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ON THE DAMAGE EVOLUTION OF REINFORCED CONCRETE I-BEAM BRIDGES DUE TO DYNAMIC VEHICLE-STRUCTURE INTERACTION

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Abstract. Reinforced concrete is used as structure system on highway bridges. The dynamic loads generated from the passage of vehicles on it can decrease the integrity of the concrete, as well as the reinforcement. For a more feasible analysis, the damage mechanics allows the representation of the concrete's crack propagation. It is adopted a constitutive nonlinear dynamic damage model based on the Mazars' damage model, which also regards the effects of the inversion of mechanical solicitations due to vibrations. Similarly, a more representative approach of the reinforcement is achieved by the account of the plasticity of the steel. Those phenomena enhance the model complexity, transforming it in a nonlinear dynamics problem, whose resolution is obtained by associating Newmark's method with Newton-Raphson's iterative technique. This works seeks to evaluate the nonlinear dynamic effects on a bridge's beam, modeled trough Finite Elements Method, using the ABXDNL 2.7 program. Additional routines were developed in C++ programming language in order to consider different cross sections, variable geometries, as well as non-constant distribution of the reinforcement.

Keywords: Bridge Computational Modelling, Nonlinear Dynamics, Damage Mechanics, Dynamic Interaction, Finite Element Method.

1. INTRODUCTION

With the recent changes in material's consumption concept, there has been a bigger concern about its resistance exploitation. Consequently, the structures became more slender and susceptible to nonlinear behavior, generating increases on the stresses (Machado, 1983).

The normative codes related to the structures project design usually suggest the increase in loads and the reduction of materials' strength, so that these structures are able to resist superior requirements than the really needed. Furthermore, for the reinforced concrete structures, the codes allow the consideration of a linear elastic behaviour of the materials in the structural analysis. For the bridge's design case, however, due to the dynamic loads generated by the passage of vehicles over it, the dynamic effects no longer can be neglected. For those cases, although, the codes accept that these effects are simplified, turning them into static loads amplified by a impact factor (Abeche, 2015).

Due to the concrete's complex behaviour, the formulation of a complete constitutive model becomes something difficult. Models have been elaborated based on the elasticity and plasticity theories, and more recently on the damage mechanics, each giving fair responses, provided that the studied situation afford a behaviour consistent with the proposed theory (Pituba, 1998).

The present work seeks to contribute for the nonlinear dynamics field, analyzing the interaction between vehicle and an I-beam cross section bridge, considering the damage mechanics and the plasticity theory for the materials behaviour. The analysis is made through the Finite Element Method, regarding the *Euler-Bernoulli*'s beam element.

2. *MAZARS'* DAMAGE MODEL

Among many constitutive models based on the damage mechanics and related to the concrete's behaviour, the *Mazars*' constitutive model (1984) presents an adequate representation of some experimental evidence, added the fact that it involves few parameters. When this constitutive model is applied to finite elements of *Euler-Bernoulli* beams, this model

uses a scalar variable that quantifies the local state of the material's deterioration.

This model has as fundamental hypothesis (Mazars, 1984):

– Locally, the damage is consequence of stretching;

– The damage is represented by a scalar variable, whose evolution occurs when a reference value for the "equivalent stretch" is surpassed;

– The damage is considered as isotropic, although some experimental analysis indicate an anisotropic behaviour of the concrete;

– The damaged concrete behaves as an elastic medium. Therefore, permanent strain in a unload situation is disregarded.

This work utilizes the damage model proposed by Abeche (2015), based on the Mazars (1984) damage model, that considers the dynamic inversion of mechanical stresses due to vibration effects. Due to the concrete's asymmetric behavior, Mazars defines two damage variables: D_T for tensile damage and D_C for compressive damage, as

$$D_T(\widetilde{\varepsilon}) = 1 - \frac{\varepsilon_{d0}(1 - A_T)}{\widetilde{\varepsilon}} - \frac{A_T}{e^{B_T(\widetilde{\varepsilon} - \varepsilon_{d0})}}, \tag{1}$$

$$D_C(\widetilde{\varepsilon}) = 1 - \frac{\varepsilon_{d0}(1 - A_C)}{\widetilde{\varepsilon}} - \frac{A_C}{e^{B_C(\widetilde{\varepsilon} - \varepsilon_{d0})}}. \tag{2}$$

where the parameters A_T, B_T, A_C and B_C are obtained through experimental tests, ε_{d0} is the parameter of the limit elastic deformation, as shown in the Fig. 1, and $\widetilde{\varepsilon}$ is the equivalent strain. The parameters ranges are (Mazars, 1984)

$$0.7 \le A_T \le 1; \qquad 10^4 \le B_T \le 10^5; \qquad 1 \le A_C \le 1.5; \qquad 10^3 \le B_C \le 2 \cdot 10^3; \qquad 10^{-5} \le \varepsilon_{d0} \le 10^{-4}. \tag{3}$$
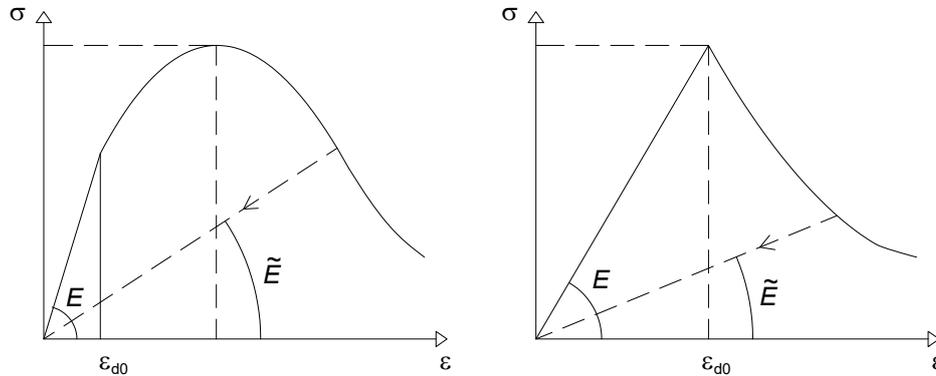

Figure 1: Mazars damage model for the compressive and tensile cases. Adapted from Abeche (2015).

The equivalent strain id given by

$$\widetilde{\varepsilon} = \begin{cases} \varepsilon, & if \ \varepsilon \ge 0 \\ -\nu\sqrt{2\varepsilon}, & otherwise \end{cases} \tag{4}$$

where ν is the Poisson's ratio of the concrete.

The damaged material *Young* modulus ($\tilde{E}$), for a continuous medium, is defined as

$$\tilde{E} = (1 - D)E, \tag{5}$$

where D is the damage variable, which can be obtained throug the linear combination

$$D(\varepsilon) = \alpha_T D_T + \alpha_C D_C, \tag{6}$$

in which α_T and α_C are coefficients obtained through the stain state of each point of the solid, regarding that $\alpha_T + \alpha_C = 1$. For the case of pure tensile stress, it is considered $\alpha_T = 1$. For pure compression stress, $\alpha_C = 1$.

3. PLASTICITY THEORY

It is adopted a bilinear elastoplastic model that considers the strain hardening by plastic deformation, with the same behavior when subjected to traction and compression. The model is presented on Fig. 2.
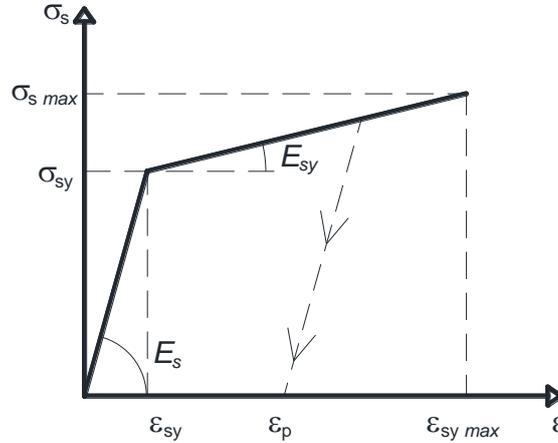
Figure 2: Steel's constitutive model. Adapted from Abeche *et al.* (2016c).

where σ_{sy} is the yield stress, E_s is the initial Young modulus, ε_{sy} is the yield extension and E_{sy} is the Young modulus after the yield of the steel bars (Abeche *et al.*, 2016a).

The steel can suffer plasticity effect upon reaching the yield stress σ_{sy}. Therefore, the material will have a plastic strain ε_p that can be between $+\varepsilon_{sy} \leq \varepsilon_p \leq +\varepsilon_{sy\ max}$, for a traction case, or $-\varepsilon_{sy} \leq \varepsilon_p \leq -\varepsilon_{sy\ max}$, for a compression case. The stress of the constitutive model, can be calculated as

$$|\sigma| = \begin{cases} |\sigma_{sy}| + E_{sy}(\varepsilon_p - |\varepsilon_{sy}|), & |\pm\varepsilon_{sy}| \leq \varepsilon_p \leq |\varepsilon_{sy\,max}| \\ E_s|\varepsilon_{sy}|, & -\varepsilon_{sy} \leq \varepsilon \leq \varepsilon_{sy} \end{cases} \tag{7}$$

4. EQUIVALENT STIFFNESS FOR DIFFERENT CROSS SECTIONS

In order to consider the beam's cross section, it is used the equivalent stiffness method (Beer and Johnston, 2008). Thereby, it is possible to divide the section into layers and apply the damage for the concrete's case, and the plastification calculations, for the steel's case, in order to modify each layer's stiffness.

As laminated composite beams, the cross section of the bridge's elements is divided into n layers, as shown in Fig. 3.
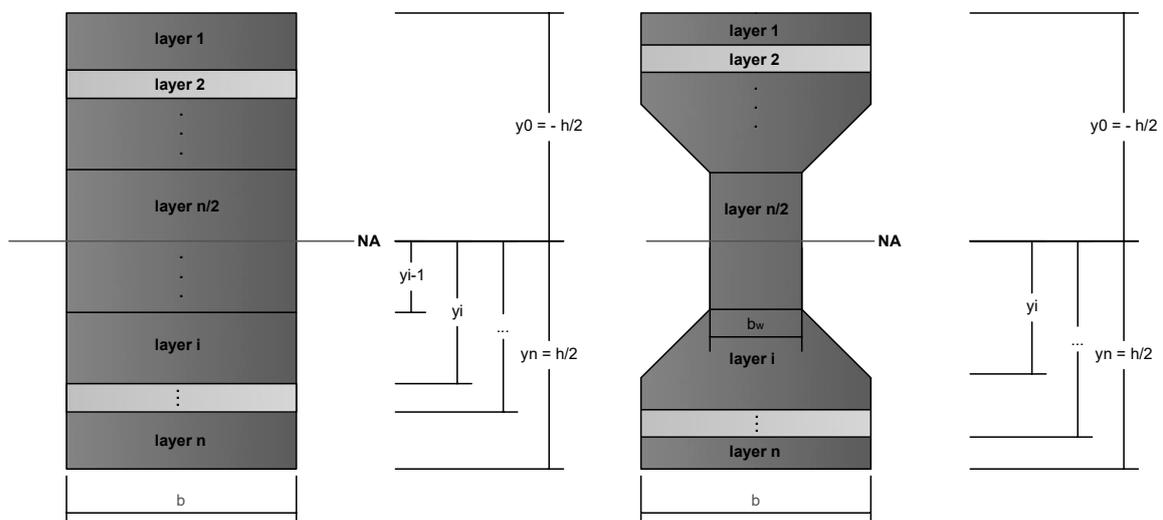
Figure 3: Rectangular and I-shape beam's cross section divided into n layers

For the particular case of constant width, the equivalent stiffness EI_{eqv} is determined by

$$EI_{eqv} = \frac{1}{3}b\sum_{i=1}^{n}E_i(y_i^3 - y_{i-1}^3) \tag{8}$$

where n is the number of layers, b its constant width, E_i is the *Young* modulus of the ith layer, for different materials. The y_i and y_{i-1} are the y axis coordinate values of the division points (Abeche, 2015).

For the I-beam cross section, adjustments are made in Eq. (8) in order to consider the different b for each layer.

5. NONLINEAR DYNAMIC STRUCTURAL ANALYSIS BY DAMAGE

Many researchers developed different methods for solving nonlinear problems, among which are noteworthy the works of Machado (1983) and Abeche (2015) for dynamic physical nonlinearities caused by damage. In this paper, the numerical method adopted follows the computational and mathematical procedures proposed by Abeche (2015), which consists in solving the global equation of motion iteratively and incrementally by combining the *Newton-Raphson*'s technique with the *Newmark*'s method.

For the case of material deterioration, the forces are no longer linearly dependent of the displacements. This model considers that the damage directly affects the stiffness of the system and consequently the structural damping (Abeche *et al.*, 2016b). The stiffness matrix, thus, becomes

$$[K_B] = [K_B(\{u_B(\overrightarrow{x},t)\})], \quad EI = f(\overrightarrow{x},t). \tag{9}$$

It can be noted that the stiffness matrix becomes dependent of the displacements which are a functional of time and position vector.

The damping matrix, according to the *Rayleigh*'s method, can be written as

$$[C_B(\{u_B(\overrightarrow{x},t)\})]_{t+\Delta t}^{(i-1)} = (\tilde{\alpha}_B)_{t+\Delta t}^{(i-1)}[M_B]_{t+\Delta t}^{(i-1)} + (\tilde{\beta}_B)_{t+\Delta t}^{(i-1)}[K_B(\{u_B(\overrightarrow{x},t)\})]_{t+\Delta t}^{(i-1)} \tag{10}$$

where $\tilde{\alpha}_B$ and $\tilde{\beta}_B$ are *Rayleigh*'s coefficients for calculating the damping matrix, $t + \Delta t$ is the time increment and $i - 1$ is the iteration step.

The damping matrix, due to its relation to the stiffness matrix, also becomes dependent of the time and the position vector.

The nonlinear dynamic equation, for a damaged beam, is shown in the Eq. (11).

$$\left(\frac{[M_B]_{t+\Delta t}^{(i-1)}}{\beta\Delta t^2} + \frac{\gamma[C_B]_{t+\Delta t}^{(i-1)}}{\beta\Delta t}\right)\{u_B\}_{t+\Delta t}^{(i)} + [K_B]_{t+\Delta t}^{(i-1)}\{\Delta u\}^{(i)} = \{F_B^{ext}\}_{t+\Delta t} + \frac{[M_B]_{t+\Delta t}^{(i-1)}}{\beta\Delta t^2}(\{u_B\}_t + \{\dot{u}_B\}_t\Delta t$$
$$+ (0,5-\beta)\{\ddot{u}_B\}_t\Delta t^2)^{(i)} + [C_B]_{t+\Delta t}^{(i-1)}(\frac{\gamma}{\beta\Delta t} + \left(\frac{\gamma}{\beta} - 1\right)\{\dot{u}_B\}_t + \left(\frac{\gamma}{2\beta} - 1\right)\{\ddot{u}_B\}_t\Delta t)^{(i)} - \{F_B^{int}\}_{t+\Delta t}^{(i-1)} \tag{11}$$

where β and γ are the *Newmark* method parameters determined for the numerical integration accuracy and stability, i is the present iteration, $\{F_B^{ext}\}_{t+\Delta t}$ the external forces vector, $\{F_B^{int}\}_{t+\Delta t}$ the internal forces vector and $\{\Delta u\}$ the incremental displacements vector. $[M_B]$, $[C_B]$, $[K_B]$ are the global matrices of mass, damping and stiffness, $\{u_B\}$, $\{\dot{u}_B\}$ and $\{\ddot{u}_B\}$ are the global vectors of displacements, velocities and accelerations, respectively.

It is used the ABXDNL 2.7 program (Abeche, 2015) for solving the Eq. (11), considering the damage mechanics and the plasticity theory.

6. RESULTS AND DISCUSSION

The vehicle-bridge system to be compared within this work is a single degree of freedom vehicle passing through a rectangular cross-section highway bridge, without irregularities, with constant reinforcement distribution, adapted from T. de O. Abeche (2016). For this paper, the vehicle's velocity is considered constant and three cases of suspended mass (m_2) are simulated, in order to observe different degrees of damaging on the bridge. The system is shown in the Fig. 4.

As results, it is presented the damage evolution and the dynamic responses for the cases of bridges with rectangular and I-shape cross sections, under the same conditions.

The parameters adopted are presented in the Tab. 1.

For both the rectangular and I-shape cross section cases, the height (h) and the initial EI_{eqv} are equal. The cross section measurements are presented in the Fig. 5.

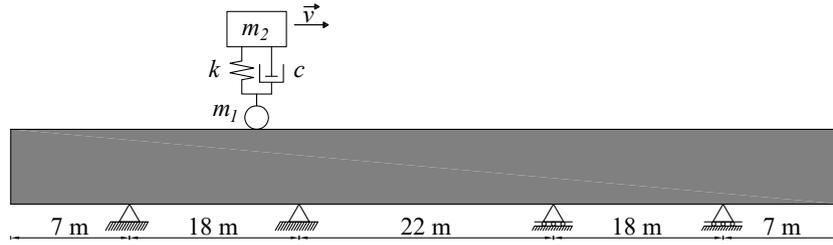The bridge's parameters are presented in the Tab. 2.

Figure 4: Vehicle-Bridge model. Adapted from (T. de O. Abeche, 2016)

Table 1: Vehicle, *Newmark*'s Method, Damage and Plasticity input data

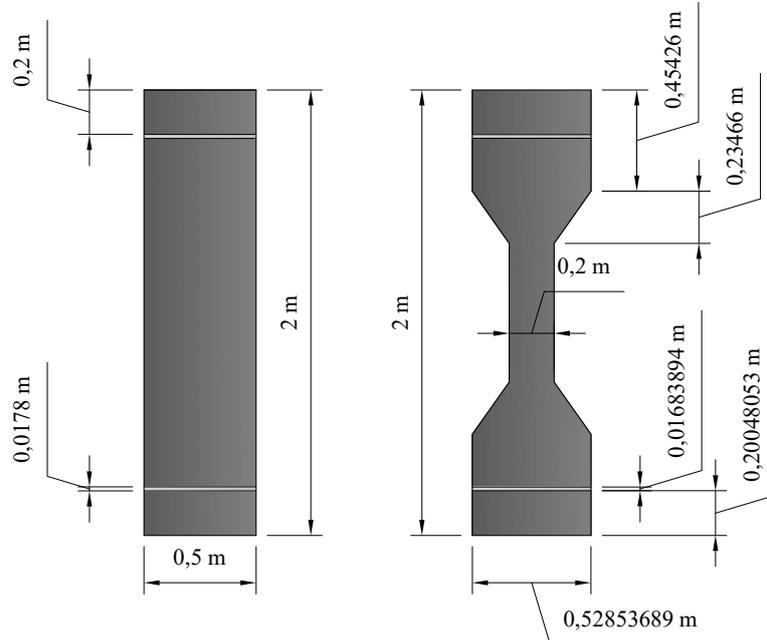| Vehicle	*Newmark*'s parameters	Damage parameters	Plasticity parameters
$m_1 = 3,000$ kgf	$\gamma = 0,5$	$A_T = 0,995$	$E_s = 210\ GPa$
$k = 9,120$ kgf	$\beta = 0,25$	$B_T = 30.000$	$\nu_s = 0,3$
$c = 86$ kNs/m	$10,200$ Time steps	$A_C = 1,2$	$\varepsilon_{sy} = 10\ \%_0$
$v = 30$ km/h	$dt = 0,0012$ s	$B_C = 1.050$	-
$7,200$ Time steps	$\zeta = 0,025$	$\varepsilon_{d0} = 5 \cdot 10^{-5}$	-

Figure 5: Rectangular and I-shape cross section measurements

Table 2: Bridge input data

Parameters	L_B (m)	b (m)	h (m)	I (m^4)	E_C (GPa)	ν_C	ρ (%)	m (kg/m)	Elements	Layers
Rectangular Beam	72	0.5	2.0	0.3333	29.43	0.2	1.80977	2,496	72	42
I-Beam		Variable[1]					2.35265	1,955		

[1] measurements in Fig. 5

In this section are presented the results for the rectangular and I-shape beams, considering the load cases of $m_2 = 12,000$ kgf, $m_2 = 15,000$ kgf and $m_2 = 19,000$ kgf.

6.1 Results for m_2 = 12,000 kgf

The final damaged configuration for the load case of $m_2 = 12,000$ kgf is presented in the Fig. 6.
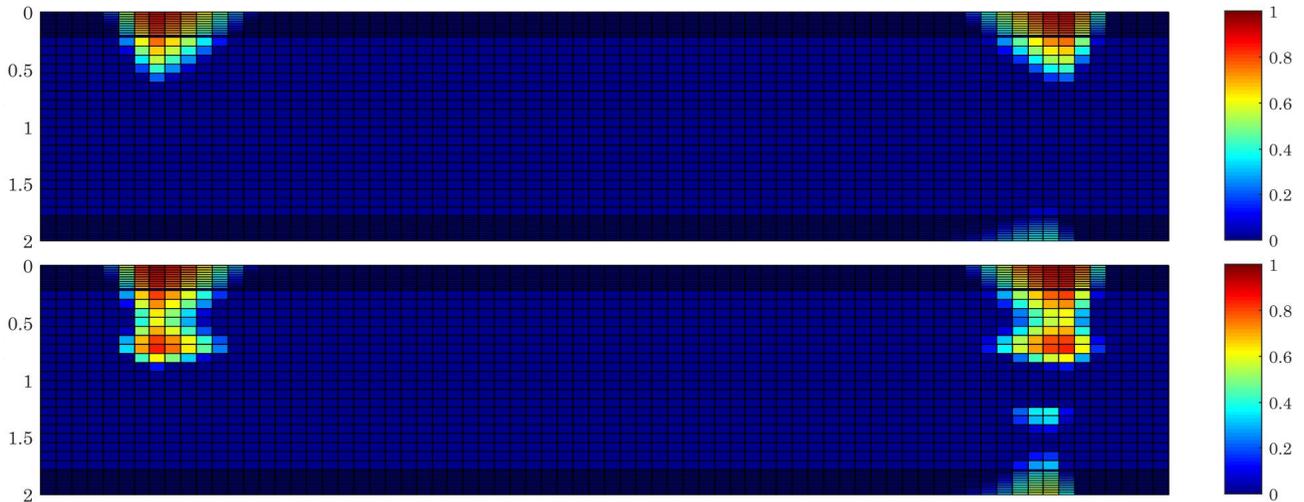

Figure 6: Final damaged configuration of the (a) rectangular and (b) I-shape beams for $m_2 = 12,000$ kgf

It can be noted that when the damage reaches the I-beam's web, it spreads more through the elements and layers aside than the rectangular case. The damage in the flange section remains similar to the rectangular case.

6.2 Results for m_2 = 15,000 kgf

For the load case of $m_2 = 15,000$ kgf, the damaged configuration for the rectangular and the I-shape cross sections become as presented in the Fig. 7.
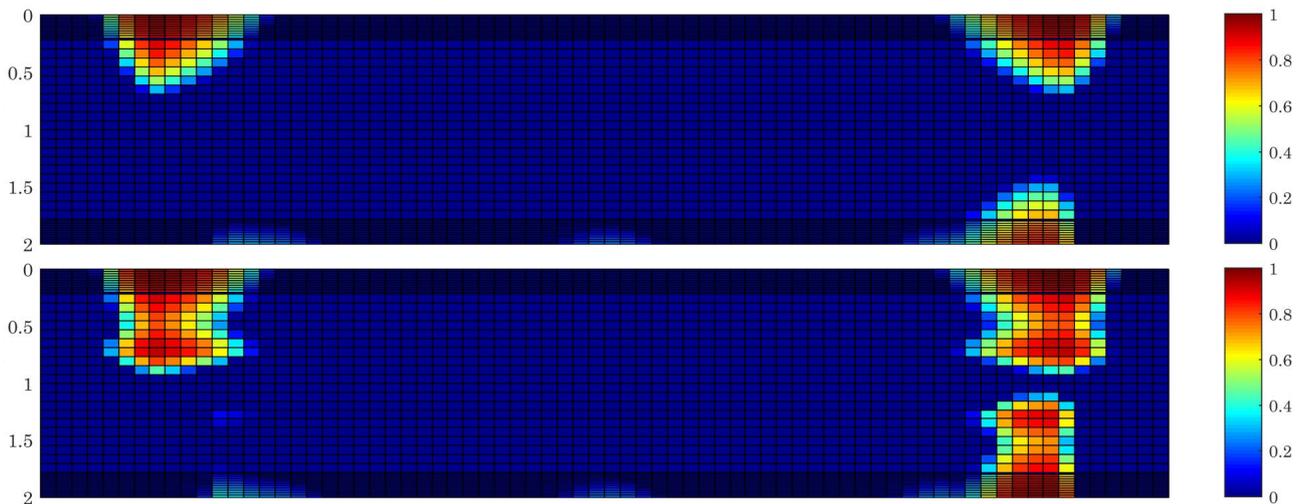

Figure 7: Final damaged configuration of the (a) rectangular and (b) I-shape beams for $m_2 = 15,000$ kgf

For this case, the damaged region in the I-beam's web becomes more evident, and is also noticeable that the damage in this region occurs even when the outermost layers are under small degree of damage.

6.3 Results for m_2 = 19,000 kgf

From the analysis which $m_2 = 19,000$ kgf, the final damaged configuration is shown in the Fig. 8.

The flange areas remain the same in the rectangular and in the I-shape beams. The web region, however, presents a very accentuated difference between the rectangular and the I-beam cases. In order to exemplify this behaviour, the Fig. 9 shows the final damaged configuration of three different elements considering both cross sections.
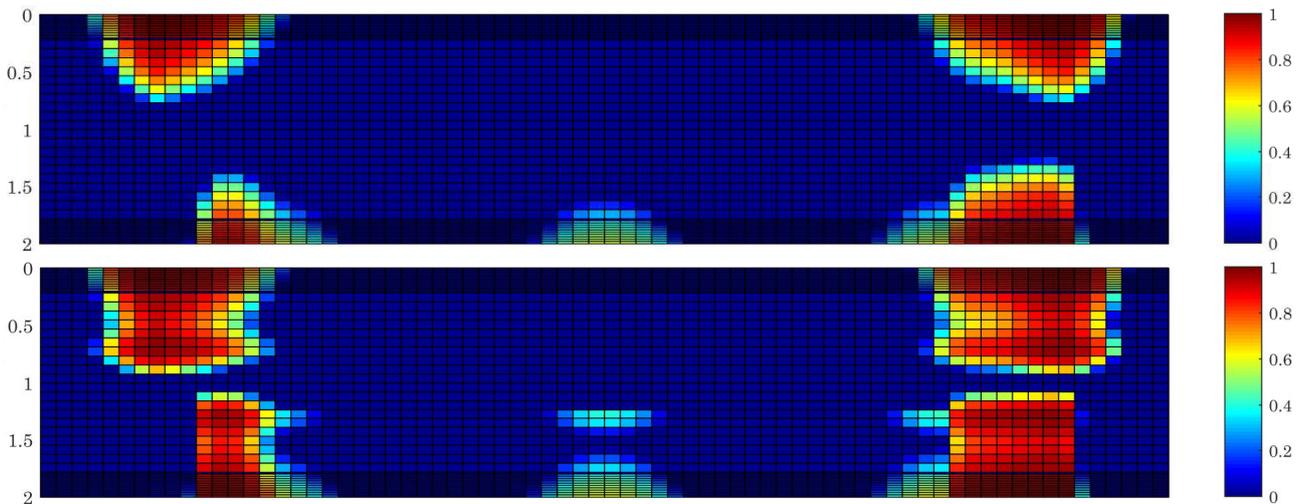
Figure 8: Final damaged configuration of the (a) rectangular and (b) I-shape beams for $m_2 = 19{,}000$ kgf

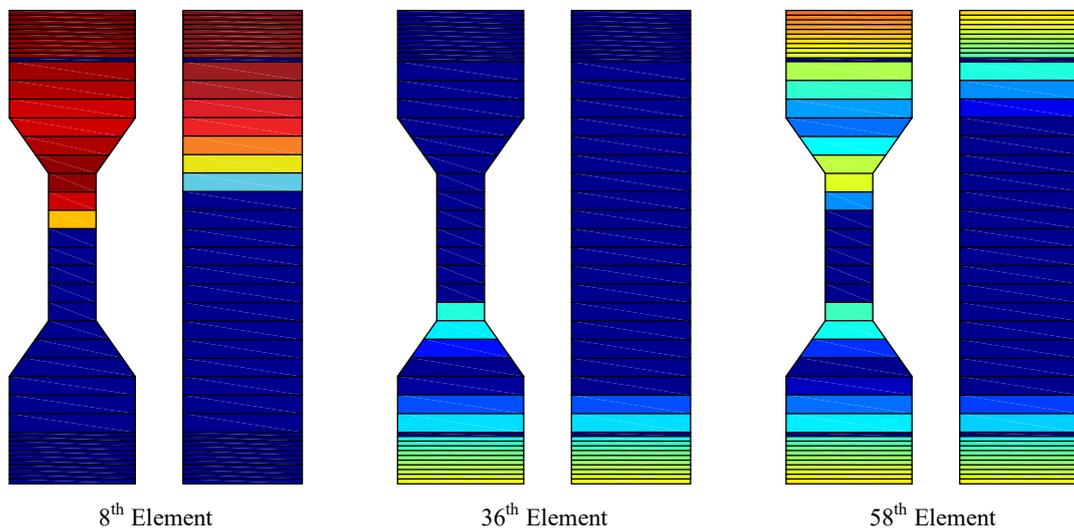8th Element   36th Element   58th Element

Figure 9: Final damaged configuration for the 8th, 36th and 58th elements in I-beam ad rectangular cross sections

The Fig. 10 shows the responses of displacement of every degree of freedom considering both cross sections and the linear and nonlinear dynamic cases.

It can be noted that the rectangular and the I-shape cross section beams present the same linear dynamic response, once the stiffness is the same.

The nonlinear dynamic response of displacement for both cross sections presents differences between them. The I-shape case present responses with larger amplitudes when compared to the rectangular case.

The Fig. 11 shows the linear and the nonlinear responses of displacement fo the mid-span node.

It is noticeable the difference between the three responses, as the I-shape cross section presents the larger displacements.
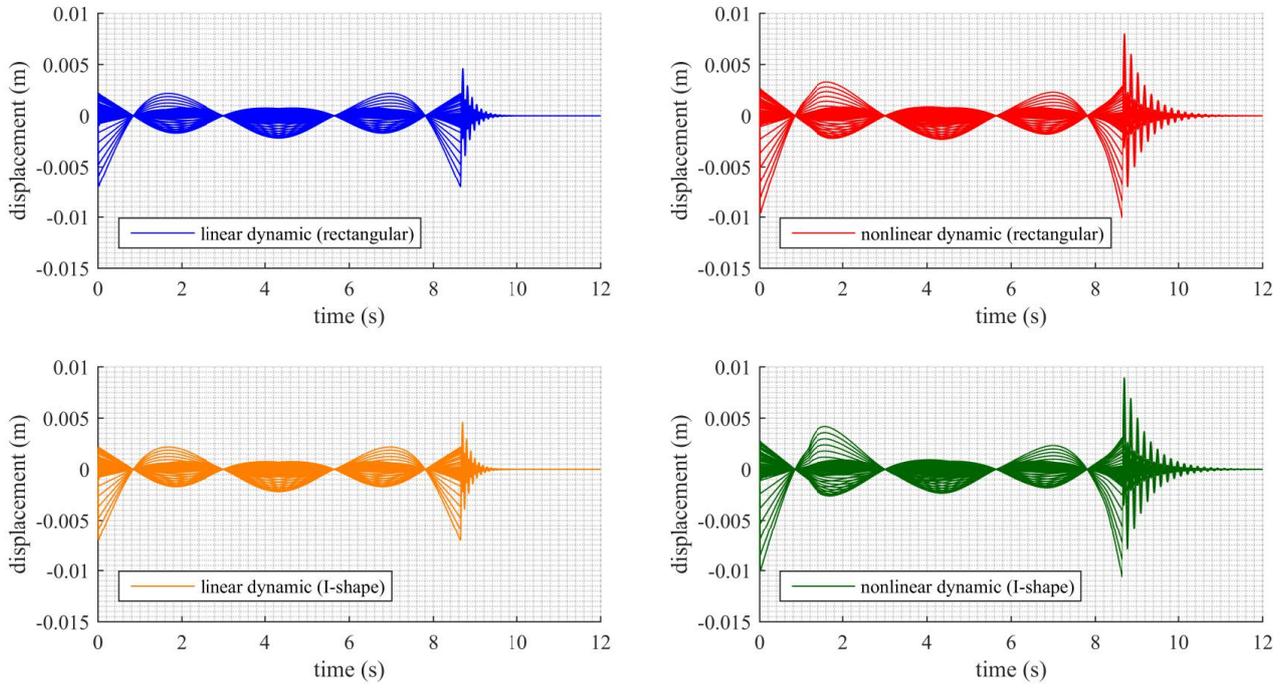
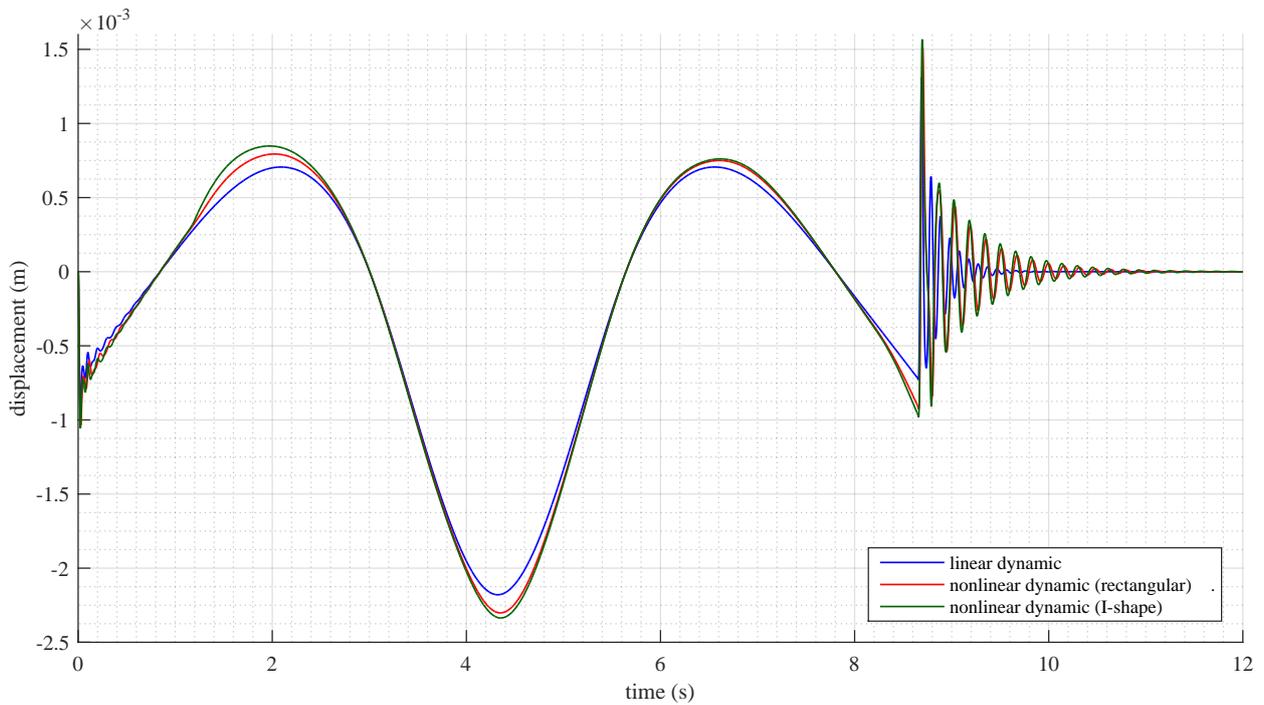Figure 10: Dynamic responses of displacement

Figure 11: Dynamic responses of displacement for the middle span node

7. CONCLUSION

With the analysis presented in this work, it is possible to notice the influence of the beam's geometry in its damage evolution and, consequently, in the dynamic responses of the bridge.

Although the rectangular and the I-shape beams have the same initial stiffness and height, the integrity loss of the first one is more accentuated than the rectangular.

8. ACKNOWLEDGEMENTS

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