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Evolutionary structural optimization for natural frequency maximization of structures with cyclic periodicity

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Abstract. *In this work, the Bi-directional Evolutionary Structural Optimization (BESO) method is extends to dynamic problems of circular periodic structures. The maximization of the natural frequency of a structure could be interpreted as the minimization of the dynamic compliance of the structure for a certain frequency. It can be of great importance in many fields of engineering project such as the automotive industry, aerospace, among others. A methodology is proposed to maximize the natural frequency of circular structures with cyclic periodicity. It is considered that the periodic structures have equal sectors forming the system. A sensitivity analysis is developed considering periodic dynamic problems and composite materials in the context of the BESO method. In order to show the efficiency of the proposed formulation, several cyclic systems has been investigated and different configurations of periodic cells are presented.*
Keywords: *Topology Optimization, BESO Method, Natural Frequency, Cyclic Periodic Structures*

1. INTRODUCTION

In the last years, the topology optimization has become a very important tool in the industry and research, a large number of different formulations has been developed based on different optimization methodologies (Bendsoe and Kikuchi, 1988), (Zhou and Rozvany, 1991), (Xie and Steven, 1993), (Xie and Steven, 1997), (Bendsoe and Sigmund, 2003), (Sethian and Wiegmann, 2000), (Wang *et al.*, 2003), (Huang and Xie, 2010). The main idea of topology optimization is to find a proper distribution of material in a given design domain that minimize an objective function taking into account a certain number of constraints.

The proposed methodology herein is based on the evolutionary topology optimization approach, where the finite element method is used in the analysis of the system and the design domain updating has a binary feature (Huang and Xie, 2010). It is presented a procedure aiming the maximization of the first natural frequency of the system subjected to a prescribed volume as the constraint. Firstly, it is considered the optimization of structures composed of one single material, thus, the optimized topologies are composed of material and void phases. Secondly, it is considered the analysis of a composite material composed of two-phase material with different elasticity modulus, and the final topologies present two phases, one for each material.

The optimization of structures with cyclic periodicity like wheel rim, flywheel, impeller of centrifugal pump, etc, has been done using different approaches as can be seen in (Cagan and Agogino, 1991), (Giger and Ermanni., 2005), (Cugini *et al.*, 2009) and an investigation considering the topology optimization approach are presented in the work of (Moses and Ryvkin, 2003) and (Zuo *et al.*, 2011) for the static cases. In the current work, the dynamic approach is developed for the maximization of the fundamental natural frequency of cyclic systems.

In the context of the BESO method, many works have already been done in the study of the natural frequency optimization and in the minimization of dynamic response of the structures (Xie and Steven, 1996), (Huang *et al.*, 2010), (Zuo *et al.*, 2013), (Vicente *et al.*, 2015), (Picelli *et al.*, 2015), (Vicente *et al.*, 2016a), (Vicente *et al.*, 2016b), (Picelli *et al.*, 2017). Herein, it is extended these applications to the case of cyclic symmetric structures.

2. COMPUTATIONAL PROCEDURE

Considering the maximization of the natural frequency of periodic structures, the BESO method can be formulated as follows (Huang and Xie, 2010)

$$Maximize \quad \omega_n^2 = \frac{\mathbf{u}_j^T \mathbf{K} \mathbf{u}_j}{\mathbf{u}_j^T \mathbf{M} \mathbf{u}_j} \quad (Objective \quad function)$$

$$Subject \quad to \quad V^* - \sum_{i=1}^{N} V_{k,i} x_{k,i} = 0 \quad (Volume \quad constraint)$$

$$and$$

$$x_{1,i} = x_{2,i} = ... = x_{m,i} \quad (Periodic \quad constraint)$$

$$x_{k,i} = x_{min} \quad or \quad x_{k,i} = 1$$

$$k = 1, 2, ..., m \quad (number \quad of \quad periodic \quad cell) \quad and \quad i = 1, 2, ..., N \quad (number \quad of \quad elements)$$

where ω_j^2 is the Rayleigh quotient, ω_j is the j^{th} natural frequency $(j = 1, 2...n)$ and $\mathbf{u}_j$ is the eigenvector (mode shape of ω_j), K and M is the stiffness and mass global matrix respectively, V^* is the current volume of the structure, V_i is the volume of the ith element and N is the total number of elements in the mesh, x_i is the design variable, if the element is removed this value is very small (x_{min}), if the element is added the value is 1.

Following the material interpolation proposed in Bendsoe and Sigmund (1999), the sensitivity number to void and solid elements can be determined as follows,

$$\alpha_i = \begin{cases} \frac{\mathbf{u}_j^T}{\omega_j} \left[\left(\frac{1-x_{min}}{1-x_{min}^p} \right) \mathbf{K}^e - \frac{\omega_j^2}{p} \mathbf{M}^e \right] \mathbf{u}_j, & for \quad x_i = 1 \\ \frac{\mathbf{u}_j^T}{\omega_j} \left[\left(\frac{x_{min}^{p-1} - x_{min}^p}{1-x_{min}^p} \right) \mathbf{K}^e - \frac{\omega_j^2}{p} \mathbf{M}^e \right] \mathbf{u}_j, & for \quad x_i = x_{min} \end{cases} \tag{1}$$

where $\mathbf{K}^e$ and $\mathbf{M}^e$ is the stiffness and mass matrix of the ith element respectively. The sensitivity number for each element is calculated and then ranked to evaluate which elements will be removed and which will be added in the structure domain, using the BESO procedure.

Extending the procedure to composite materials, the sensitivity number to analysis with two-phase material can be determined as follows (Bendsoe and Sigmund, 1999),

$$\alpha_i = \begin{cases} \frac{\mathbf{u}_j^T}{\omega_j} \left[\left(\frac{1-x_{min}}{1-x_{min}^p} \right) (\mathbf{K}_1^e - \mathbf{K}_2^e) - \frac{\omega_j^2}{p} (\mathbf{M}_1^e - \mathbf{M}_2^e) \right] \mathbf{u}_j, & for \quad material \quad 1 \\ \frac{\mathbf{u}_j^T}{\omega_j} \left[\left(\frac{x_{min}^{p-1} - x_{min}^p}{1-x_{min}^p} \right) (\mathbf{K}_1^e - \mathbf{K}_2^e) - \frac{\omega_j^2}{p} (\mathbf{M}_1^e - \mathbf{M}_2^e) \right] \mathbf{u}_j, & for \quad material \quad 2 \end{cases} \tag{2}$$

where $\mathbf{K}_1^e$ and $\mathbf{M}_1^e$ is the stiffness and mass matrix of the material 1, and $\mathbf{K}_2^e$ and $\mathbf{M}_2^e$ is the stiffness and mass matrix of the material 2. In this case, the interpolation of the two-phases materials was considered with $E_1 > E_2$, therefore, the material 1 is stiffer than the material 2. The optimization starts from full design composed of material 1 and the evolution of topology begins with a prescribed evolutionary volume ratio (ER), which is defined as the proportion of volume reduction of material 1 relative to the total volume, concurrently the volume of material 2 gradually increases until reach the volume constraint.

The periodic subdivision is used to create layout periodicity in the structure. To apply the periodic conditions for a structure divided into cells, a mean value of the sensitivities of the corresponding elements in each cell must be calculated using Eq. (3) (Moses and Ryvkin, 2003).

$$\bar{\alpha} = \frac{1}{m} \sum_{k=1}^{m} \alpha_i^k \tag{3}$$

where $\bar{\alpha}_i$ is the periodic sensitivity of the ith element in each cell, k is the cell number and m is the total number of cell. In this work a circular periodic structure with *m* number of cells is considered, as shown in Fig. 1.

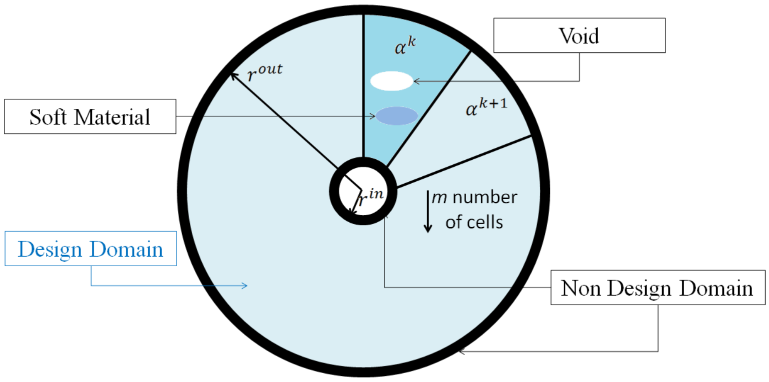


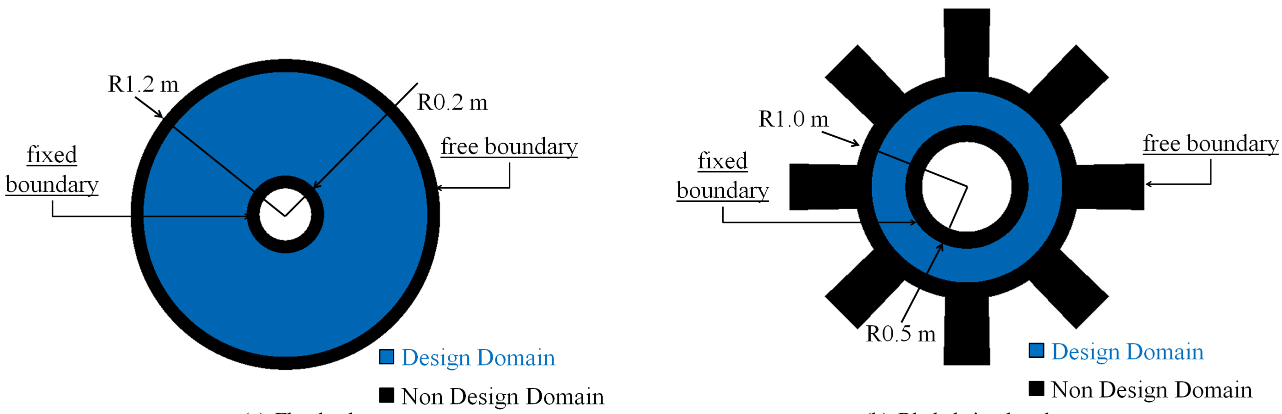
Figure 1. Considered design domain, cyclic periodic structure with *m* cells.

3. NUMERICAL RESULTS

In this numerical analysis, the proposed methodology is used to maximize the first natural frequency of the periodic system composed of *m* equal cells in two different geometrical cyclic domain.

The first case is shown in Fig. 2. The model is a flywheel disc with cycle period, the dimensions are present in Fig. 2.a. The structural optimization of the design domain consider a periodic structure composed of 4 cells in the first case and 8 cells in the second case. The blue area is the design domain and the black area is the non-design domain. The inner ring is fixed as the boundary condition.

The second case analyzed is a model of a disc with 8 blades. In Fig. 2.b is shown the geometry, dimensions and boundary conditions. Aiming the maximization of the first mode of vibration the proposed procedure is applied. The blue area is the design domain and the black area is the non-design domain. As in the first case, the inner ring is fixed as the boundary condition.



(a) Flywheel (b) Bladed circular plate

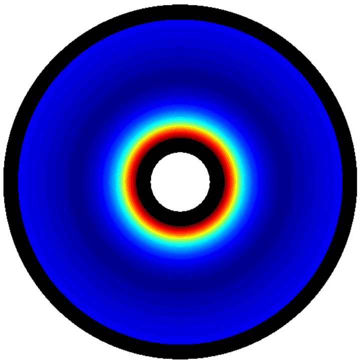
Figure 2. The two different geometrical cyclic domains studied in this current paper

3.1 One material result

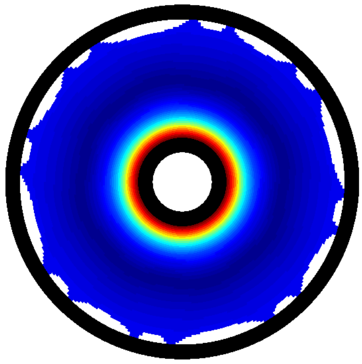
The topology optimization procedure is used to maximize the natural frequency of the system taking into account void and solid phases distributed into the design domain.

Firstly, it is presented the optimization of flywheel (Fig. 2.a), subject to volume constraints of 50%. The used material has the follow properties, Young's Modulus in 200 GPa and Poisson's ratio is 0.3.

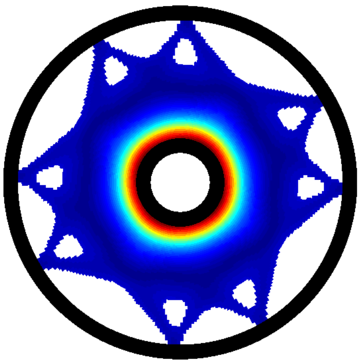
Figure 3 shows the final topology and the evolutionary history of the flywheel 4-cells. Initially, the fundamental frequency was 203.6 Hz and after the optimization its value increases to 247.6 Hz.



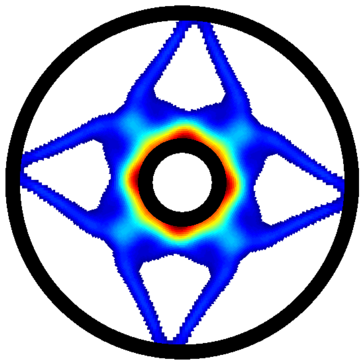
(a) Initial design domain



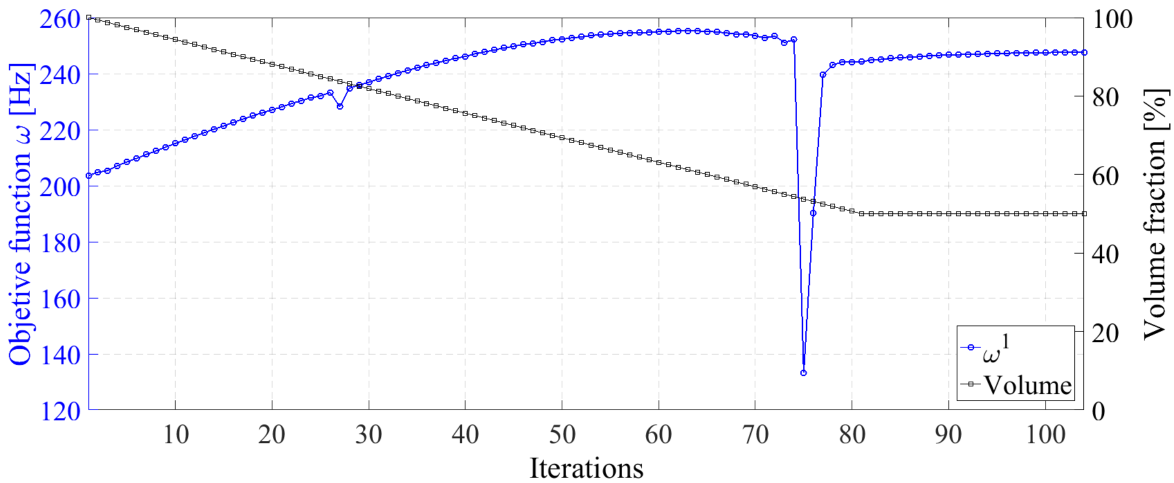
(b) Iteration 10



(c) Iteration 40



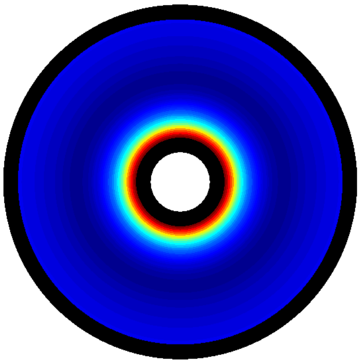
(d) Final topology



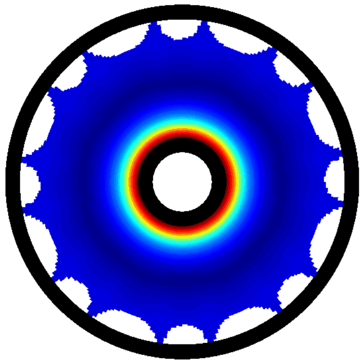
(e) Objective function and volume fraction histories.

Figure 3. Flywheel disc with 4-cells periodicity

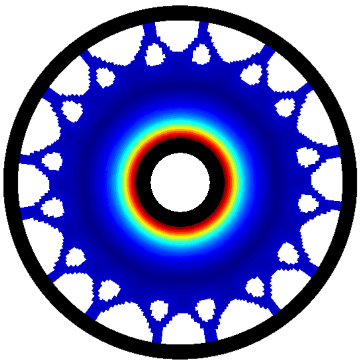
Figure 4 shows the final topology and the history of the objective function to the flywheel with 8-cells. The fundamental frequency was 203.6 Hz and after the optimization it is increased to 250.8 Hz.



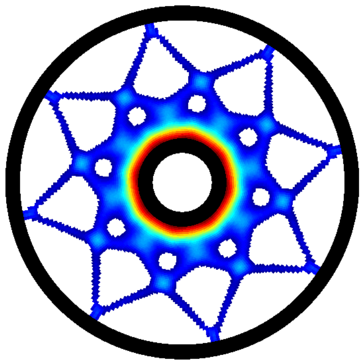
(a) Initial design domain



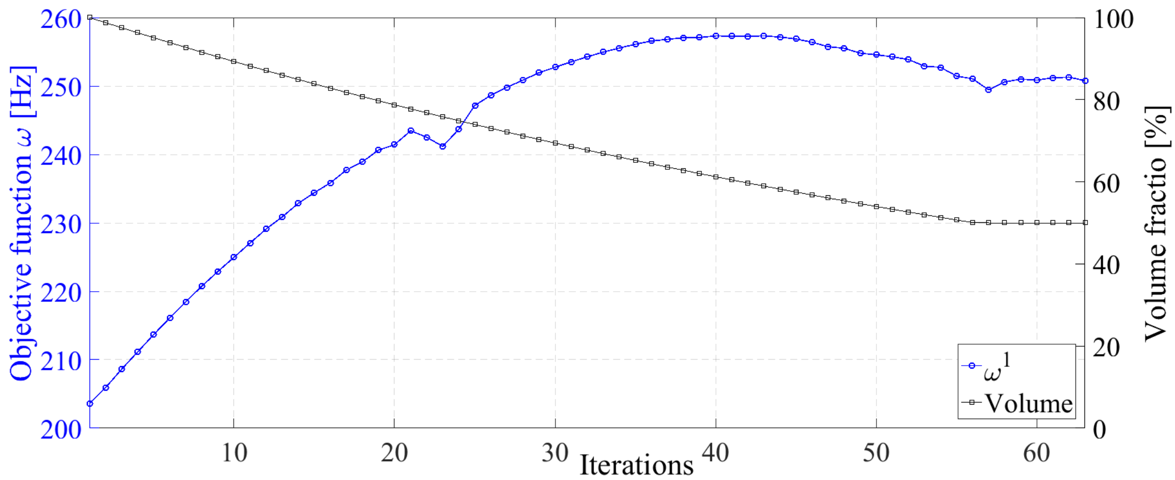
(b) Iteration 11



(c) Iteration 21



(d) Final topology

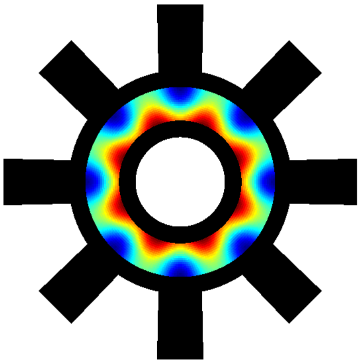


(e) Objective function and volume fraction histories.

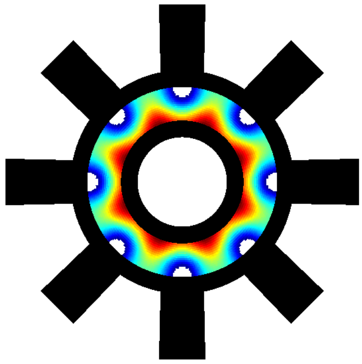
Figure 4. Flywheel disc with 8-cells periodicity.

In the optimization of the disc with 8 blades, Fig. 2.b it is considered the 8-cells periodicity, each one with one blade. The objective is maximize of fundamental frequency subject to a volume constraint of 50%. The considered material has the Young's Modulus of 200 GPa and Poisson's ratio of 0.3.

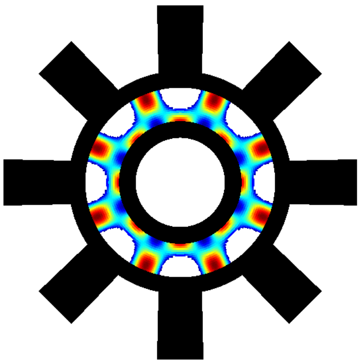
Figure 5 presents the evolution histories of the objective function, the topologies and the volume fraction.



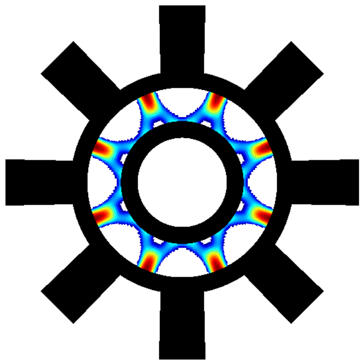
(a) Initial design domain



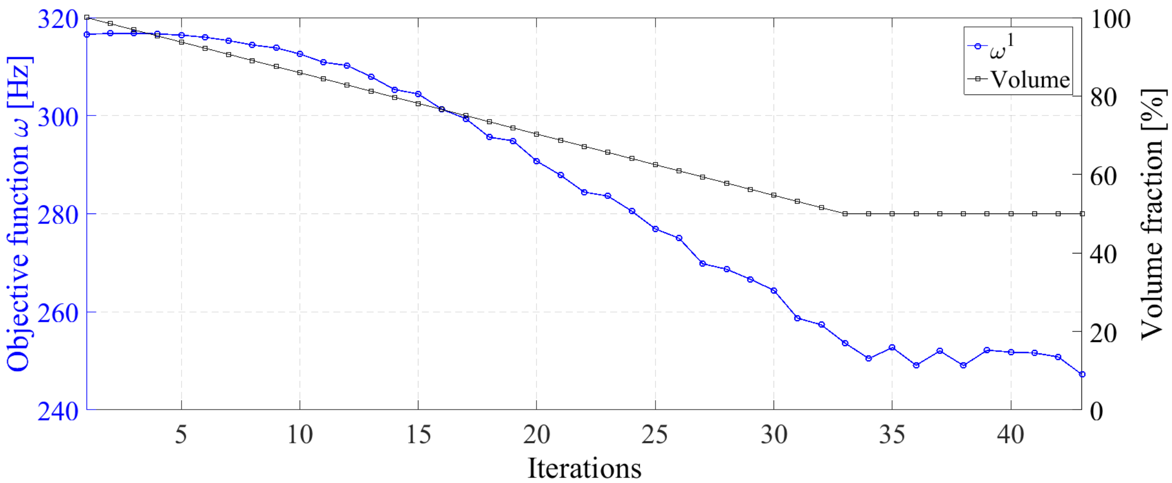
(b) Iteration 5



(c) Iteration 20



(d) Final topology



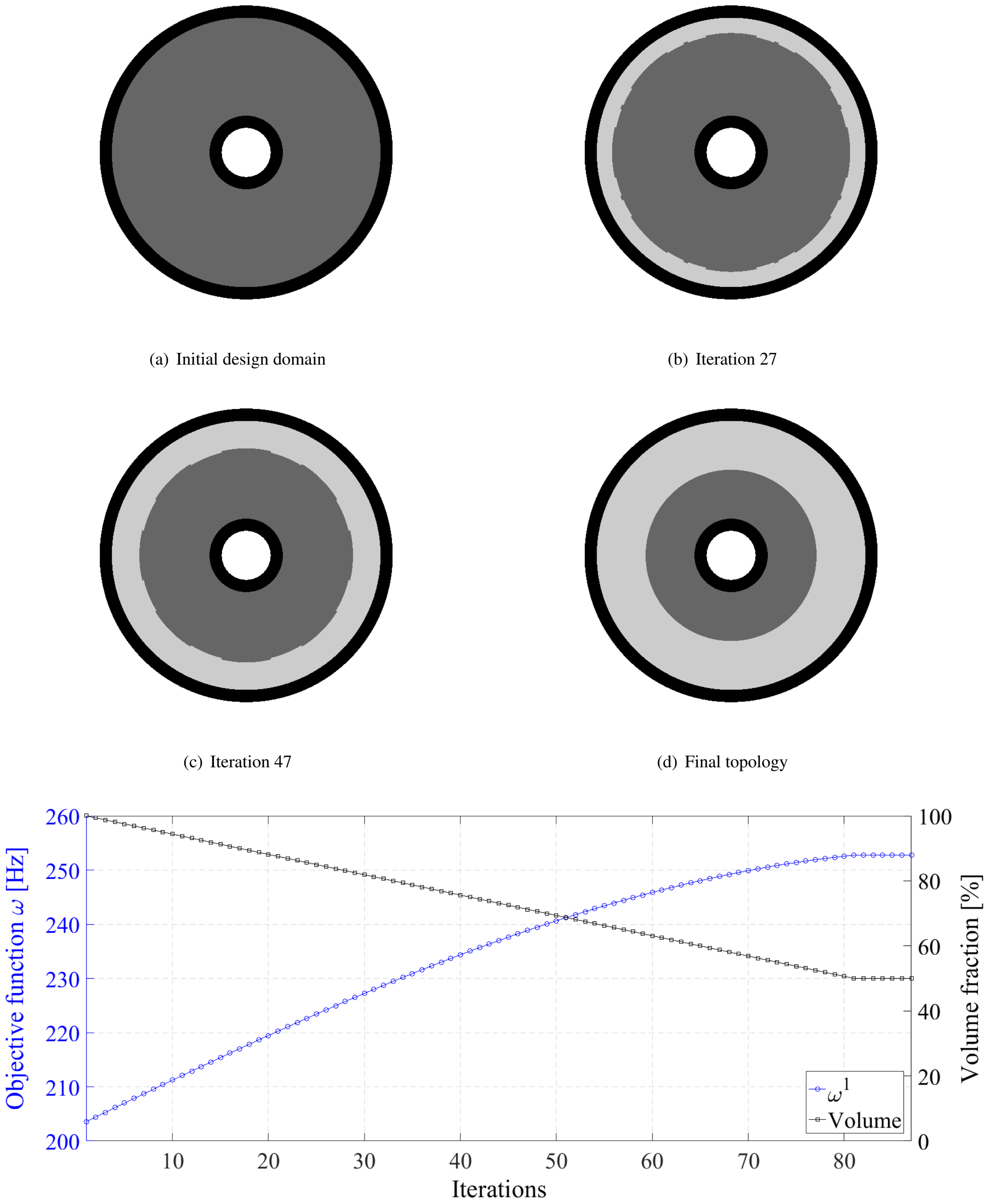
(e) Objective function and volume fraction histories.

Figure 5. Bladed circular plate with 8-cells periodicity.

3.2 Two-phase material result

The analysis with two materials was considered in this topology optimization case. The objective was evaluate the distribution of a composite material with two different phases. The material stiffer is has Young's Modulus of 200 GPa and Poisson's ratio of 0.3, and the softer material with Young's Modulus of 70 GPa and Poisson's ratio of 0.3.

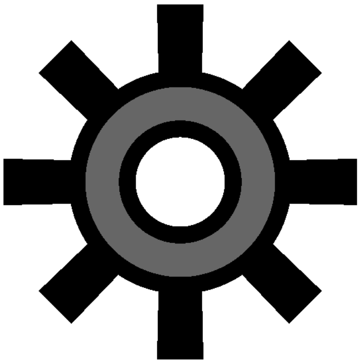
The same flywheel analyzed in case of one material is considered. Fig. 6 shows the final topology and the evolutionary history, the color dark gray represents the stiffer material and the color light gray is the softer material. In the first iteration, all elements were composed with material stiffer. In Fig. 6.e, the volume fraction is related with the stiffer material. The fundamental frequency was 203.6 Hz and after the optimization it is increment to 252.6 Hz.



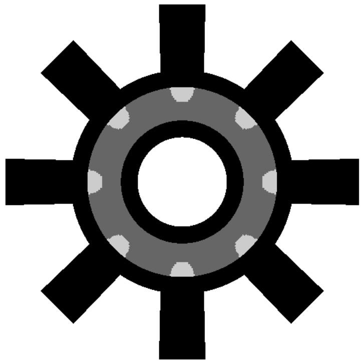
(e) Objective function and volume fraction histories.

Figure 6. Flywheel disc with 8-cells periodicity and two-phase material.

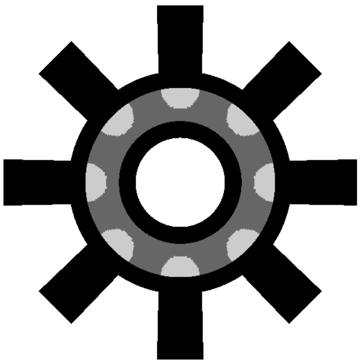
The optimization with two materials for the disc with 8 blades is shown in Fig. 7. The color dark gray is the material stiffer and the color light gray is the softer. The initial fundamental frequency was 324.4 Hz and after the optimization, its value reduced to 306.4 Hz.



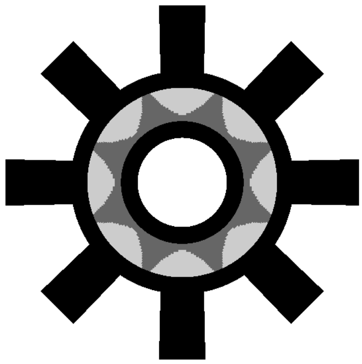
(a) Initial design domain



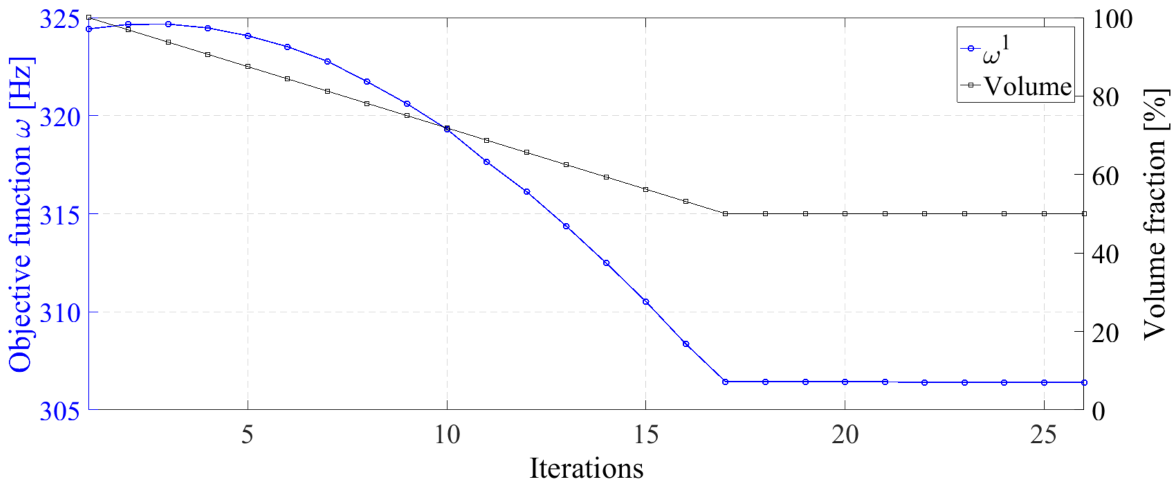
(b) Iteration 5



(c) Iteration 10



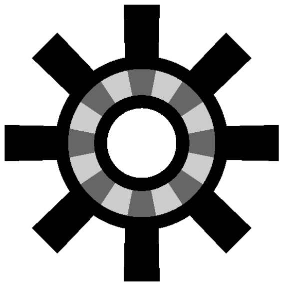
(d) Final topology



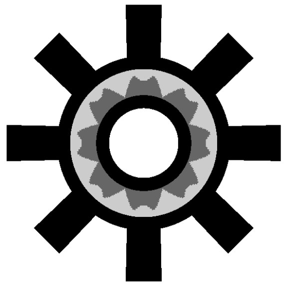
(e) Objective function and volume fraction histories.

Figure 7. Bladed circular plate with 8-cells periodicity and two-phase material.

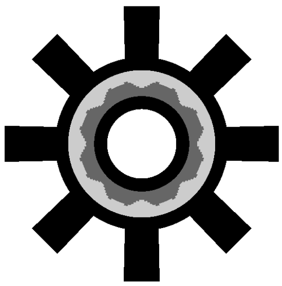
It was also considered that the structure starts with 50% of the stiffer material and 50% of softer material, the objective is evaluable optimization procedure with a constant volume. The result is shown in Fig. 8. The initial fundamental frequency was 285.5 Hz, after the optimization the value increased to 301.8 Hz and the total number of iterations was 26.



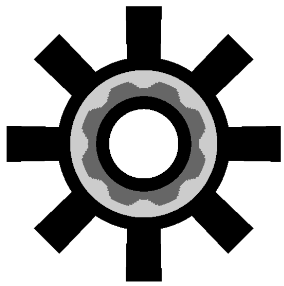
(a) Initial guess design



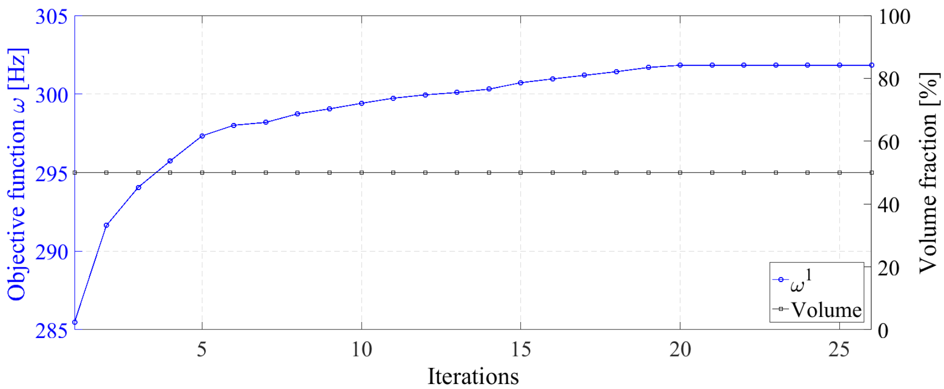
(b) Iteration 5



(c) Iteration 16



(d) Final topology



(e) Objective function and volume fraction histories.

Figure 8. Bladed circular plate with 8-cells periodicity and two-phase material with constant volume.

4. CONCLUSIONS

In this work, a topology optimization approach has been implemented for the maximization of the natural frequency of circular structures taking into account the cyclic periodicity of the systems. A sensitivity analysis is presented for the maximization of the natural frequency of circular structures composed of symmetric periodic cells. A prescribed final volume is imposed as the constraint in the optimization formulation. It was considered system composed of one and two materials in the optimization cases. Numerical examples are presented using the proposed methodology considering free vibration problems. Different designs has been obtained in the optimization of circular plates depending of the periodicity constraint. In the analysis of a disc with blades at constant volume, the objective function evolution has presented a smoothed convergence. The results indicate the efficiency of the proposed algorithm, as converged the solutions to the analyzed cases.

5. ACKNOWLEDGEMENTS

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