24th COBEM - 2017

24th ABCM International Congress of Mechanical Engineering
December 3-8, 2017, Curitiba, PR, Brazil

COBEM-2017-2772
FRACTIONAL PI^{Λ} (PD^{M}) CONTROLLER DESIGN FOR A CMG STABILIZING SYSTEM FOR TWO-WHEELED VEHICLES

Paulo Roberto Oliveira Martins
Gustavo Cunha da Silva Neto
Danilo de Santana Chui
Junout Martins Louzada Neto
Federal University of Amazonas, Faculty of Technology, Department of Mechanical Engineering, General Rodrigo Octávio Jordão Ramos Ave, 6200, Coroado I, Senator Arthur Virgílio Filho Campus – North Sector, 69077-000 Manaus, AM, Brazil.
pmartins@ufam.edu.br
gustavoneto@ufam.edu.br
danilochui@ufam.edu.br
junout@ufam.edu.br

Abstract. *Road traffic injuries are one of the leading causes of death in the world. The most frequent victims are pedestrians, cyclists and motorcyclists. It's worthy of attention design safer vehicles as a mitigating measure against these fatalities. Besides, stabilize two-wheeled vehicles is a challenging task, and it has been drawing the attention of the control research community for years. This work aims to concatenate the study of a fractional control and a stabilizing system. It's proposed to adopt a control moment gyroscope (CMG) stabilizing system with the purpose of balancing the vehicle in the upright position, even if it undergoes side impact forces. The model of the actuator was obtained using the Lagrangian mechanics, and then it was linearized around the balance position. It was used a generalization of the proportional-integral-derivative controller (PID) associated with the fractional calculus theory to design a* $PI^{\lambda}(PD^{\mu})$ *type controller for the stabilizing system's control. Two different systems were simulated to demonstrate the actuator's model applicability, one with bicycle parameters and another with motorcycle ones. The performance of classical and fractional PID were compared and the robustness of the* $PI^{\lambda}(PD^{\mu})$ *controller was evaluated for gain variations, that is, how the step response velocity of the stabilizing system changes, given a variation in the angular setpoint, maintaining the constant angular overshoot tolerance condition. The stabilizing system proposed has shown to be a promising technological solution for design safer vehicles.*

Keywords: *Two-wheeled vehicles stabilization, Lagrangian modelling, Fractional PID Controllers, Fractional-Order Control, PID Controller.*

1. INTRODUCTION

Road traffic injures have become a worldwide public health issue being responsible for the death of nearly 1.3 million of people every year around the world, and about a half of them are pedestrians, cyclists or motorcyclists (WHO, 2011).

In Brazil, accidents involving motorcycles lead the statistics (DPVAT, 2015). Besides, motorcycles are unstable in low speed and when they undergo collisions causing the vehicle to fall contributes to aggravate motorcyclist's injuries. Thus, design a stabilizing system that avoids the unbalancing of motorcycles even when they are completely stopped would be a way to reduce these fatalities.

There are a few stabilizing systems for two wheeled vehicles, but the Control Moment Gyroscope (CMG) actuated one's, as represented in Fig.1, combine torque amplification output and fast response, being be the most suitable for this application (KIM and BRETNEY, 2014; LAM, 2012; MARTINS, 2016).

In the last decades, studies regarding the association of fractional calculus and control theory, known as fractional control, has shown to be a promising approach because they are able to describe real phenomena better and achieve greater robustness of the control systems.

A specific case of fractional controllers is the extent of classic PID ones. They were firstly presented by Igor Podlubny, in 1994, as $PI^{\lambda}D^{\mu}$, (PODLUBNY, 1994; SHAH, 2016), and focus on combine the PID simplicity (ÅSTRÖM, 1995; NISE, 2011; OGATA, 2010) to the robustness and flexibility of fractional theory of control.

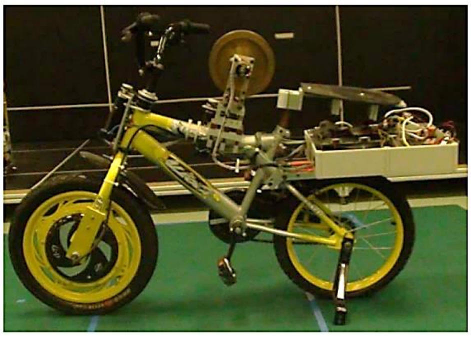

Figure 1. Bicycle with CMG

This work aims to model and control a Control Moment Gyroscope stabilizing system for two-wheeled vehicles in terms of the rolling angle with respect to the upright position using a $PI^{\lambda}(PD^{\mu})$ controller and demonstrates its better performance in comparison with classic PIDs controllers.

2. FRACTIONAL PID CONTROL

The generalized transfer function of a fractional PID controller can be represented by:

$$C(s) = P + \frac{I}{s^{\lambda}} + Ds^{\mu} \tag{1}$$

Fractional PID controllers are a generalization of classical PID, then, instead of moving along four fixed points in the plane $[\mu,\lambda]$, as shown in Fig.2 , it is possible to select any real values continuously in the quarter plane of λ and μ.

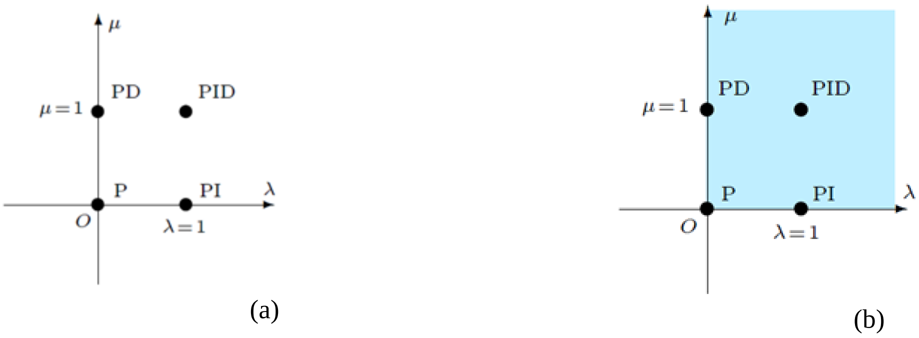

Figure 2. Fractional order PID *vs* classical PID: from points to plane: (a) integer- and (b) fractional-order

Moreover, $PI^{\lambda}D^{\mu}$ controllers require five parameters to be completely tuned, they also allow to accomplish five design specifications. Because of that this type of controllers, when compared to classical one, can better address high-order systems or system with non-linearities, being also possible to achieve a desired response for different points of operation of a linearized model by a single fractional controller. Additionally, those controllers are less sensible to gain variations easily attaining the isodamping condition.

2.1 Fractional PID Behavior

According to Valerio and Costa (2013), and is represented in Fig. 3, a fractional PID behaves like $\frac{I}{s^{\lambda}}$,at low frequencies, and like Ds^{μ}, at high frequencies. Furthermore, if P is larger enough than I and D, there will be an intermediate range where it behaves like P.

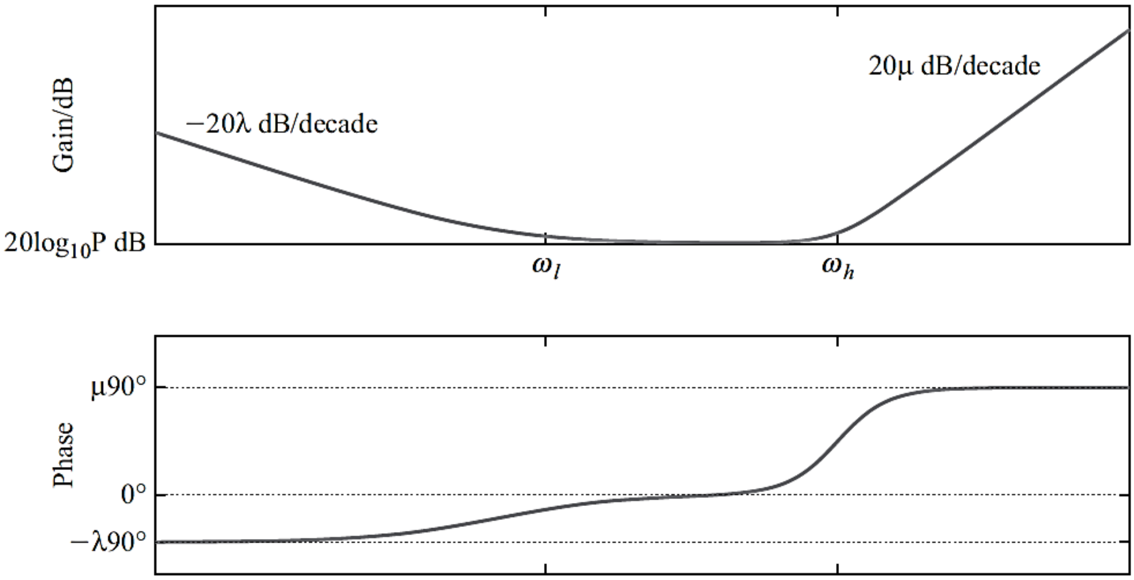

Figure 3. Frequency behavior of a $PI^{\lambda}D^{\mu}$ controller

2.2 Design Procedure for PD^{μ} or PI^{λ} controllers

Fractional PD or PI controller have three parameters to be tuned, so three different design requirements can be achieved, being, in this work, two of them related to a stability condition of an interdependent desired phase margin at a specified gain-crossover frequency (NISE, 2011).

The third criterion is determined, so that, it ensures the robustness of the controller regardless gain variations. According to Chen (2003) apud Valerio and Costa (2013) this condition is achieved by making the phase of the system be constant at a range close to the gain-crossover frequency. Thus, even though, there may be gain variations or uncertainties in the gain-crossover frequency, the phase margin will remain the same, once the phase has a constant behavior.

$$\pi+\angle\left[C\left(j\omega_{cg}\right)G\left(j\omega_{cg}\right)\right]=\phi_m \quad (2.1)$$

$$\left|C\left(j\omega_{cg}\right)G\left(j\omega_{cg}\right)\right|=1 \quad (2.2)$$

$$\left.\frac{d\angle\left[C\left(j\omega\right)G\left(j\omega\right)\right]}{d\omega}\right|_{\omega=\omega_{cg}}=0 \quad (2.3)$$

To simplify solving the equation system, Monje et al. (2010) suggests rewrite the controller transfer function as follow:

$$C\left(s\right)=K_p\left(1+K_d s^{\mu}\right) \quad (3)$$

Therefore, from Eq. 2.1 its obtained that, for a given phase margin and gain-crossover frequency, in Eq. 4, the parameter K_d is a function of μ only.

$$K_d=\frac{\tan\left(-\pi+\phi_m-\angle\left[G\left(j\omega_{cg}\right)\right]\right)}{\omega_{cg}^{\mu}\text{sen}\left(\frac{\mu\pi}{2}\right)-\omega_{cg}^{\mu}\cos\left(\frac{\mu\pi}{2}\right)\tan\left(-\pi+\phi_m-\angle\left[G\left(j\omega_{cg}\right)\right]\right)} \quad (4)$$

At the same manner, from Eq. 2.3:

$$K_d = \frac{-Q \pm \sqrt{Q^2 - 4P^2\omega_{cg}^2}}{2P^2\omega_{cg}^{2\mu}} \tag{5}$$

where $P = \left.\frac{d\left[\angle G(j\omega)\right]}{d\omega}\right|_{\omega=\omega_{cg}}$ and $Q = \mu\omega^{\mu-1}\text{sen}\left(\frac{\mu\pi}{2}\right) + 2\omega^{\mu}\cos\left(\frac{\mu\pi}{2}\right)\cdot\left.\frac{d\left[\angle G(j\omega)\right]}{d\omega}\right|_{\omega=\omega_{cg}}$.

Again, given the gain cross-over frequency ω_{cg} ,from Eq. 5, K_d is also a function of μ only.

Finally, for the second design specification of the controller, that is Eq. 2.2, it follows that:

$$K_p = \frac{1}{\left|G(j\omega_{cg})\right|\sqrt{\left(1+K_d\omega^{\mu}\cos\left(\frac{\mu\pi}{2}\right)\right)^2 + \left(K_d\omega^{\mu}\text{sen}\left(\frac{\mu\pi}{2}\right)\right)^2}} \tag{6}$$

From the system of two equations, namely Eq. 4 and Eq. 5, which relate the value of K_d functions of μ, by plotting them and finding the concurrent point of the graphs, that is, the pair of value that satisfy both equations these two parameters are tuned. Then, we find the value of K_p plying the design value of ω_{cg}, K_d and μ, found in the previous step, in Eq. 6. Thus, all three design parameters of the PD$^{\mu}$ controller are specified given conditions Eq. 2.1, Eq. 2.2 and Eq. 2.3. It is observed that it is still possible to design a PI$^{\lambda}$ controller using the same methodology, but including the condition that $\lambda = -\mu$.

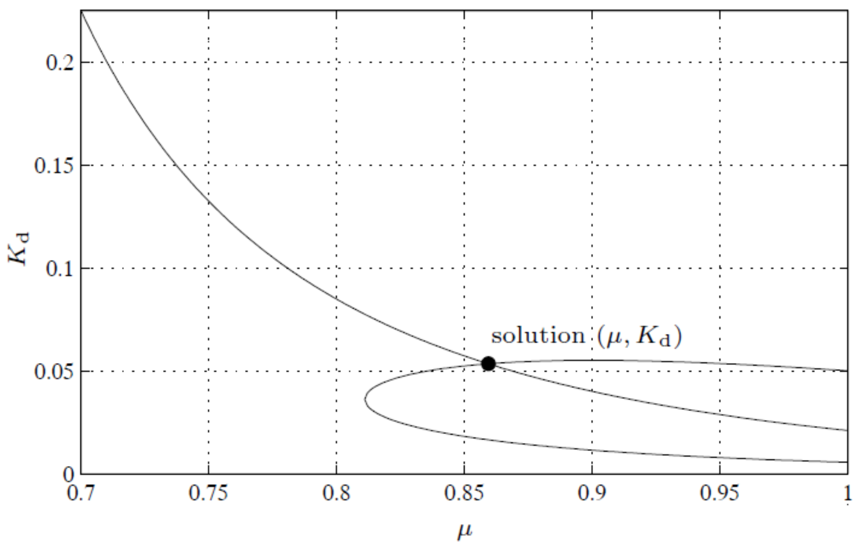

Figure 4. Graphs for the functions $K_d(\mu)$ from Eq. 4 and Eq. 5.

2.3 CRONE Approximation

Despite designing fractional controller, usually integer-order transfer functions that approximate the fractional behavior are adopted to take advantage of software and numerical methods for integer-order functions only.

The CRONE (Commande Robuste d'Ordre Non-Entier) approximation is the most commonly used for s^{α} ,$\alpha \notin \mathbb{Z}$. It makes use of N stable real poles and N stable real zeros, being valid for the interval of frequencies of $[\omega_l, \omega_h]$.

The superposition of the effect of each pole and zero results in a phase oscillating around $\alpha \cdot 90°$ and gain with a slope of $\alpha \cdot 20$ dB/decade. Therefore, the approximated fractional controller can be interpreted as a higher order filter.

3. STABILIZING SYSTEM MODEL

The CMG stabilizing system was modelled based on the simplification of two-wheeled vehicle as an inverted pendulum with a CMG coupled to it using similar methodology found in (LAM,2012).

The model's equations were obtained using the Lagrangian equation of motion, which consist of an alternative formulation to Newtonian mechanics, broadly used for complex systems of rigid bodies.

$$\frac{\partial L}{\partial q_i} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} = Q_i^{'} \quad (7)$$

where L is the Lagrangian function which is the difference between the kinetic and potential energy of the system, respectively, and $Q_i^{'}$ represents the net generalized constraint force.

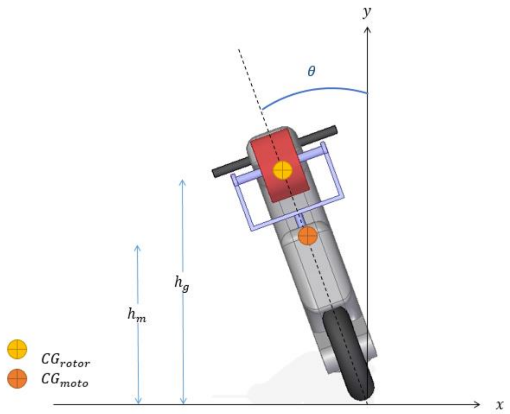

Figure 5. Model's diagram

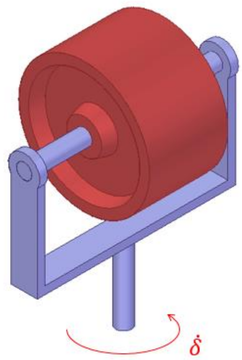

Figure 6. CMG diagram

$$\ddot{\theta}\left[m_m h_m^2 + m_g h_g^2 + I_m + I_p \text{sen}^2\delta + I_r \cos^2\delta\right] + 2\text{sen}\delta\cos\delta\left(I_p - I_r\right)\dot{\theta}\dot{\delta} - g\left(m_m h_m + m_g h_g\right)\text{sen}\theta = I_p\omega\dot{\delta}\cos\delta + M(t) \quad (8.1)$$

$$\ddot{\delta}I_r - \dot{\theta}^2\left(I_p - I_r\right)\text{sen}\delta\cos\delta = NK_{motor}i - I_p\omega\dot{\theta}\cos\delta - B_{motor}\dot{\delta} \quad (8.2)$$

$$U = L\frac{di}{dt} + Ri + K_e\dot{\delta} \quad (8.3)$$

Then, a linearization around the equilibrium position ($\delta = 0°$ and $\theta = 90°$) was proceeded, and the transfer function of the linearized system relating the output upright position angle with the input electric tension was found. Two different sets of parameters were simulated to demonstrate the actuator's model applicability, one with bicycle parameters and another with motorcycle ones.

The control loop used is represented in the Figure 4. The angular reference $\theta(s)_{ref}$ is set to the balance position. The feedback loop error is then converted, by the K gain, to a voltage $U(s)$ that will be the input for the controller $C(s)$, which will be modulated according to the fractional PID strategy control to regulate the system around the balance position. The disturbance $D_T(s)$ represents a side impact. The extra $H(s)$ loop represents a feedback of the angular speed.

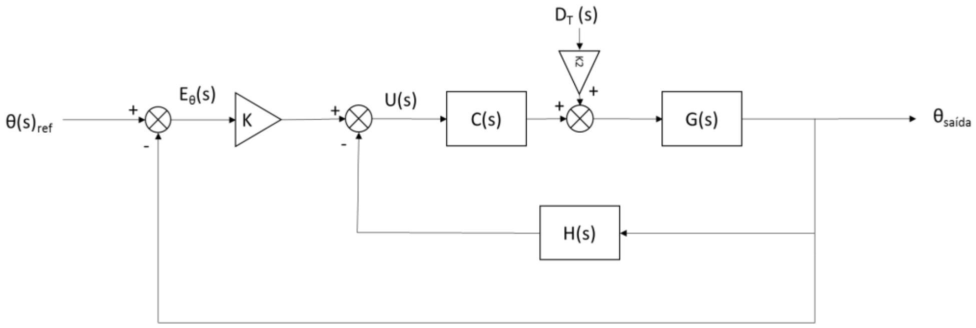

Figure 7. Control Loop

Based on the transfer function of each application, a fractional PD controller was designed to assure a suitable time response by a desired phase margin at a specific crossover margin gain frequency (DORF, 2016) using the methodology found in (MONJE et al.,2010; MARTINS, 2016). Nevertheless, the proportional and derivative control actions are not able to reduce the steady state error to zero. Saying that, a fractional PI controller was designed to address this issue. Both fractional controllers were put together in a cascade configuration resulting in a $PI^{\lambda}(PD^{\mu})$ controller.

Then, an CRONE approximation was used to obtain a high-order integer controller that maintain the same desired fractional dynamic behavior. After that, the controlled system was simulated in MATLAB® to evaluate if the fractional PID controller proposed would present superior performance in comparison with classical (ÅSTRÖM,1995; LUYBEN, 1997) ones as expected.

Table 1. Parameters

Parameter	Bike	Moto	Unit	Description
m_r	$2,02$	7	kg	Mass of flywheel
m_m	$20,6$	$276,98$	kg	Mass of vehicle
h_r	$0,58$	$0,75$	m	Flywheel CG upright height
h_m	$0,49$	$0,636$	m	Vehicle CG upright height
I_m	. 2,1 .	32	$kg \cdot m^2$	Vehicle moment of inertia about ground contact line
I_p	$0,0088$	$0,070$	$kg \cdot m^2$	Flywheel polar moment of inertia about CG
I_r	$0,0224$	0,185	$kg \cdot m^2$	Flywheel radial moment of inertia about CG
ω	469	1570	rad / s	Flywheel angular velocity
L	$0,000119$	$4 \cdot 10^{-4}$	H	Motor inductance
R	$0,61$	$0,8$	Ω	Motor resistance
N	65	50	-	Gear ratio
B_m	$0,003$	$7,4 \cdot 10^{-5}$	$kg \cdot m^2 / s$	Motor viscosity coefficient
K_m	$0,0259$	$7,63 \cdot 10^{-3}$	Nm / A	Motor torque constant
K_e	$0,0027$	$7,63 \cdot 10^{-3}$	$V \cdot s / rad$	Motor back emf constant
g	$9,81$	$9,81$	m / s^2	Gravitational acceleration

Source: LAM, 2012; MARTINS,2016.

4. RESULTS AND DISCUSSION

Applying the set of parameters for bicycle, the following transfer function was obtained:

$$G(s)=\frac{\Theta(s)}{U(s)}=\frac{3,364\cdot 10^{5}}{s^{4}+5126s^{3}+2476s^{2}+4,3\cdot 10^{5}s-3,412\cdot 10^{4}} \tag{9}$$

The transfer function of the PI^{λ} (PD^{μ}) controller designed is:

$$C(s)=10^{-\frac{92}{20}}\left(1+\frac{10000}{s^{\frac{1}{3}}}\right)\left(6,3165\left(1+0,0128s^{1,96}\right)\right) \tag{10}$$

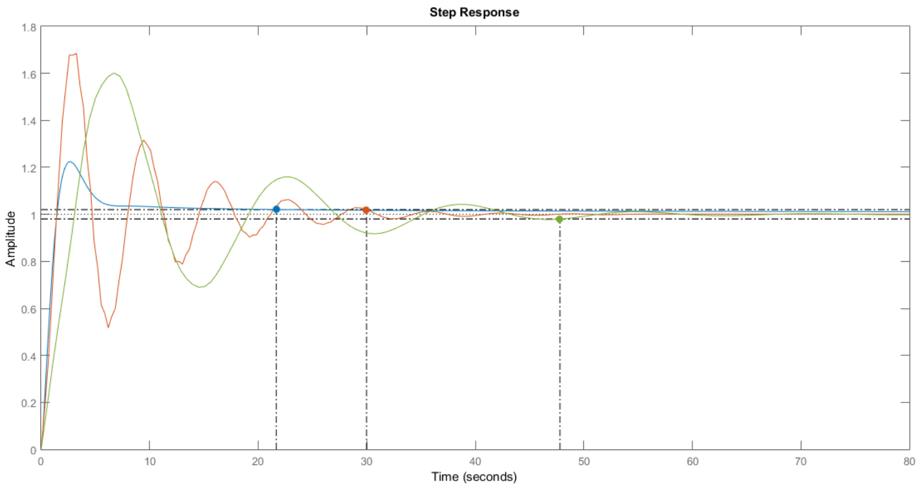

Figure 8. Step response of the bicycle system for PI^{λ} (PD^{μ}), ZN and TL controllers.

From the Figure 8, we can perceive that the $PI^{\lambda}(PD^{\mu})$ controller presented settling time and peak amplitude smaller than classical ones.

In addition, the $PI^{\lambda}(PD^{\mu})$ controller has achieved the isodamping condition for gain variations, as seen in Fig. 9.

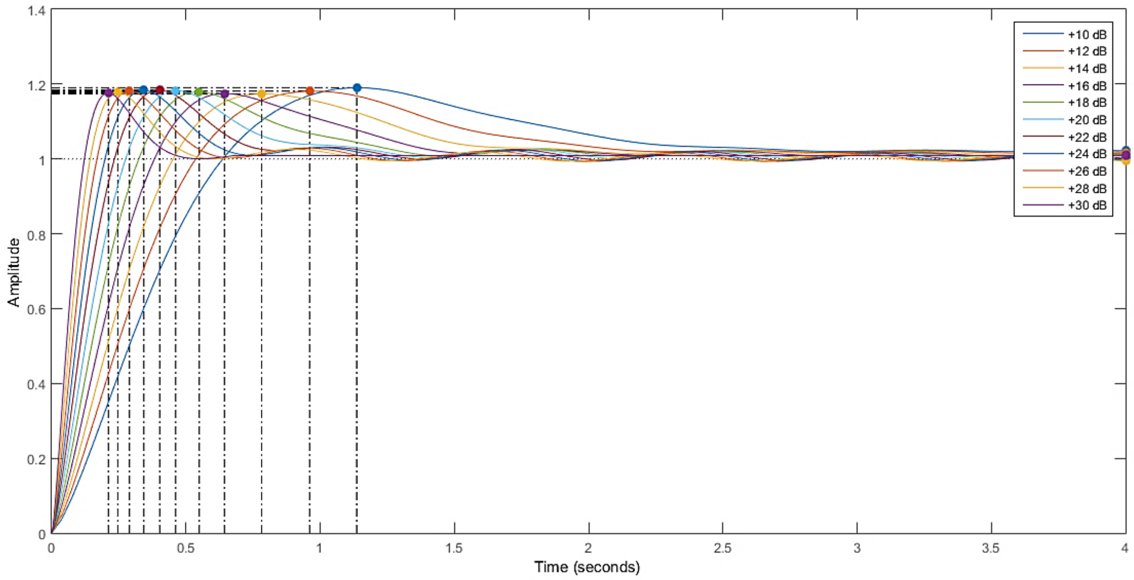

Figure 9. Robustness test of PI^{λ} (PD^{μ}) controller to gain variation (+10 a 30 dB)

Likewise, applying the set of parameters for motorcycle, the transfer function obtained is:

$$G(s)=\frac{3731}{s^4+1951s^3+467.8s^2+8.364e05s-470.3} \quad (11)$$

And the transfer function of the PI$^{\lambda}$ (PD$^{\mu}$) controller designed is:

$$C(s)=\left(10^{-\frac{50}{20}}\left(1+\frac{100}{s^{\frac{1}{3}}}\right)\right)\left(547,4744\left(1+2,3371\cdot 10^{-3}s^{1,9994}\right)\right) \quad (12)$$

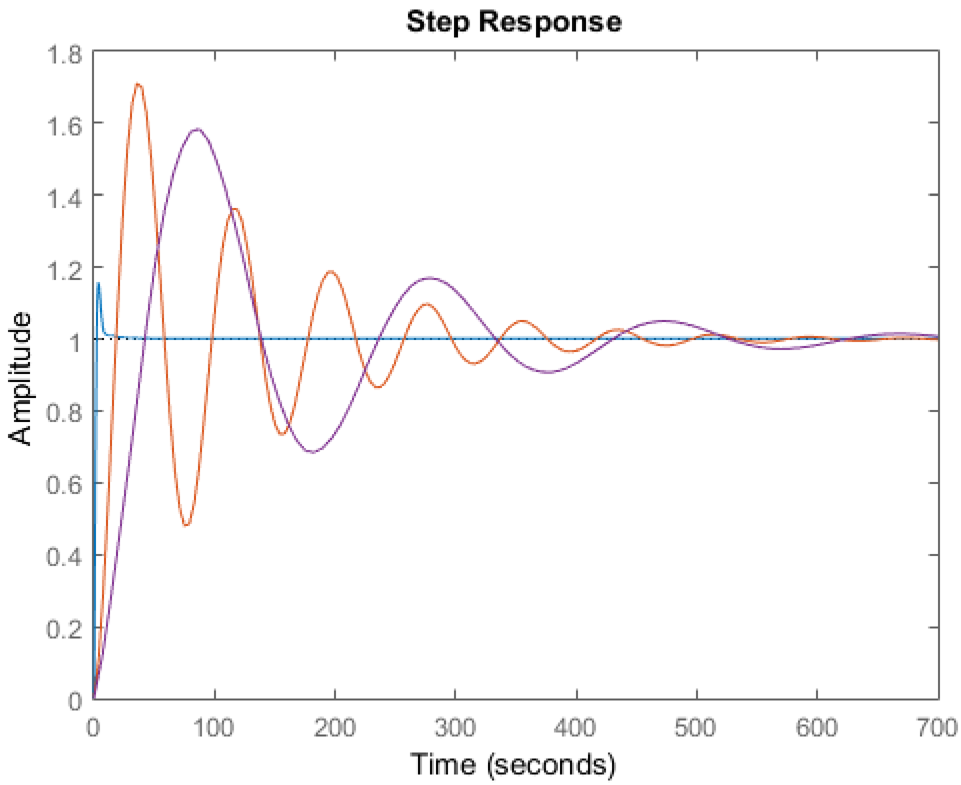

Figure 10. Step response of the bicycle system for PI$^{\lambda}$ (PD$^{\mu}$), ZN and TL controllers.

Once more, the controller proposed achieved the isodamping condition for gain variation, as we observe in Fig. 11

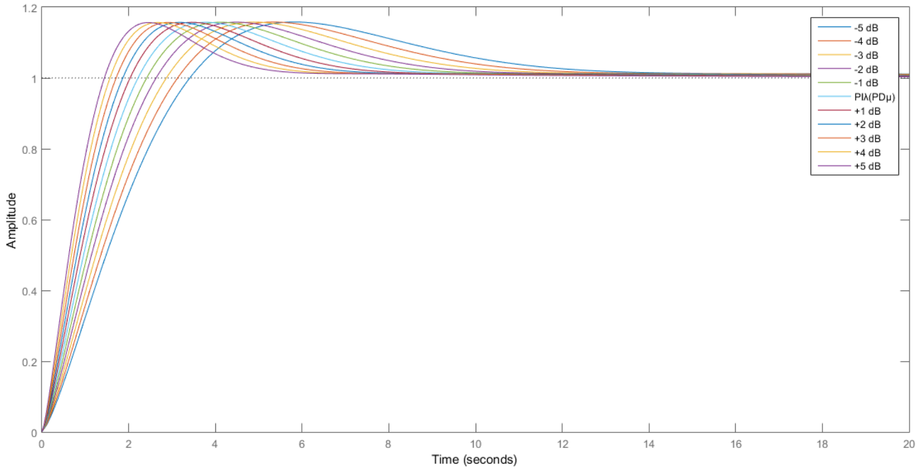

Figure 11. Robustness test of PI^{λ} (PD^{μ}) controller to gain variation ()

A simulation, in Fig.12, was proceeded considering the vehicle with a small initial inclination. Then the controller drives the system to the balance position. Because of the low effect of integral action, the error is reduced, but it does not go to zero in a finite interval of time. An impulsive disturbance momentum is applied in the roll direction to evaluate if the system remains stable even if it undergoes side impacts.
It is observed that the designed controller can achieve the system stability. However, the intense oscillating disturbance response may be an undesired feature.

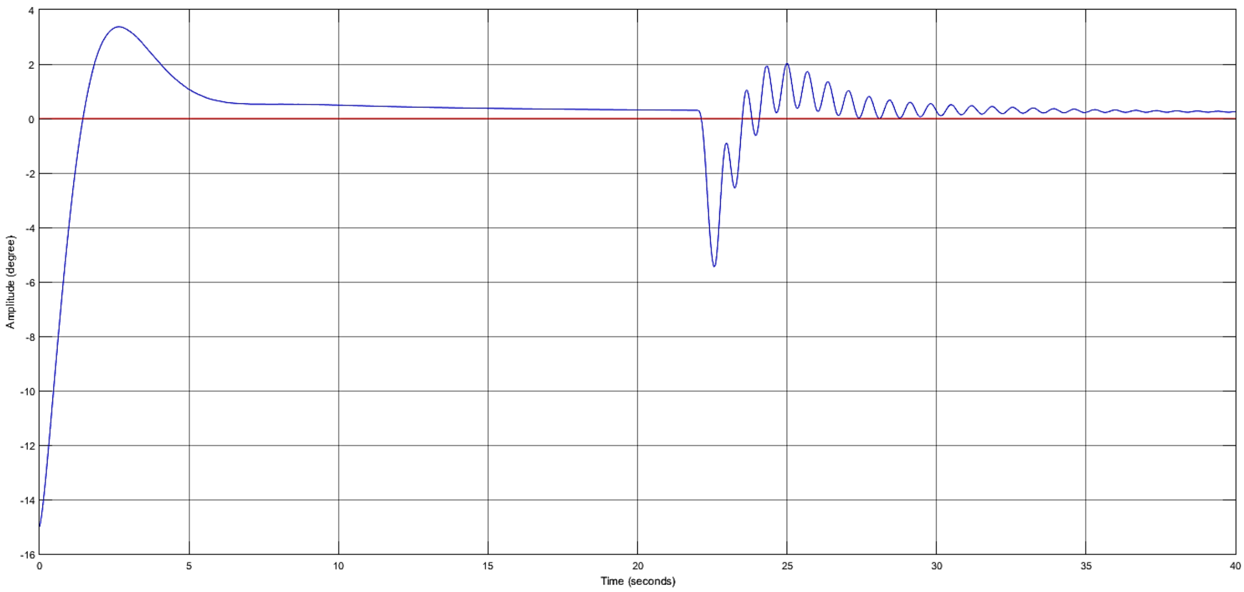

Figure 12.Simulation of side impact for the bicycle stabilizing system.

5. CONCLUSIONS

The model adopted in this work has shown to be valid for two-wheeled vehicles indistinctly.

Additionally, the PI^{λ} (PD^{μ}) controllers presented faster response and smaller overshoot than classic PID controllers. Besides, the former has achieved the isodamping condition even when gain variations were imposed, that is, it's possible to adjust the speed response of the stabilizing system, without compromising the maximum angle variation safety condition.

Therefore, the PI^{λ} (PD^{μ}) controller has shown to be a very suitable alternative for CMG stabilizing systems' control and can be used in future works to reduce the effects of traffic collisions involving motorcycles.

6. REFERENCES

ÅSTRÖM, K. J.; HÄGGLUND, T.; PID Controllers: Theory, Design, and Tuning. 2nd ed. EUA: Instrument Society of America, 1995.

DORF, Richard C.; BISHOP, Robert H. Modern control systems. Prentice Hall, 2011.

DPVAT, S. L. Boletim Estatístico de Janeiro a Dezembro de 2015.

KIM, Daniel Kee Young; BRETNEY, Kevin. Electrical system for gyroscopic stabilized vehicle. U.S. Patent n. 8,918,239, 23 dez. 2014.

LAM, Pom Yuan. Design and Development of a Self-Balancing Bicycle Using Control Moment Gyro. 2012. Tese de Doutorado.

LEMOS, Nivaldo A. Mecânica analítica. Editora Livraria da Física, 2007.

LIMEBEER, David JN; SHARP, Robin S. Bicycles, motorcycles, and models. IEEE control systems, v. 26, n. 5, p. 34-61, 2006.

LUYBEN, Michael L.; LUYBEN, William L. Essentials of process control. McGraw-Hill, 1997.

MARTINS, Paulo R. O. Modelagem e Controle de um Sistema de Estabilização por GCM de veículos de duas rodas para o caso estático utilizando controlador PIλ (PDμ).

MONJE, Concepción et al. Fractional-order systems and controls: fundamentals and applications. Springer Science & Business Media, 2010.

NISE, Norman S. Control system engineering, John Wiley & Sons. Inc., New York, 2011.

OGATA, K. Modern control engineering 5th edition. Lugar: Upper Saddle River, New Jersey 07458. 2010.

PODLUBNY, Igor. Fractional-order systems and fractional-order controllers. Institute of Experimental Physics, Slovak Academy of Sciences, Kosice, v. 12, n. 3, p. 1-18, 1994.

SHAH, Pritesh; AGASHE, Sudhir. Review of fractional PID controller. Mechatronics, v. 38, p. 29-41, 2016.

WORLD HEALTH ORGANIZATION et al. Decade of Action for Road Safety, 2011–2020: Saving millions of lives. 2011.

7. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.