24th COBEM - 2017

24th ABCM International Congress of Mechanical Engineering
December 3-8, 2017, Curitiba, PR, Brazil

COBEM-2017-1524
THERMOELASTIC TOPOLOGY OPTIMIZATION WITH MULTIPLE MATERIALS FOR MAXIMUM STIFFNESS USING MATERIAL INTERPOLATION SCHEME WITH PENALIZATION.

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Abstract. *In several cases, thermal and mechanical loads occur simultaneously in a structure. Due to this load condition, the structural shape and topological design procedures become complex and sometimes nonintuitive. The thermal loads are dependent of the shape and topology of the structure, for this reason the design must be an iterative procedure. In this paper a method for the stiffness maximization subject to volume constraints on the phase materials considering thermomechanical loads is proposed, based on the thermo-elastic finite element theory and on the material interpolation schemes with penalization for multiple materials. The nodal thermal load of each element is applied only if the element is presented in the design domain. Therefore, the thermomechanical loads are considered design-dependent body loads. The optimization problem is solved using a Bi-directional evolutionary structural optimization method (BESO), with the soft kill approach. Two examples are presented to show the influence of the temperature changes and the mechanical loads on the obtained topologies.*

Keywords: *Bi-directional evolutionary structural optimization (BESO), Thermoelastic, Stiffness maximization.*

1. INTRODUCTION

Topology optimization is a versatile tool for the design that is widely used in the solution of many engineering problems. Structural Topology optimization methods, such as SIMP, BESO, and level set methods, have been developed based on the finite element analysis, and used to improve the structural characteristics (Bendsoe and Sigmund, 2003; Huang and Xie, 2010; Allaire *et al.*, 2004).

This methods are being utilized by few authors to minimized the mean compliance of thermoelasctic structures considering, temperature's distribution independent of the thermoelastic design (Rodrigues and Fernandes, 1995), material volume constraints (Xia and Wang, 2008), thermoelastic stress loads (Gao and Zhang, 2010), a multi-scale and multi-material analysis (Xu *et al.*, 2016). Moreover, the BESO method has been used to minimized the compliance of structures under design-dependent loads (Picelli *et al.*, 2014, 2017). Furthermore, Li *et al.* (2001) consider thermoelastic problems under varying temperature fields to solve thermoelastic problems, Deng *et al.* (2013) used the thermoelastic approach to do a multiobjective design of a thermoelastic structure and Rodríguez and Pavanello (2015) considering thermo-mechanical multi-objective problems.

In this work, the BESO method and the material interpolation scheme with penalization are applied to two-dimensional thermomechanical problems, when multiples materials are allowed on the design domain. The influence of the temperature changes and the loading conditions in the final topology is also verified.

2. PROBLEM STATEMENTS

The formulation of the topology optimization problem, for minimizing the mean compliance or strain energy can be defined as follow:

$$minimize: \qquad \{C\} = \frac{1}{2}\{P\}^T\{u\} = \frac{1}{2}(\{F_m\}+\{F_T\})^T\{u\} \tag{1}$$

$$subject \quad to: \quad V_j^* - \sum_{i=1}^{N} V_i x_{ij} - \sum_{i=1}^{j-1} V_i^* = 0 \quad j = 1, 2, ..., n-1 \tag{2}$$

$$x_{ij} = x_{min} \quad or \quad 1 \tag{3}$$

where $\{C\}$ is the mean compliance, $\{P\}$ the force applied on the structure, $\{F_m\}$ the mechanical load, $\{F_T\}$ the thermal load, $\{u\}$ the displacement vector, n the number of material phases, V_j^* the prescribed volume for each material j and x_{ij} the design variable for the ith element and the jth material.

The design variable x_{ij} is defined as (Huang and Xie, 2009) :

$$x_{ij} = \begin{cases} 1 & for \quad E \geq E_j \\ x_{min} & for \quad E \leq E_j \end{cases} \tag{4}$$

where the elasticity moduli of different material phases and the thermal expansion coefficients are E_1, E_2, ... , E_n and α_1, α_2, ... , α_n, respectively.

3. MATERIAL INTERPOLATION

The interpolation of the Young's modulus (E) and the thermal expansion coefficient (α) for materials with n phases, could be developed using the material interpolation schemes with penalization (Bendsoe and Sigmund, 2003), which must be applied between two neighboring phases as follows:

$$E(x_{ij}) = x_{ij}^p E_j + (1 - x_{ij}^p) E_{j+1} \tag{5}$$

$$\alpha(x_{ij}) = x_{ij}^q \alpha_j + (1 - x_{ij}^q) \alpha_{j+1} \tag{6}$$

where p and q are the penalties exponents and $j = 1, 2, ..., n-1$.

4. SENSITIVITY ANALYSIS

The sensitive number can be found by the gradient of the objective function (C) with respect to the design variable x_{ij} (Xu *et al.*, 2016). Assuming that $E_1 > E_2 ... > E_n$, the sensitive numbers are calculated as follows:

$$sn_{ij} = -\frac{\partial C}{\partial x_{ij}} = \frac{1}{2}\{u_i\}^T \frac{\partial [k(x_{ij})]}{\partial x_{ij}}\{u_i\} - \frac{\partial \{P(x_{ij})\}}{\partial x_{ij}}^T\{u_i\} = \frac{1}{2}\{u_i\}^T \frac{\partial [k(x_{ij})]}{\partial x_{ij}}\{u_i\} - \frac{\partial \{F_T(x_{ij})\}}{\partial x_{ij}}^T\{u_i\} \tag{7}$$

where $[k(x_{ij})]$, the elemental stiffness matrix; $\{F_T(x_{ij}\}$, the elemental thermal force are given by the following equations:

$$[k(x_{ij})] = \int_{\Omega_i} [B_i]^T [D(x_{ij})] [B_i] \, d\Omega \tag{8}$$

$$\{F_T(x_{ij}\} = \int_{\Omega_i} [B_i]^T [D(x_{ij})] \{\alpha(x_{ij})\} \Delta T d\Omega \tag{9}$$

where Ω_i is the elemental domain, $[B_i]$ is the strain-displacement matrix, $[D(x_{ij})]$ is the elasticity matrix and ΔT is the temperature change. Then, the derivative of the stifness matrix and the thermal force is given by Eq. (10) and Eq. (11), respectively.

$$\frac{\partial [k(x_{ij})]}{\partial x_{ij}} = \int_{\Omega} [B_i]^T \frac{\partial [D(x_{ij})]}{\partial x_{ij}} [B_i] \, d\Omega \tag{10}$$

$$\frac{\partial \{F(x_{ij})\}}{\partial x_{ij}} = \int_{\Omega} [B_i]^T \left(\frac{\partial [D(x_{ij})]}{\partial x_{ij}} \{\alpha(x_{ij})\} + [D(x_{ij})] \frac{\partial \{\alpha(x_{ij})\}}{\partial x_{ij}} \right) \Delta T d\Omega \tag{11}$$

Using plane stress assumption, considering Poisson's ratio constant for all material phases, the therm $\frac{\partial D}{\partial x_{ij}}$ can be written as follow:

$$\frac{\partial\left[D(x_{ij})\right]}{\partial x_{ij}} = \frac{\partial E(x_{ij})}{\partial x_{ij}} \begin{bmatrix} \frac{1}{1-\nu_j^2} & \frac{\nu_j}{1-\nu_j^2} & 0 \\ \frac{\nu_j}{1-\nu_j^2} & \frac{1}{1-\nu_j^2} & 0 \\ 0 & 0 & \frac{1}{2(1+\nu_j^2)} \end{bmatrix} \tag{12}$$

Equation (13) and Eq. (14) are the derivatives with respect to the design variable x_{ij} of Eq. (5) and Eq. (6), respectively.

$$\frac{\partial E(x_{ij})}{\partial x_{ij}} = p x_{ij}^{p-1}(E_j - E_{j+1}) \tag{13}$$

$$\frac{\partial \alpha(x_{ij})}{\partial x_{ij}} = q x_{ij}^{q-1}(\alpha_j - \alpha_{j+1}) \tag{14}$$

Using Eq. (7) to Eq. (14) it is possible to calculate the sensitive number for each element. The calculated sensitive number is ranked to evaluate which element will be removed and which will be added on the structure domain, using BESO procedure (Huang and Xie, 2010).

5. NUMERICAL EXAMPLES

The first design domain, illustrated in Fig. 1, is a roller-supported beam subjected to a concentrated load F and submitted to a constant temperature change ΔT. The structure is meshed using 160X60 four node plane stress elements, and it is heated uniformly. The proposed multi-material material algorithm is used. The used material properties are: Young's moduli $E_1 = 210\ GPa$ and $E_2 = 70\ GPa$, thermal expansion coefficients $\alpha_1 = 1.2e^{-5}$ °C^{-1} and $\alpha_2 = 2.3e^{-6}$ °C^{-1} and the Poisson's ratio 0.3 for both phases. The adopted final volume fractions are $V_{f1} = 0.15$ and $V_{f2} = 0.25$. The force applied on the structure is $1000\ N$ and different values of the temperature change ΔT are considered: 0, 25, 50 and 100 °C.

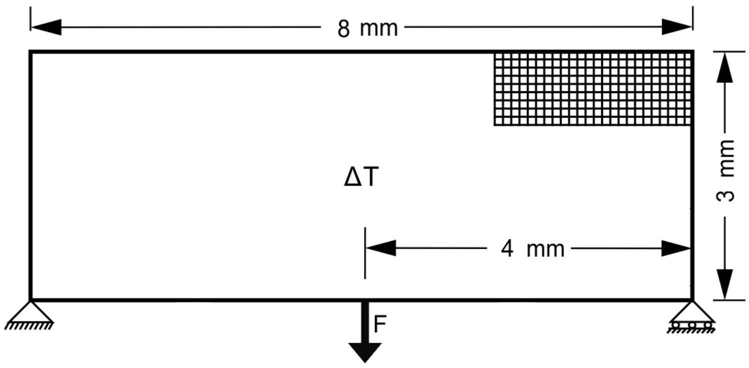

Figure 1: Design domain of a roller-supported beam: one concentrated load.

The soft-kill BESO method is applied with an evolutionary ratio $ER = 2\%$, lower bound of the density $x_{min} = 0.001$, penalties exponents $p = 3$ and $q = 3$, volume addition ratio $AR = 1\%$ and length scale of the filter $r_{min} = 0.17\ mm$. Figure 2. shows the obtained optimal topologies, where the black and the pink elements represent the material 1 and material 2, respectively. Figure 3. shows the evolutionary histories of the mean compliance and the volume fraction.

In the Fig. 3 the compliance variation is smooth on the iterative process, and it final value increases with the increase of the temperature changes. In addition, the final topologies vary with the temperature change ΔT. For high values of ΔT the algorithm concentrated the material 1 at the center of the structure creating dissimilar topologies.

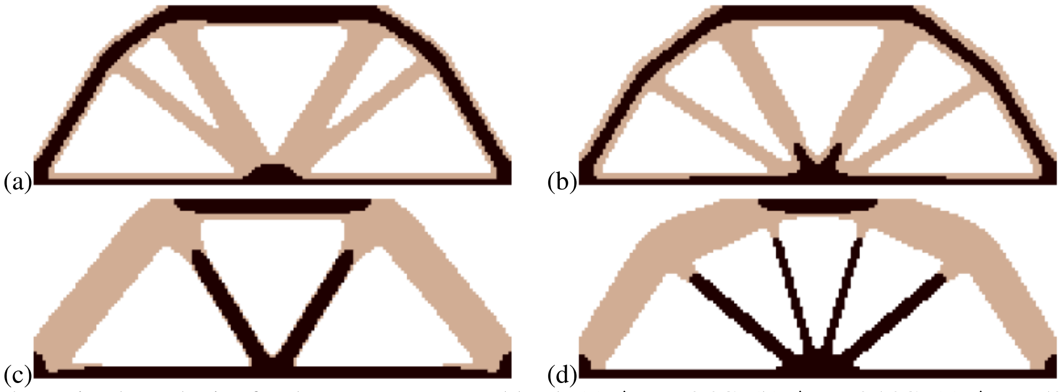

Figure 2: The optimal topologies for the one concentrated load. (a) Δ T = 0 °C; (b) ΔT = 25 °C; (c) ΔT = 50 °C; (d) Δ T =100 °C

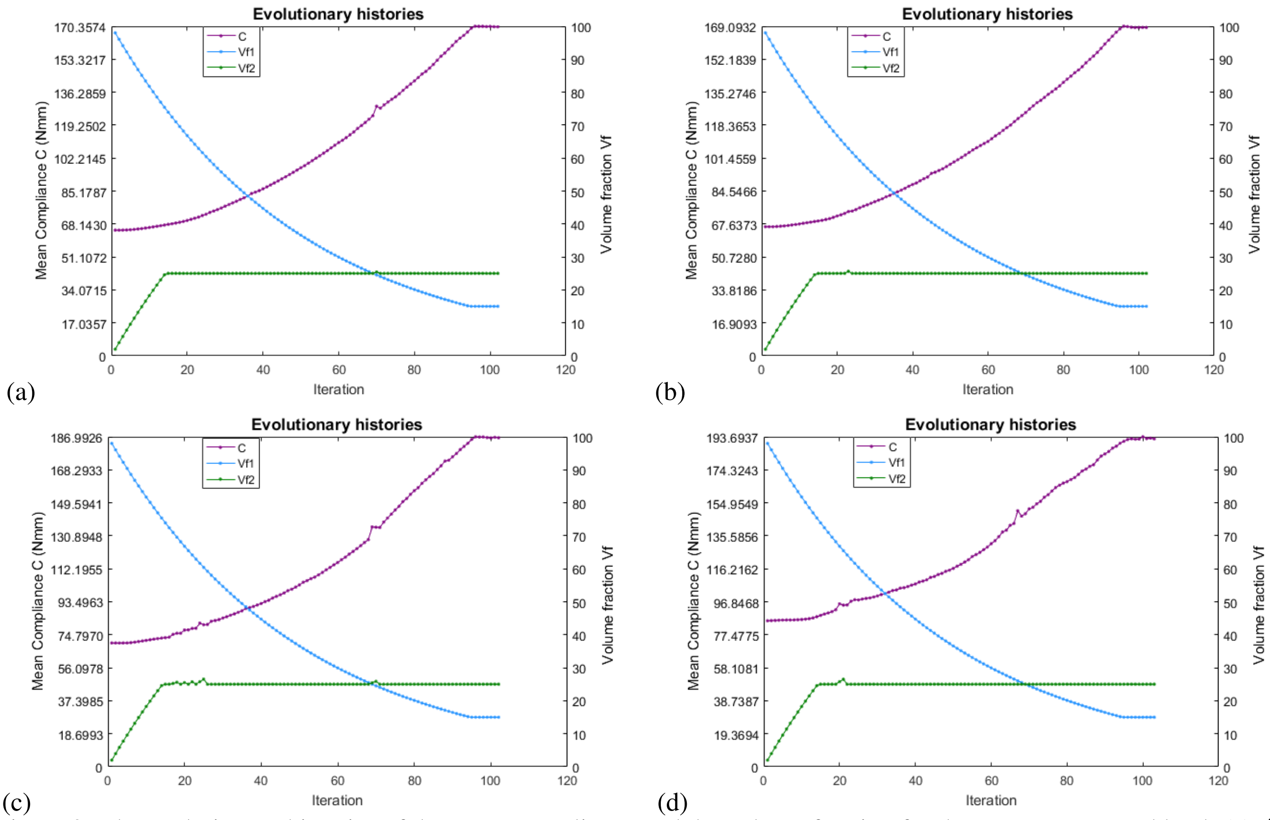

Figure 3: The evolutionary histories of the mean compliance and the volume fraction for the one concentrated load. (a) Δ T = 0 °C; (b) ΔT = 25 °C; (c) ΔT = 50 °C; (d) Δ T =100 °C

In the second load case, illustrated in Fig. 4, three concentrated mechanical loads are considered, in order to analyze the influence of the load condition in the final topology. Using the same materials properties and algorithm parameters of the load case 1, the optimal topologies and the evolutionary histories of the mean compliance and the volume fraction are presented in Fig. 5. and Fig. 6, respectively.

For this case the compliance variation is also smooth along the iterations, and it final value increases with the increase of the temperature changes, as shown in Fig. 6. The final topology depends on the magnitude of the temperature change. In Fig. 5 can be observed that inasmuch as the temperature change increases, the algorithm concentrated the material 1 on the bottom of the structure where the loads are applied.

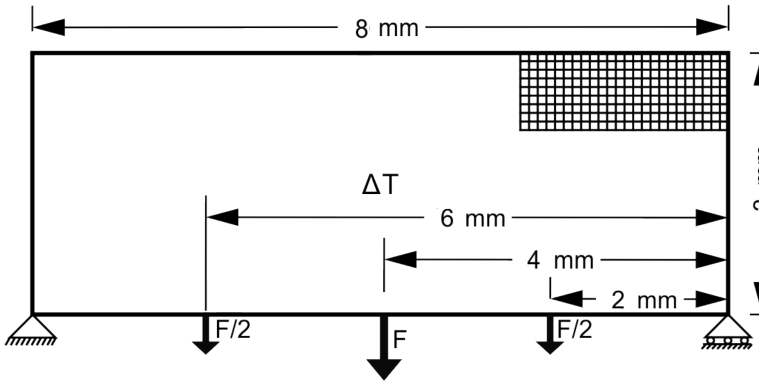

Figure 4: Design domain of a roller-supported beam: three concentrated loads.

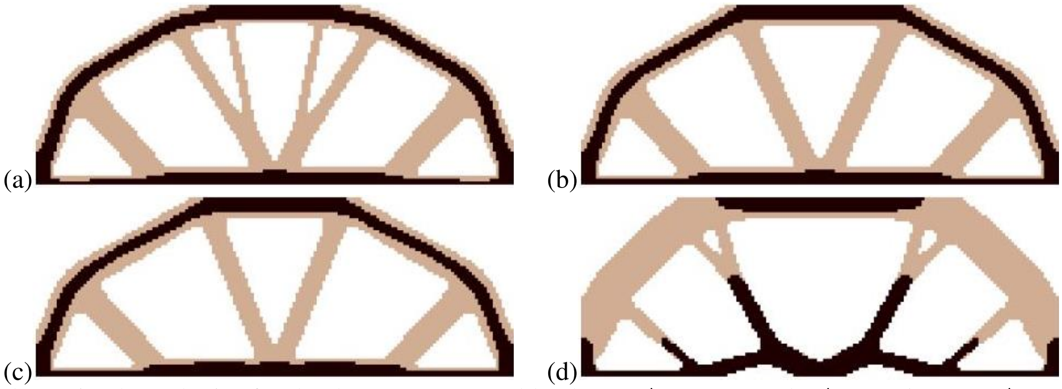

Figure 5: The optimal topologies for the three concentrated loads. (a) Δ T = 0 °C; (b) ΔT = 25 °C; (c) ΔT = 50 °C; (d) ΔT = 100 °C

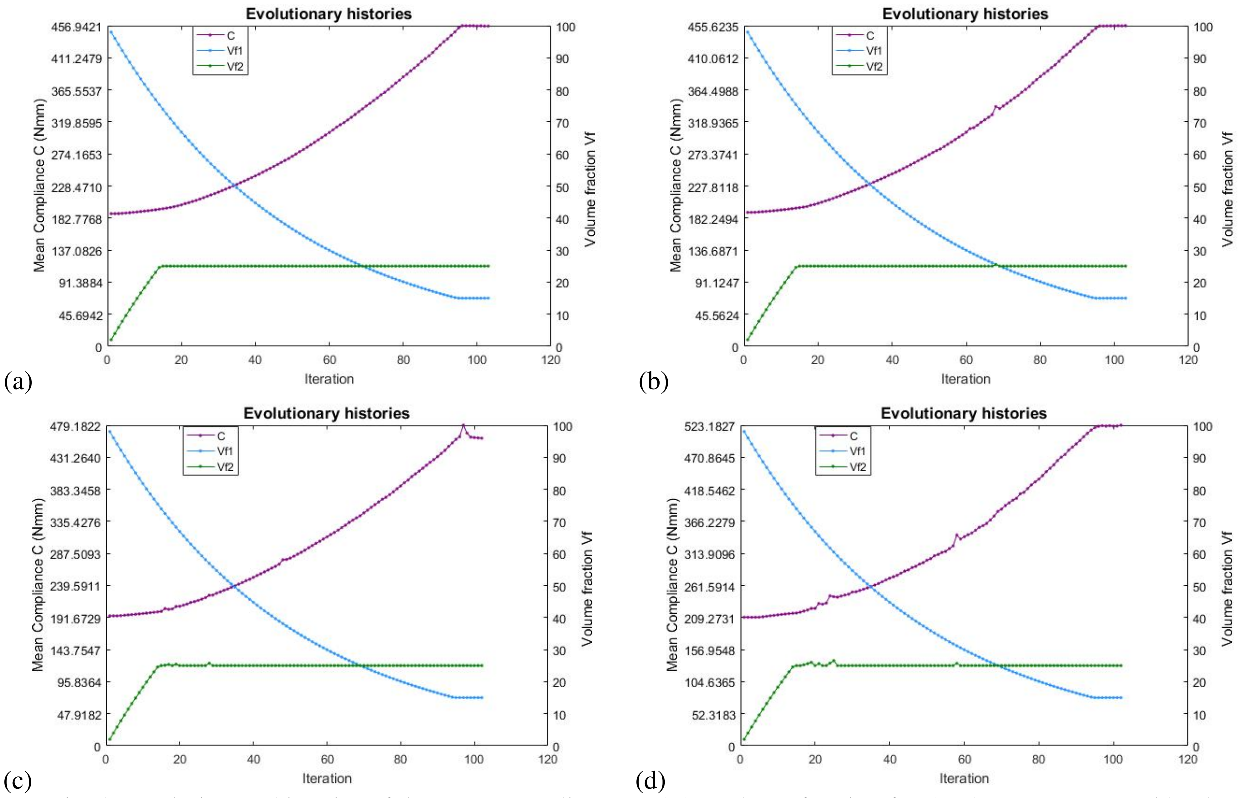

Figure 6: The evolutionary histories of the mean compliance and the volume fraction for the three concentrated loads. (a) Δ T = 0 °C; (b) ΔT = 25 °C; (c) ΔT = 50 °C; (d) Δ T =100 °C

6. CONCLUSIONS

A procedure for the design of thermo-mechanical structures with multiple materials is proposed. The interpolation of Young's modulus and the thermal expansion coefficient has been done, using the material interpolation scheme with penalization between two neighboring phases. The sensitive of the mean compliance with respect to the designs variables is derived.

Two numerical examples are presented to show the effectiveness of the proposed procedure to design thermo-mechanical structures with multiple materials. The results indicate that the final value of the objective function increases with the increase of the temperature change, the final topologies present a sandwich of material 2 over a core of material 1. Moreover, the final topologies depend on the magnitude of the temperature changes and on the loading conditions; as can be seen in Fig. 2, the core of the material 1 start to appear in the center of the structure, for a ΔT greater than 0 °C, while for the topologies shown in Fig. 5, the material 1 appear in the center of the structure for a $\Delta T = 100$ °C. It can be concluded that inasmuch as the temperature change increases, the algorithm places the material 1 (which is the stiffer material) between the supports and the concentrated loads.

7. ACKNOWLEDGEMENTS

The authors are grateful to CAPES, FAPESP (São Paulo Research Foundation, grant number 2013/08293-7) for the financial support of this work.

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