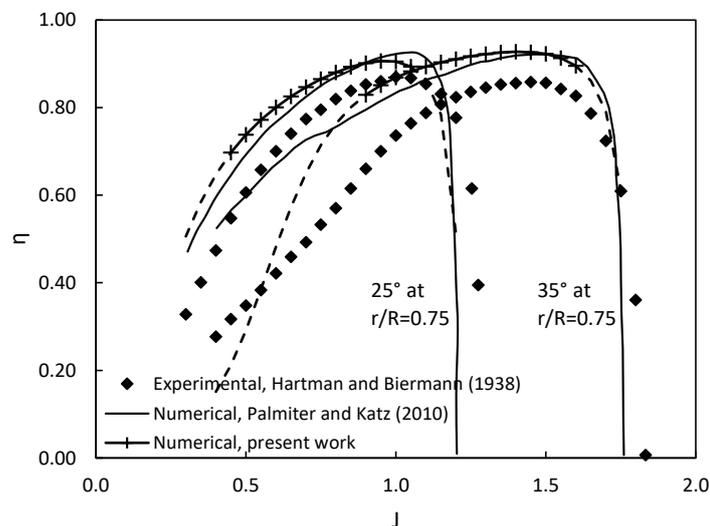


Figure 3. Variation of the efficiency with the advance ratio for propeller Clark Y 5868-9

Table 1 presents the aerodynamic characteristics calculated by the present method for the advance ratio which produces maximum efficiency for two blade propellers Clark Y 5868-9 with pitch angle of 25° and 35° . The numerical predictions are compared with the experimental values obtained by Hartman and Biermann (1938). Since the experimental results are in graphical form free access software “Tracker” was used to digitalize the graphs. The error presented in Tab. 1 is calculated as the difference between the predicted value and the experimental result divided by the experimental value (reference value). One can observe that the relative error is around 7% for the thrust coefficient and of order of -12% and -2% for the power coefficients, for pitch angles of 25° and 35° .

Table 1. Aerodynamic characteristics of the propeller Clark Y 5868-9 with two blades, diameter 3.048 m and pitch of 25° and 35° at 0.75R

Pitch	25°			35°		
$N(rpm)$	1000			800		
	Numerical (present)	Experimental	Error (%)	Numerical (present)	Experimental	Error (%)
J	0.950	0.950	---	1.400	1.400	---
η	0.906	0.856	5.4	0.927	0.856	8.4
k_T	0.049	0.052	-7.0	0.064	0.060	6.9
k_P	0.051	0.058	-11.8	0.096	0.098	-1.9
$T(N)$	1434	1542	-7.0	1198	1120	6.9
$Q(Nm)$	729	827	-11.8	877	894	-1.9
$P(W)$	76358	86630	-11.8	73495	74910	-1.9

Both the panel method due to Palmiter and Katz (2010) and the present method allow calculate the loading along blade as can be seen in Fig. 4 for the case of pitch angle of 25°. As can be seen the loading curves follow the same tendencies but the present method shows higher values from about 60% of the rotor radius and also higher values at the blade tip. Perhaps this is due to the fact the panel method represent better the local geometry of the airfoil section than the present method.

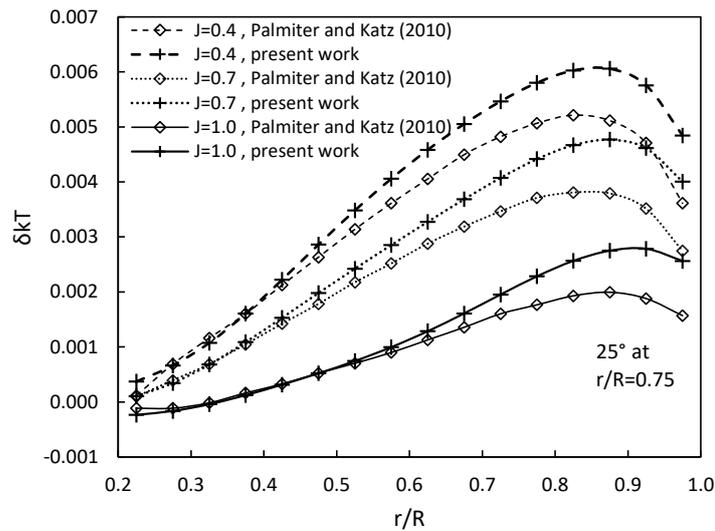


Figure 4. Comparison between the present predictions of the radial loading along the blade and Palmiter's and Katz results

Palmiter and Katz (2010) reported that a big part of the blade showed inadequate pressure distribution as calculated by the panel method. Hence, they suggested modifying the blade geometry of Clark Y 5868-9 to obtain better flow conditions along the blade. Their proposed propeller has a diameter of 3.048 m (10 ft) and profile Clark Y with a rectangular blade with thickness varying linearly between 20% at the root and 9% at the tip of the blade. The local pitch angles change linearly from the root to the tip, and is defined as that at $r = 0.75R$. The taper ratio λ is varied between 0.25 and 1.25.

We used this information to calculate the blade loading with taper ratio 0.75, pitch angle of 25° and compared the predictions with those due to Palmiter and Katz (2010). As can be seen in Figure 5 the predicted results seen to agree well for advance ratio $J = 1.0$. One can observe, Figure 5, that the predicted results indicate that the tip region of the modified propeller is less loaded in comparison with the blade of the reference propeller, and that the loading is heavier in the central region of the blade.

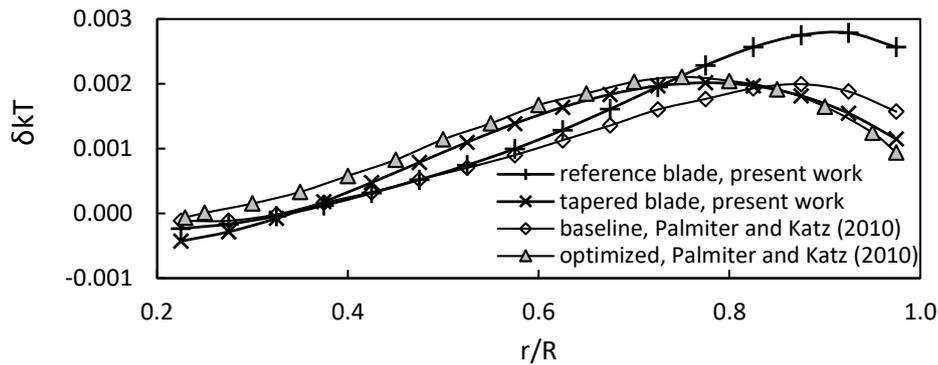


Figure 5. Comparison of the radial loadings along the reference blade with the tapered blade for advance ratio $J = 1.0$

The above results show acceptable agreement and indicate that the numerical code can be used to predict reasonably well the aerodynamic characteristics of propeller blades.

3.2 Influence of the chord and pitch angle on the blade performance

Further calculations were realized to investigate the effects of chord and pitch angle distributions on the blade loading and on the performance characteristics of the propeller.

The reference propeller assumed here for comparison is that for which Hartman and Biermann (1938) presented the geometry and the experimental results, the Clark-Y 5868-9 propeller. It has Clark-Y as an airfoil for the blades, two blades, 3.048m (10 ft) diameter and pitch of 25° . The present code used the data for the Clark-Y(B) available in Lyon *et al.* (1998) for Reynolds number of 3.0×10^5 .

The validated code was used to calculate a proposed propeller with the same geometry and operational conditions as the reference propeller, except that the airfoil section is Göttingen 796 instead of Clark-Y, similar airfoils. The characteristics of Göttingen 796 were determined from XFOIL for various Reynolds numbers. The propeller calculation uses the local Reynolds number according to Ismail and Rosolen (2017).

Figure 6 shows a comparison between the proposed propeller and the reference propeller for the advance ratio $J=1$ corresponding to maximum efficiency. The root region is defined between $r/R=20\%$ and $r/R=40\%$, while the tip region is defined between $r/R= 80\%$ and the blade tip. The respective contributions of the three regions of proposed propeller to the total thrust are: -1% for root region, 48% for middle region and 53% for the tip region.

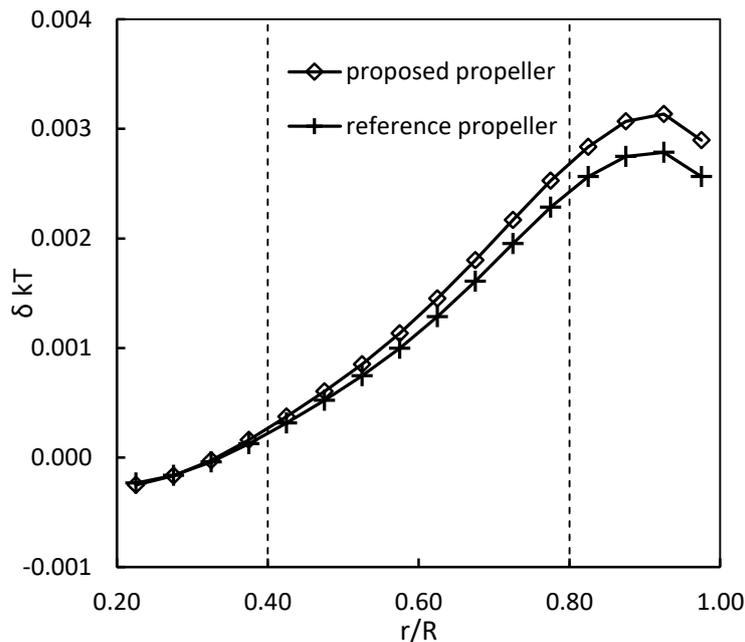


Figure 6. Comparison of radial distribution of proposed propeller with Göttingen 796 and reference propeller

The propeller with the airfoil Göttingen 796 is calculated for different values of chord and pitch angle distributions in order to investigate their effects on the blade loading and the results are shown for advance ratio $J = 1.0$ in Fig. 7.

The blade loading is calculated for $\lambda = 1.0$, $\lambda = 0.75$ and $\lambda = 0.50$ and keeping the chord length equal to the value corresponding to that in the middle of the blade height. The distribution of the pitch angle is linear having pitch angle of 25° at $r = 0.75R$ and with 47° at the root section which produces best results at $J = 1.0$.

One can observe from Fig. 7, propellers (c) to (e), reduction of local loading at the tip region and more blade loading at mid region of the blade in comparison with the reference blade. One can also notice that these changes become more noticeable with the increase of the blade taper, decrease of λ .

Adopting a linear distribution for the pitch angle and maintain the chord distribution as in the reference blade, propeller (f), one can observe strong reduction of blade loading at the tip region and strong increase of blade loading in the middle part of the blade.

Finally, from Fig. 7, propeller (g), the elliptical chord and linear pitch angle distributions resulted in severe loading reduction in comparison with the tapered blades.

Effects of varying the chord and pitch angle distributions on the aerodynamic performance of the propellers under analysis and the respective loadings of their blade for advance ratio $J = 1.0$ are shown in Tab. 2 as identified in Fig. 7.

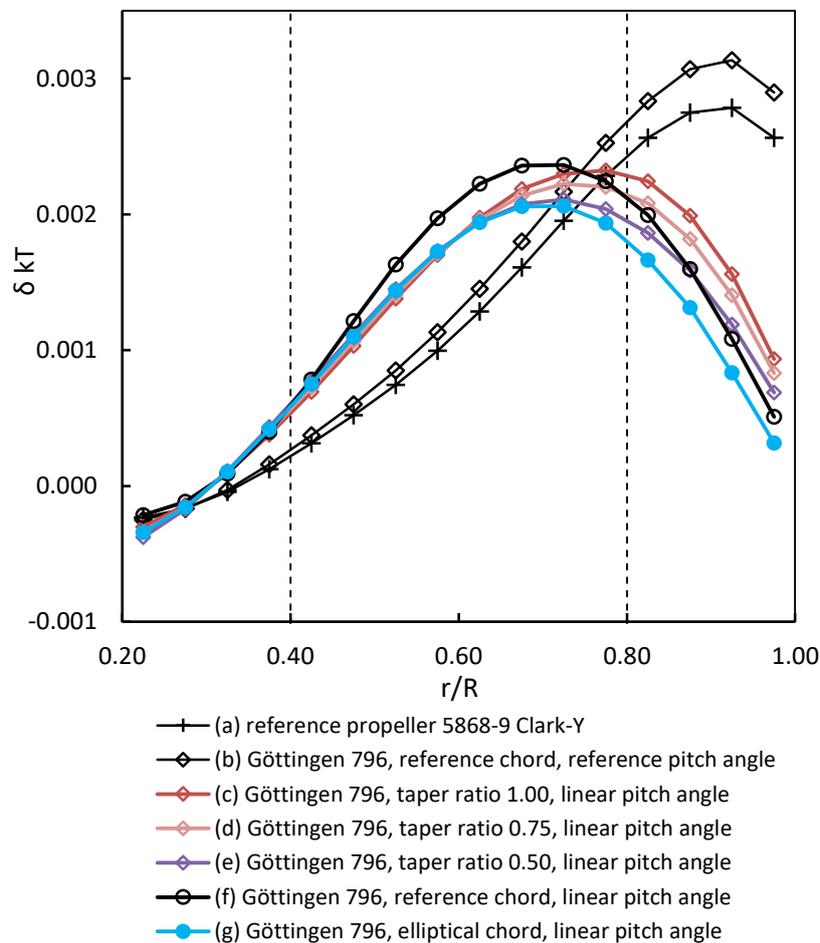


Figure 7. Comparison of the radial loadings along the blade of propellers with Göttingen 796 airfoil and different distributions of chord and pitch for advance ratio $J = 1.0$ compared with reference Clark Y 5868-9 propeller

Table 2. Performance characteristics and blade loadings of the propellers with Göttingen 796 airfoil and different chord and pitch angle distributions on the blade for $J = 1.0$ compared with reference Clark Y 5868-9 propeller.

Propeller ⁽¹⁾	Airfoil	Chord	Pitch angle	η	$T(N)$	$Q(Nm)$	Thrust loading of the blade (%)		
							root	middle	tip
(a)	reference Clark-Y	reference	reference	0.9050	1177	631	-1.59	48.41	53.18
(b)	Göttingen 796	reference	reference	0.9238	1324	695	-1.28	48.35	52.93
(c)	Göttingen 796	straight $\lambda=1.00$	linear	0.9154	1195	633	0.15	66.78	33.07
(d)	Göttingen 796	tapered $\lambda=0.75$	linear	0.9169	1151	609	0.07	68.61	31.32
(e)	Göttingen 796	tapered $\lambda=0.50$	linear	0.9182	1090	576	-0.02	71.31	28.71
(f)	Göttingen 796	reference	linear	0.9182	1183	625	0.79	73.47	25.74
(g)	Göttingen 796	elliptical	linear	0.9222	1008	530	0.13	75.84	24.03

⁽¹⁾ nomenclature according to Fig. 7

Figure 8 shows the predicted results of the thrust, power and torque coefficients as well as the efficiency for the propeller (b) with Göttingen 796 airfoil, reference distributions of chord and pitch angle on the blades and the results for the propeller (g) with Göttingen 796 airfoil, elliptical chord and linear pitch angle distributions on the blades. One can observe that propeller (g) shows lower coefficients of thrust, torque and power with nearly the same efficiency, than the initial proposed propeller (b). This result agrees with the values presented in Tab. 2. Hence, these changes in the geometry of the blade reduce the blade loading near the tip but also reduce the propeller thrust since the blade loading at the central part is not sufficiently big.

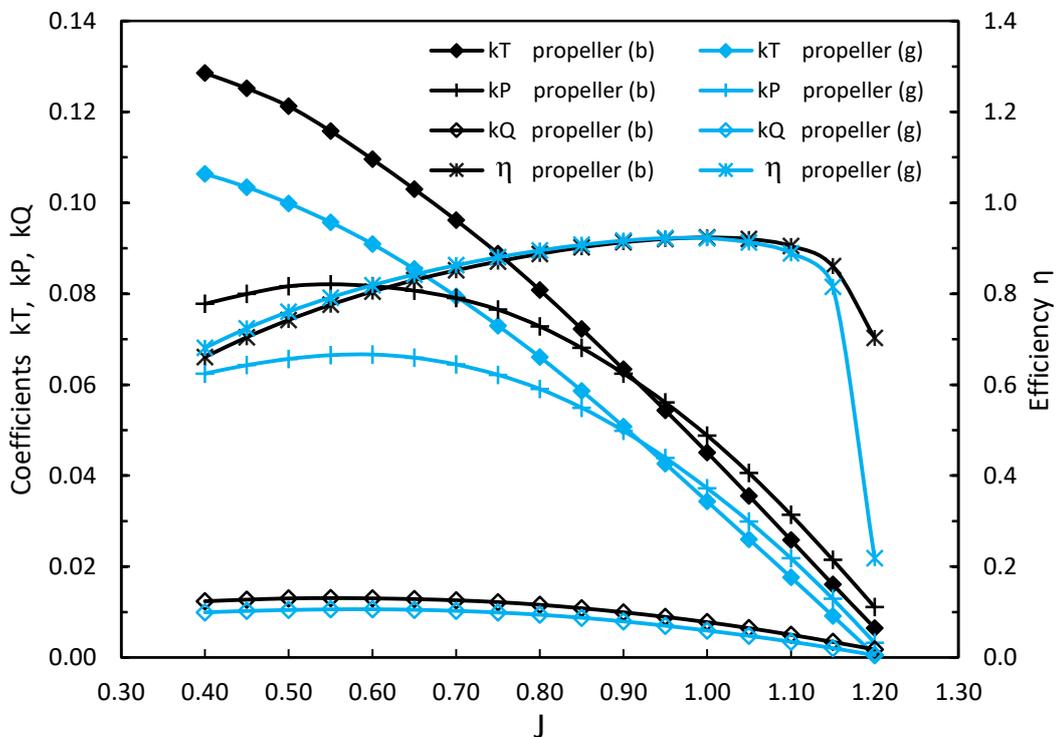


Figure 8. Comparison of the radial loadings along the blade of propellers (b) and (g) with airfoil Göttingen 796 and different distributions of chord and pitch (according to Tab. 2)

4. CONCLUSIONS

This study presents a procedure for propeller calculation based on momentum theory of actuator disc and blade element theory. From the geometry and aerodynamic characteristics of the blade sections it was possible to determine the performance parameters of the propeller including thrust loading, blade torque, thrust and power coefficients, and efficiency of the propeller as functions of the advance ratio. Predictions from the present method are in agreement with both experimental results and numerical calculations by the panel method. The proposed model is adequate for predicting the performance of small propellers with acceptable accuracy. The method is simple and consumes small CPU time of around 0.7 s.

The predicted results for the reference propeller Clark Y 5868-9 with 25° and 35° pitch were compared with experimental results and numerical results from panel method. The thrust and power coefficients predicted by the present method are close to the experimental results for intermediate advance ratios, while the predicted efficiency are close to the panels method predictions, except for low values of the advance ratio and 35° pitch. It is important to mention that the predictions from both methods are higher than the experimental results.

The loading along the blade of the reference propeller calculated by the present method follows the same tendencies of that calculated by the panel method but the present method shows higher values at the blade tip. Perhaps this is due to the fact that the panel method represents better the local geometry of the airfoil section than the present method.

For a propeller with tapered blade and linear distribution of the pitch angle the predicted results of the blade loading calculated by the present method agree well with the panel method results for the best advance ratio.

One can observe that the reduction of blade loading at the tip region and the increase of blade loading in the middle part of the blade can be obtained by means of linear distribution of the pitch angle or by tapered blade. One can also note that the use of the elliptical chord distribution instead of tapered blade accentuates these effects. However, the changes in the geometry of the blade reduce the blade loading near the tip but also reduce the coefficients of thrust, torque and power at nearly constant efficiency, in comparison with the propeller with the reference chord and pitch angle distributions.

5. ACKNOWLEDGEMENTS

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