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EFFECTS OF IMPELLER DESIGN ON THE CHARACTERISTICS OF CENTRIFUGAL PUMPS

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Abstract. *In this work, a research on mathematical models and impeller projects was carried out, whose application focus on centrifugal pumps, with emphasis on backward-facing blades. As Euler's Law states, said hydraulic turbomachine transforms mechanical energy into pressure energy to the working fluid, which in this case is water. As for the details of this project, the goal was to design a high-efficiency rotor, able to overcome a pressure head of 15 mH2O, using an already known pump. The mathematical model was elaborated based on the speed diagram, considering a steady state system, in addition to constant inlet and outlet speeds between sections. With the defined geometric aspects, a CAD model of real dimensions was crafted, aiming to build a 3D printed prototype.*

Keywords: *Impeller, centrifugal pump, hydraulic turbomachine.*

1. INTRODUCTION

A hydraulic pump is a machine that yields or utilizes mechanical energy and transfers it to a fluid in the form of hydraulic energy (FAIZULLY, 2013) Among the several components that constitute a pump, the impeller is the one responsible for the main role during the energy transference – transforming mechanical energy into pressure energy to the fluid.

An impeller is a rotating component with blades, which exert force on the fluid, resulting in an acceleration gain by the work fluid. The impeller rotates at a high speed, in a continuous movement, due to the force provided by the engine, causing the fluid to be sucked. This causes a vacuum zone in its center and a high-pressure zone in its boundaries.
The present work aims at designing a centrifugal radial hydraulic turbomachine impeller, which operates with water.

2. ROTOR MODELS

Rotors can be configured in three different ways: open, half open and closed. Open rotors are used together with overall small pumps. However, it shows low structural resistance and low efficiency. This model's advantage lies in its difficult-to-clog design, therefore being adequate to pump particle-containing liquids. Half open rotors have only one disc, in which blades are attached. Finally, closed rotors are mainly used with particle-free fluids, in which the blades are attached to both, preventing recirculation (PINTO, 2010).

2.1 Velocity Diagrams

For the calculations, a few considerations were made, such as infinite blade number, infinitely thin, wherein the current lines are congruent with the blades and the flow is one-dimensional. Thus, the speed triangle will be the same in every point located in the same diameter. Besides, a steady state condition is assumed, and both the inlet and outlet speeds, are uniform on all sections.

The fluid relative speed refers to the particle relative trajectory, along the blade direction, as if the rotor was at rest and the fluid flowing through the blades. This speed is designated by (W).

The absolute speed is the velocity of the fluid relative to the frame located outside the rotor. Thus, the motion of the particle results from two motions, the first within the rotor channels, and the second from the rotational motion of the rotor. Therefore, the velocity tangent to this trajectory is the absolute velocity (C).

Finally, the tangential velocity is the velocity that accompanies the rotational motion of the rotor. Therefore, knowing the diameter and the rotation of the rotor it is possible to obtain the tangential velocity, represented by (U).

2.2 Ideal Velocity Triangle

From general mechanics, there's the relation between the absolute velocity C, the relative velocity W and the tangential velocity U. So, the vectoral equations for the velocities that compose the triangle are given by:

$$\vec{C} = \vec{W} + \vec{U} \quad (1)$$

The absolute and relative velocity vectors can be separated into axial velocity and tangential velocity, which are important to the development and analysis of the velocity triangle. The graphical representation of all vectors that compose the velocity triangle at both the inlet and outlet can be seen from the figure below.

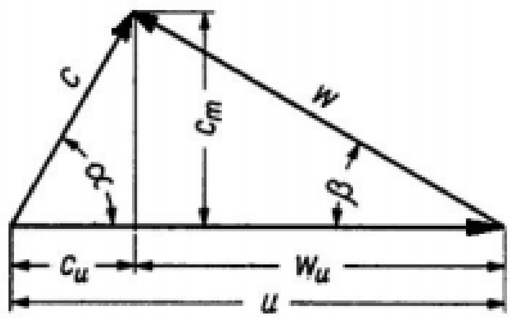

Figure 1. Generic Velocity Triangle

The "u" and "m" subscripts refer to those tangential and axial components, respectively. The alpha (α) angle corresponds to the absolute flow angle, whereas beta (β) refers to the relative flow angle.

Given the rotation of the engine that will drive the pump, it is possible to attain its angular velocity. Therefore, from Eq. (2), it is possible to calculate the working fluid flow tangential velocity.

$$u = \omega \cdot r = \frac{\pi \cdot D \cdot n}{60} \quad (2)$$

In the equation above, D [meters] is the diameter at the considered point, n [RPM] is rotation, ω [rad/s] is radial velocity, r [m] is radius and u [m/s] is tangential velocity.

Besides the considerations preceding the calculation of the velocity triangle components, it is necessary to consider that the inlet and exit flows obey to the mass conservation principle, that is, the mass remains unchanged throughout the rotor. The need to obey that principle is related to the fluid axial absolute velocity (Cm), which will provide the blade width at the inlet and exit (FOX, 2006).

The flow rate (Q) is given by the product of the velocity perpendicular to the control surface (V) by the cross-sectional area (A) of the control surface through which the flow passes. The flow obtained in Eq. (3) is given in cubic meters per second (m³/s) (GUIMARÃES,1991).

$$Q = V \cdot A \quad (3)$$

Applying the mass conservation principle, the mass flow that passes through the control surface is given by:

$$\oiint_{SC} \rho \cdot \vec{v} \cdot \overrightarrow{dA} = 0 \quad (4)$$

Solving the velocity triangle, we have:

$$\overrightarrow{C_M} = \vec{C} \cdot cos\,\alpha \tag{5}$$

Substituting Eq. (5) in Eq. (4), we have:

$$\oint\!\!\!\int_{SC} \rho \cdot \overrightarrow{C_M} \cdot \overrightarrow{dA} = 0 \tag{6}$$

Equation (6) takes this form because the angle between vectors $\overrightarrow{c_m}$ and $\overrightarrow{dA}$ equals zero. Therefore, the cosine of said angle equals 1. Hence, solving the integral for both fluid inlet and exit cases, assuming it incompressible, as well as considering the control volume to be non-deformable and showing no changes in size, results in:

$$C_{m4} \cdot A_4 = C_{m5} \cdot A_5 = Q \tag{7}$$

The calculation of the axial impeller inlet and outlet velocities is essential to obtain the rotor blades inner and outer widths. The cross-sectional area is given by:

$$A = \pi \cdot D \cdot b \tag{8}$$

In the equation above, “b” indicates blade width. Equation (8) does not consider the area occupied by fins, because they were assumed to have negligible thickness at first.

Blade width at the impeller inlet is a parameter that can be defined by the designer based on similar design data found on literature or by arbitrarily choosing it. Therefore, it is possible to find the output width given by Eq. (7).

Pump design must meet a certain operating condition, in which the pump will be driven by a designated engine and should reach a certain pressure height. The pump will have to supply power to the fluid, so it is necessary to provide work to it. The theoretical work (Y_u) developed by the pump to overcome the theoretical pressure height (H_u) is given by:

$$Y_u = (U_5 \cdot C_{u5} - U_4 \cdot C_{u4}) \tag{9}$$

Equation (10) gives the theoretical height, as follows:

$$H_u = {}^{Y_u}\!/_{g} \tag{10}$$

Equation (9) shows that there is a fraction of work that is lost during pump operation. When the alpha angle at the inlet is 90°, it causes the tangential component of the absolute velocity to become zero, so the lost parcel of work is zeroed as well. This causes the pump to operate with greater efficiency, which should be taken into consideration to optimize the rotor sizing process, to increase its overall efficiency (GUIMARÃES, 1991). The work output is calculated by:

$$Y_u = U_5 \cdot C_{u5} \tag{11}$$

After determining all parameters of the input triangle, the exit velocity triangle analysis begins. One of the first parameters obtained is the tangential velocity at the rotor exit (U_5), as in Eq. (1), only changing the diameter to the rotor outer diameter. The absolute outlet axial velocity (C_{m5}) is obtained by Eq. (7), since the principle of conservation of mass is considered.

The constructive angle β_5 is a parameter that can be selected to determine the outlet width of the rotor blade (b_5). Determining that angle can be done by manipulating Eq. (10), which determines the impeller work. The manipulation is based on the triangle of velocities at the exit of the rotor, resulting in:

$$H_u = \frac{U_5^2}{g} - \frac{U_5 \cdot Q}{g \cdot \pi \cdot D_5 b_5 \tan\beta_5} \tag{12}$$

The equation above generates the theoretical rotor operating curve. When the pressure height is minimum, the flow rate is maximum, and when the flow is minimum, the pressure height will be maximum. Since the angle β_5 is an arbitrarily chosen input parameter, the exit thickness b_5 is then attained by solving Eq. (11).

2.3 Real Velocity Triangle

During the design stage of flow machines, based on the one-dimensional flow tube theory, which assumes the impeller to have an infinite number of blades, infinitely close to each other and of infinitesimal thickness. Such conditions do not match reality. Rotors have a finite number of blades, of finite thickness, which emphasizes the need to study how these factors influence the inlet and exit impeller velocity triangles (SILVA, 2000).

Equation (13) gives the fundamental equation for flow machines with an infinite number of blades.

$$Y_{pá\infty} = u_5 c_{u5} - u_4 c_{u4} \quad (13)$$

Similarly, the fundamental equation shown in Eq. (13) can be written for a finite number of blades as follows:

$$Y_{pá} = u_5 c_{u6} - u_4 c_{u3} \quad (14)$$

In the equation above, $Y_{pá\infty}$ is the specific energy provided by a rotor with an infinite number of blades. As for $Y_{pá}$, it represents the specific energy provided by a rotor with a finite number of blades.

When comparing Equations (13) and (14), as $c_{u6} < c_{u5}$ and $c_{u3} \cong c_{u4}$, it is concluded that Ypá < Y_pá∞.

$$Y_{pá} < Y_{pá\infty} \quad (15)$$

This allows us to define what is called Power Deficiency Factor, ε:

$$\varepsilon = {}^{Y_{pá}}\!/_{Y_{pá\infty}} \quad (16)$$

Considering the finite thickness of the blades, the cross section of the region through which the flow passes is reduced with respect to the condition established prior to the rotor blades. This fact does not imply energy variation, so the component C of the absolute velocity remains unchanged, whereas the component Cm, related to the flow, is influenced by the blades thickness (SILVA, 2000).

Applying the continuity equation to a point immediately before the inlet and then to a point immediately after the input, it is possible to determine the following relation, knowing that the flow through those points is the same:

$$Q = \pi \cdot D_4 \cdot b_4 \cdot C_{m3} = (t_4 - e_{t4}) \cdot b_4 \cdot N \cdot c_{m4} \quad (17)$$

In which:

- Q = flow through the rotor;
- D4 = rotor inlet diameter;
- b4 = rotor inlet width;
- t4 = step in the inlet, measured between the edges of two blades next to each other;
- et4 = thickness of the blades at the inlet, measured in the tangential direction;
- N = number of blades in the rotor;
- cm3 = axial component of the absolute velocity of the fluid stream at a point immediately before the rotor inlet (not yet influenced by blade thickness);
- cm4 = axial component of the absolute velocity of the fluid current at a point immediately after the rotor inlet (already in the space between blades).

Note the equation below:

$$t_4 = \frac{\pi \cdot D_4}{N} \therefore N = \frac{\pi \cdot D_4}{t_4} \quad (18)$$

Replacing the result above in the previous equation, we find the following:

$$\pi \cdot D_4 \cdot b_4 \cdot c_{m3} = \pi \cdot D_4 \cdot b_4 \cdot \frac{t_4 - e_{t4}}{t_4} \cdot c_{m4} \tag{19}$$

wherein, it is concluded that:

$$e_{t4} = \frac{e_4}{sin\, \beta_4} \tag{20}$$

in which:

- e4 = inlet blade thickness, measured along a normal vector;
- β4 = inlet inclination angle of the blades.

$$c_{m3} = \frac{t_4 - e_{t4}}{t_4} \cdot c_{m4} \tag{21}$$

Hence:

$$f_{e4} = \frac{t_4 - e_{t4}}{t_4} \tag{22}$$

in which f_{e4} is called rotor inlet choke factor. So, we have:

$$c_{m3} = f_{e4} \cdot c_{m4} \tag{23}$$

Thus, the rotor exit region is reached:

$$c_{m6} = f_{e5} \cdot c_{m5} \tag{24}$$

in which:

- cm6 = axial component of the absolute velocity of a point in space placed immediately after the rotor exit;
- cm5 = axial component of the absolute velocity of a point in space placed immediately before the rotor exit.

Rotor exit choke factor:

$$f_{e5} = \frac{t5 - e_{t5}}{t_5} \tag{25}$$

In which:

- Rotor exit step and:

$$t_5 = \frac{\pi \cdot D_5}{N} \tag{26}$$

- Tangential thickness of the blades at the rotor exit:

$$e_{t5} = \frac{e_5}{sin\, \beta_5} \tag{27}$$

Since the choke factor is always less than 1, it is noticed that the axial components of the absolute velocity that are situated out of the space between the blades present values lower than those within that space.

Combining the influence of the finite number of blades and their finite thickness in the velocity triangles and presenting the inlet triangle in the condition of maximum efficiency (radial inlet, $\alpha_3 = \alpha_4 = 90^o$), we obtain what is represented in Figure (2):

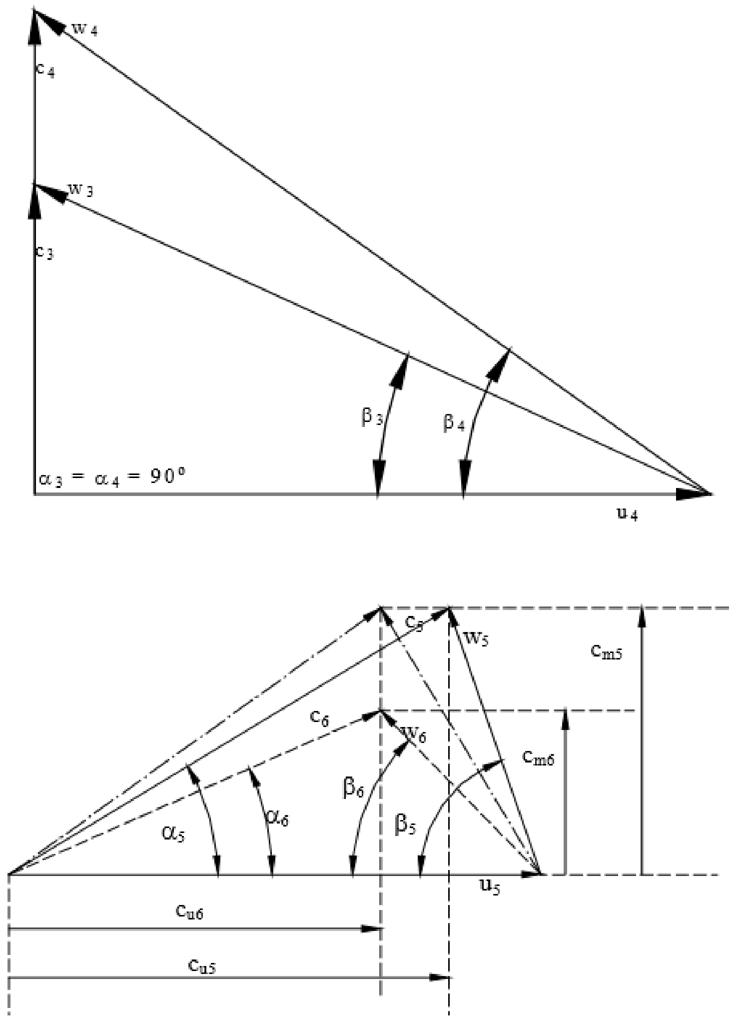

Figure 2. Modification of the inlet and exit velocity triangles of a rotor, considering the influence of a finite number of blades of finite thickness.

2.4 Pump Design Considerations

Based on equations provided by literature, a few considerations on the impeller design have been made. The first of which was to focus on approximately radial, generating turbomachines, in which fluid enters perpendicularly and is expelled parallel to the surface of the impeller, while the blade width increases as it goes further outward. The flow obeys the principle of mass conservation and through the absolute meridional speed, the blade width at its outer end can be determined (FOX, 2006), while its inner end width is a pre-determined project parameter.

For the ideal project conditions, the design was based on the one-dimensional flow tube theory, which assumes an infinite number of blades in the rotor, infinitely close to each other and of infinitesimal thickness. In a real operational condition, the flow, which has no viscosity, causes a movement called relative vortex, due to the number of blades. Based on these theories, the impeller ideal and real triangles of velocities were calculated. Other considerations essential to the project were made (FAIZULLY, 2013):

- The impeller has infinitely thin blades;
- The blades have equal length and equal spacing;
- The blades outlet angle is less than 90°, so that the potential flow can be used to determinate the impeller performance characteristics;
- The blades edges, on both ends, are parallel to the impeller axis;
- The blades have simple curvature;

- The blades have radially variable width;
- The impeller angular velocity (ω) is constant;
- The relative flow through the impeller is constant;
- The fluid flow is constant.

2.5 Results and Discussion

The impeller was designed for a specific operating condition: flow rate of 0.00111 m³/s, pump rotation of 3500 RPM, an efficiency of 33.33% and pressure head (Hb) of 14.95 mH2O. The number of blades was set at 5 and, according to its relationship with the power deficiency factor, an ideal project condition was met (Hu = 35.48 mH2O), validating the one-dimensional flow tube theory. The results obtained from the operating conditions are shown in the table below:

Table 1. Operating Condition

Y_{pumb}	146,5886958	J/kg
Y_{real}	239,870106	J/kg
Y_U	347,9171081	J/Kg
H_{pump}	14,95	m
H_{real}	24,46340126	m
H_U	35,4826867	m

The equations for the ideal inlet and exit velocity triangles were solved. The results are presented both in Table 2 and Figure 3.

Table 2. Ideal Inlet Velocity Triangle

ω	366,5191429	rad/s
U_4	4,874704601	m/s
W_4	5,370418132	m/s
C_4	2,253585136	m/s
$C_{\theta 4}$	0	m/s
$W_{\theta 4}$	4,874704601	m/s
C_{m4}	2,253585136	m/s
W_{m4}	2,253585136	m/s
α_4	90	°
β_4	24,8111898	°

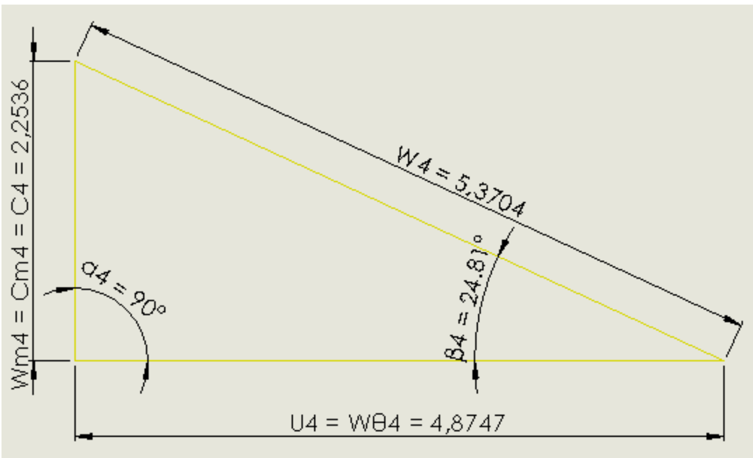

Figure 3. Ideal Inlet Velocity Triangle.

Regarding the exit triangle, the calculations resemble those of the inlet triangle, with a small particularity in determining the angle β_5 and the width b_5 of the exit blade. Those two parameters have an interdependence, which was delimited from 15 to 40º for angle β_5, which is an ideal design range, according to literature. Thus, a curve was generated through C_{m5}, which is shown below. A value in the curve that adequately presented favorable results to the construction of the volute, as well as an optimal working condition, was then chosen.

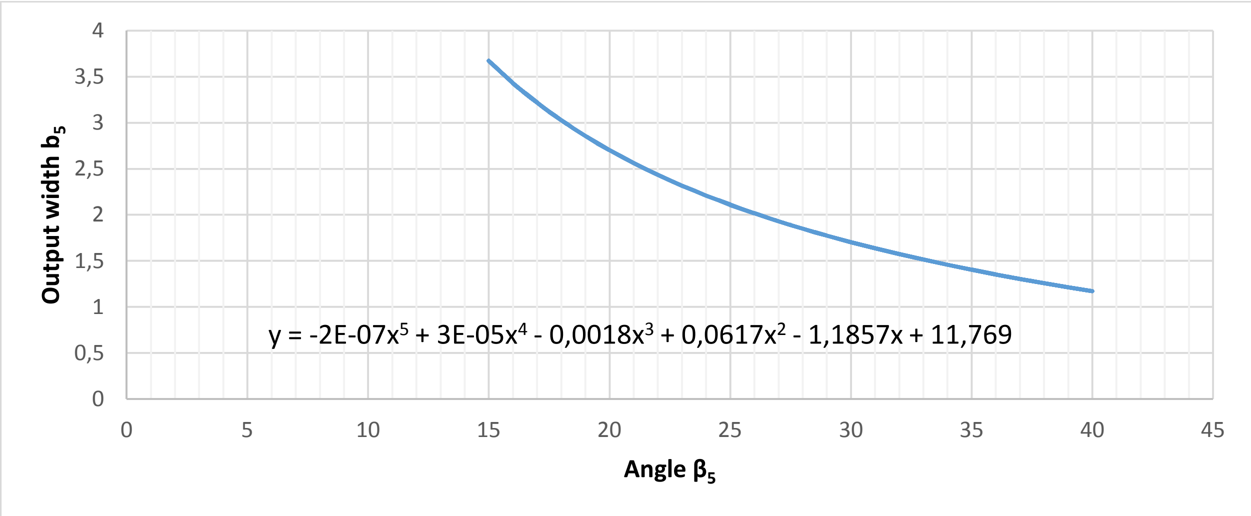

Figure 4. Relationship between width b5 and angle β5.

Therefore, based on that curve, it is defined that β5 = 21 ° and b5 = 2.56 [mm]. Once these parameters were set, it became possible to solve the entire rotor exit velocity triangle.

Table 3. Ideal Exit Velocity Triangle

ω	366,5191429	rad/s
U_5	20,34181243	m/s
W_5	3,468653746	m/s
C_5	17,14865715	m/s
$C_{\theta5}$	17,10354519	m/s
$W_{\theta5}$	3,238267243	m/s
C_{m5}	1,243054331	m/s
W_{m5}	1,243054331	m/s
α_5	4,156843593	°
β_5	21	°

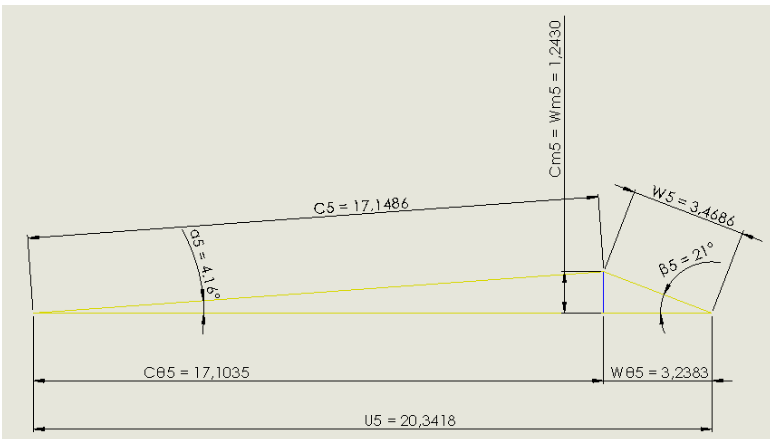

Figure 5. Ideal Exit Velocity Triangle.

To solve for the real velocity triangle, it is necessary to determine the choke factor (f), which depends on the power deficiency factor. However, it is not correct to directly use values disclosed in literature. Through the pre-defined number of blades, the necessary iterative calculations were made, so that the value defined in the project were the same as calculated. After six iterations, this adjustment was made. Thus, the results obtained for the real inlet and exit velocity triangles are shown below:

Table 4. Real Inlet Velocity Triangle

U_3	4,874704601	m/s
W_3	4,931844422	m/s
C_3	0,748561592	m/s
$C_{\theta3}$	0	m/s
$W_{\theta3}$	4,874704601	m/s
C_{m3}	0,748561592	m/s
W_{m3}	0,748561592	m/s
α_3	90	º
β_3	8,73016768	º
Z_{br}	5	-
λ_n	1,531282201	Rad
MS	0,001452814	-
Φ	1,0125	-
P	0,429339618	-
□	0,699623789	-
E	0,004	m
e_u	0,011161712	m
T	0,016713273	m
f_{e4}	0,332164771	-

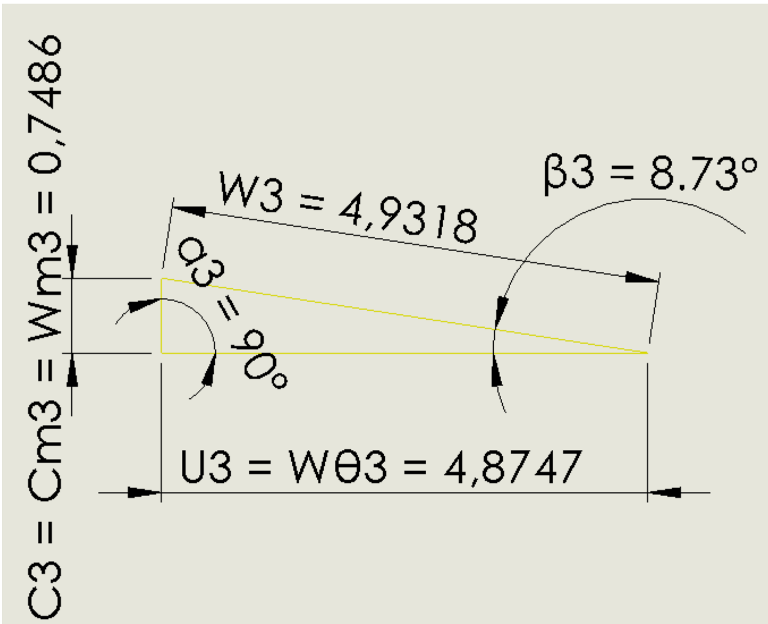

Figure 6. Real Inlet Velocity Triangle.

Table 5. Real Exit Velocity Triangle

U_6	20,34181243	m/s
W_6	2,913531117	m/s
C_6	11,83810835	m/s
$C_{\theta6}$	11,79197315	m/s
$W_{\theta6}$	8,549839282	m/s
C_{m6}	1,044116172	m/s
W_{m6}	1,044116172	m/s
α_6	5,060038626	º

β_6	6,962550676	º
Z_{br}	5	-
λ_n	1,531282201	rad
MS	0,001452814	-
φ	1,0125	-
P	0,429339618	-
□	0,699623789	-
e	0,004	m
e_u	0,011161712	m
t	0,069743357	m
f_{e5}	0,839960206	-

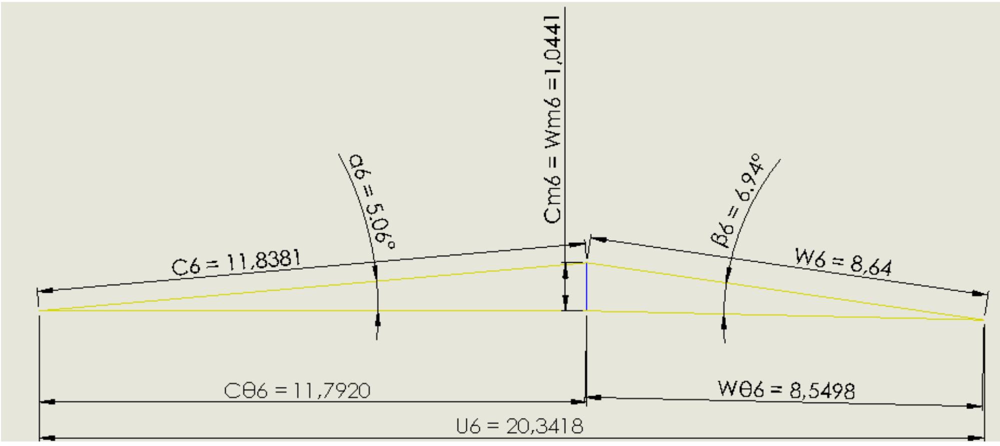

Figure 7. Real Exit Velocity Triangle

With these design parameters, it was possible to determine the constructive aspects of the impeller and, following the design methodology of a rotor blade (Figure 8), the 2D (Figure 9) and 3D (Figure 10) prototypes were designed in CAD.

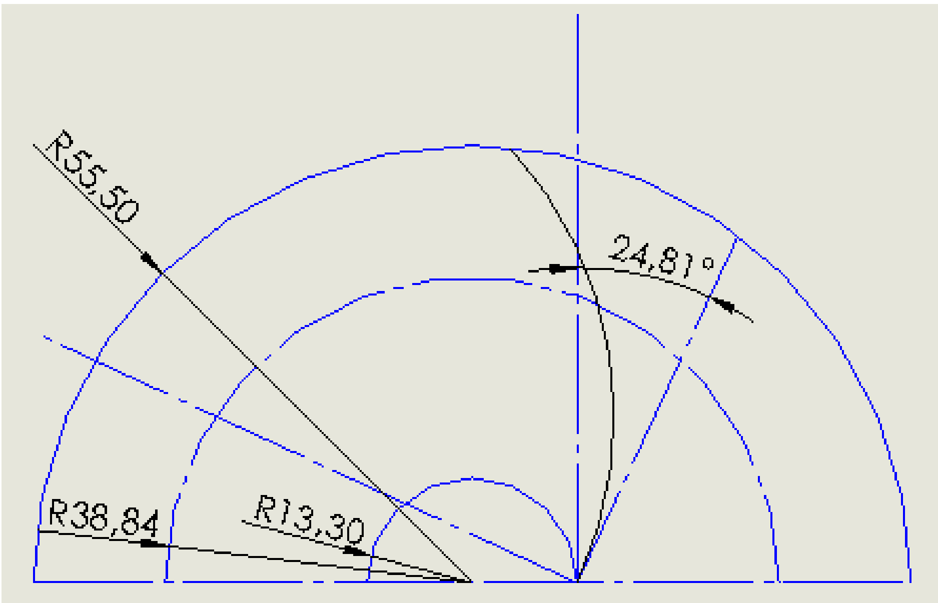

Figure 8. Design of a backward-facing blade.

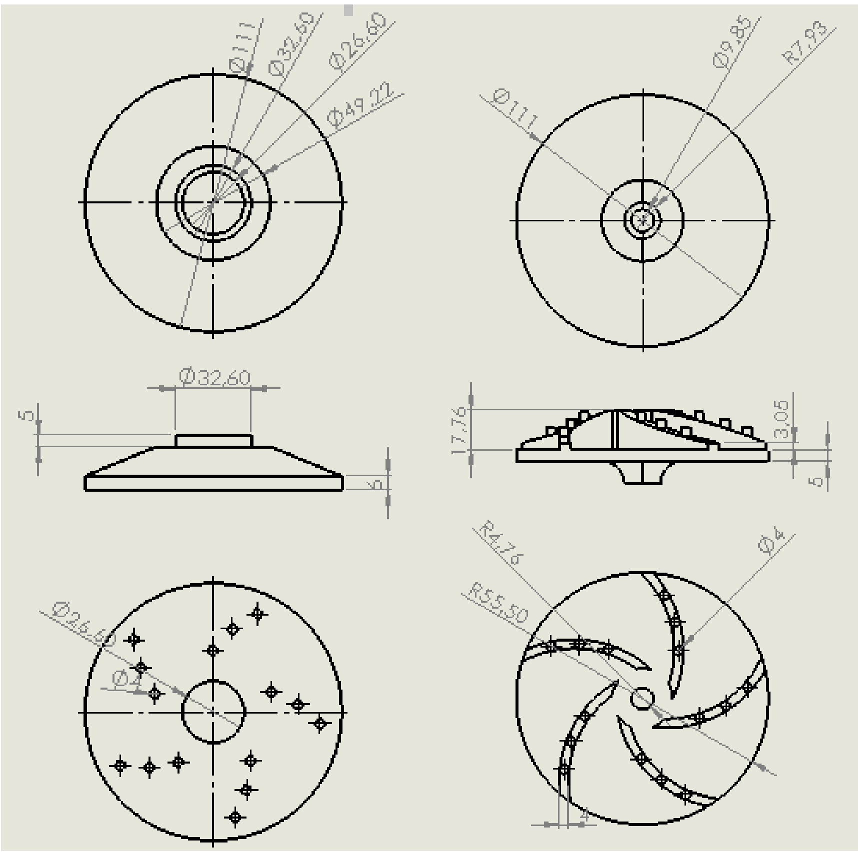

Figure 9. Top and side views of the 2D impeller.

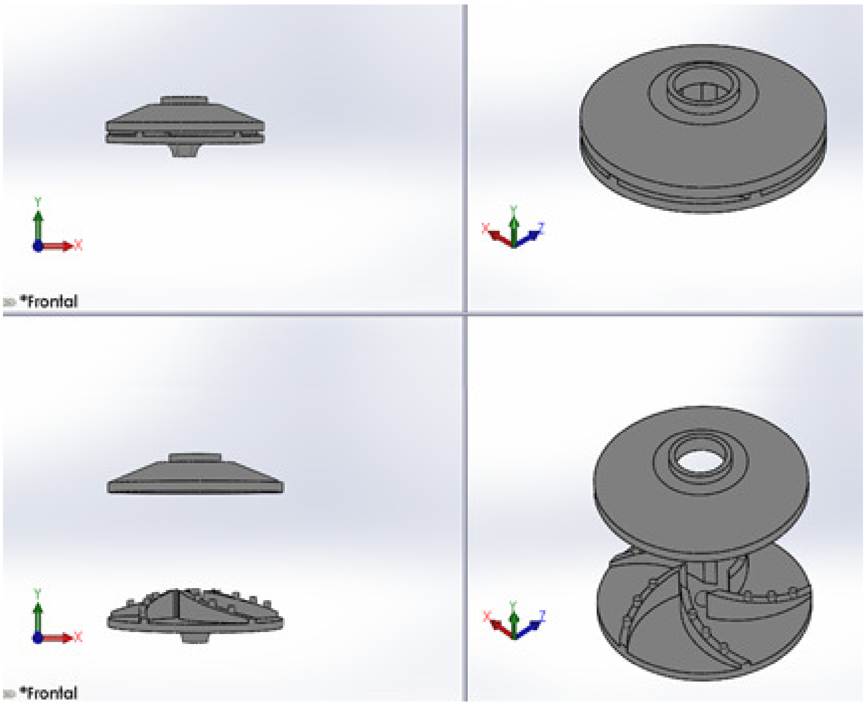

Figure 10. Impeller final 3D version.

In addition to the computational model, the impeller was printed (Figure 11) in VeroWhite Plus, which is a rigid and durable material, used in the aeronautical industry.

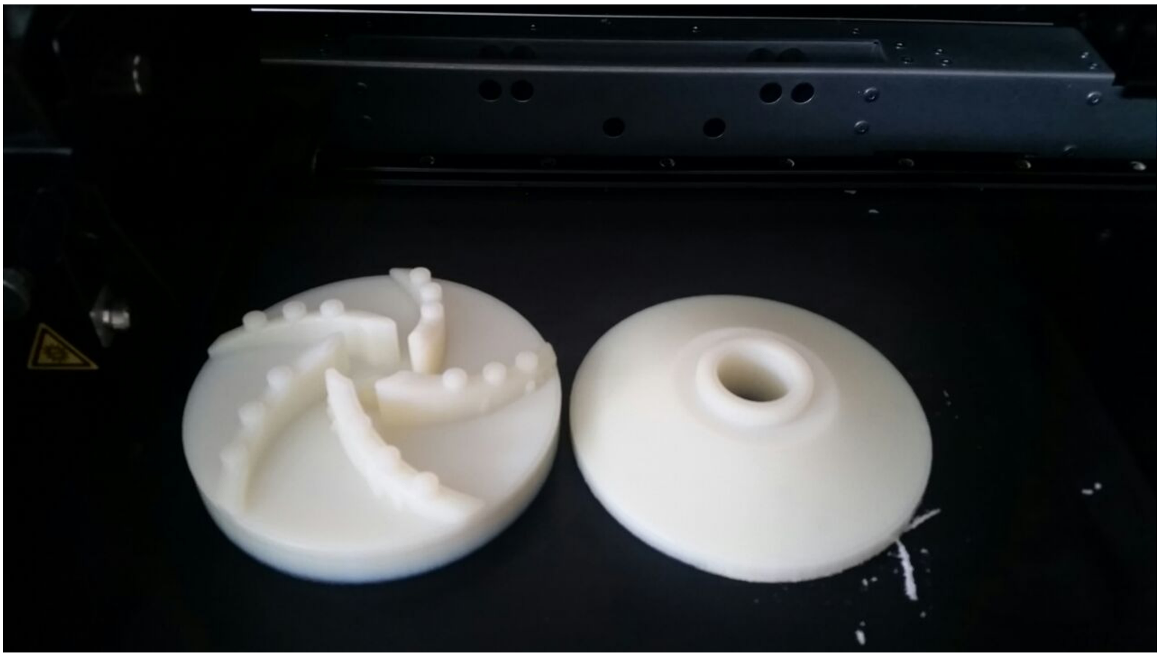

Figure 11. 3D printed impeller.

Then, computational simulations were carried out with the rotor/volute assembly. The necessary project parameters for the simulation were used to generate the 3D geometry using SolidWorks® 2016. Once the parts were made for the whole set, with the Flow Simulation® tool, a computational mesh was elaborated and the boundary conditions, set. Said conditions are:

- Wall condition: "Real Wall" option was chosen, taking into consideration the principle of adherence and fluid relative speed near the walls equal to zero;
- Inlet condition: the total pressure, total temperature and total flow conditions were used;
- Outlet condition: ambient pressure condition was used.

As for the mesh, the hexahedral type was chosen, structured all around the domain, as shown in Figure 12.

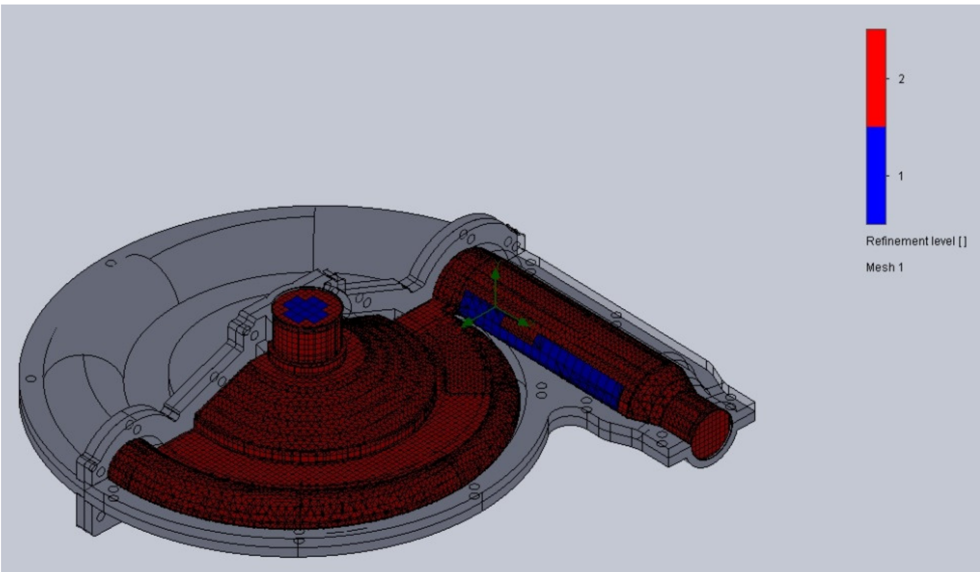

Figure 12. Generated mesh.

Results show the fluid speed through the impeller increases along its radius and shows a slight variation at the exit. Figure 13 shows the distribution of speed in the impeller/volute set.

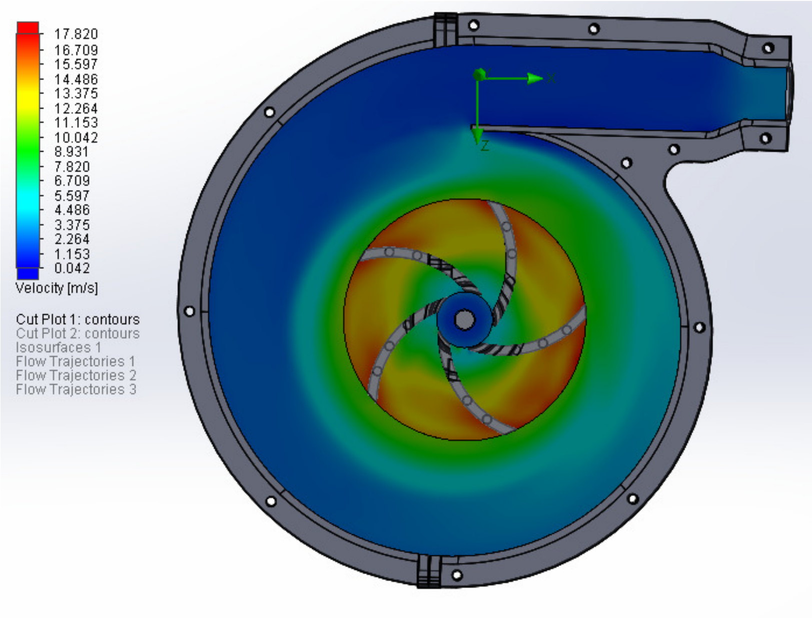

Figure 13. Speed distribution.

3. CONCLUSIONS

It was expected to design an impeller for high-efficiency centrifugal pumps, as well as building it and perform computational analysis of its behavior during certain operational conditions. Such goal was attained and validated with the one-dimensional tube flow theory, thus proving its effectiveness. The results obtained are extremely satisfactory and a key factor for this success are the considerations made along the middle phases of the project, considering the existing iterations among the variables and the relations each had on one another.

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