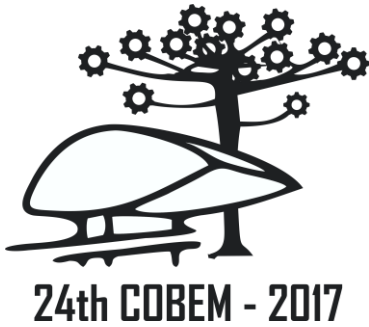


**COBEM-2017-2508****SE(B) SPECIMENS UNDER 4-POINT BENDING – RECOMMENDED
ROLLER SPAN AND SOLUTIONS FOR K_I AND ELASTIC COMPLIANCE****Gustavo Henrique Bolognesi Donato**^{1,2}¹ gdonato@fei.edu.br**Kaue Lucon Carvalho**²**Victor Lucas Götl**²² Centro Universitário FEI, Humberto de Alencar Castelo Branco Av., 3972,
São Bernardo do Campo, SP, ZIP 09850-901

Abstract. This work aims to evaluate and develop good practices for fatigue crack growth and fracture toughness testing of SE(B) specimens loaded under 4-point bending. Specifically, this study: i) investigates the most appropriate upper rollers' span based on applied loads and Hertzian stresses; ii) and develops numerical solutions for K_I and elastic unloading compliance (C) considering varying loading schemes and geometrical features. The aforementioned solutions derive from highly refined finite element models including 3D effects and containing cracks - the results are compared to available data from the literature in order to validate the methodology and to improve the knowledge in the field. The obtained results revealed that, considering the studied conditions, the recommended span between upper rollers is $L = W$, which minimizes local indentation and stress disturbance in the near-tip crack region, while at the same time guarantees high bending moment to induce severe crack driving forces. Both K_I and elastic unloading compliance solutions could be obtained as expected, but formulae (or normalization approaches) for SE(B) under 4-point bending differ from those available for 3-point bending, thus deserving attention and further experimental validation.

Keywords: SE(B) specimens, 4-point bending, Rollers' span, Elastic Unloading Compliance, K_I

1. INTRODUCTION

The presence of cracks or crack-like defects in high-responsibility components and structures can lead to unexpected (and sometimes catastrophic) failures, even when the remote applied stress is lower than the yield strength. The reduction of the resistant area caused by the crack, combined to the severe stress concentration (singularity) at the crack-tip, can activate (considering monotonic loading) the phenomenology of plastic collapse, ductile tearing or fracture, depending on the elastic-plastic response and fracture toughness of the employed material. If cyclic loading exists, fatigue crack growth can also be considered a relevant failure mechanism. In common, all aforementioned failure types must be predicted and if possible avoided, which demands accurate mechanical testing and properties.

Testing fatigue and fracture specimens containing cracks is not trivial, since demands good understanding of the loading schemes, stress fields in the near-tip region and many other effects that can alter the obtained mechanical properties, such as those caused by plasticity, friction, localized indentation, large displacements, among others. Several standards are available for laboratory testing, but in some cases, new geometries or testing conditions are considered unsolved and key-issues for accurate testing. The current research effort focus in one of such situations; tries to understand the limitations of SE(B) specimens under 3-point bending (ANDERSON, 2017) and develops the technology for testing 4-point SE(B) specimens as an alternative solution that is less investigated in the literature.

Considering modern high toughness structural steels and aluminum alloys, fatigue crack growth response (da/dN vs. ΔK or ΔJ) and fracture toughness against ductile tearing (quantified by R -curves) are of paramount relevance for reliable structural integrity evaluations (SURESH, 1998; ANDERSON, 2017). A revision of the recommended practices presented by API 579 (2016), BS 7910 (2013), among other guides for assessing the acceptability of flaws in metallic structures is interesting and can reveal to the reader the relevance and use of such mechanical properties. In terms of the related mechanical testing, they are based on fracture mechanics theoretical background and depends on appropriate specimens containing cracks. The main geometries employed by ASTM-E1820 (2016) and ASTM-E399 (2012) are the compact under tension C(T) (Fig. 1(a)) and the single-edge notched under 3-point bending SE(B) (Fig. 1(b)).

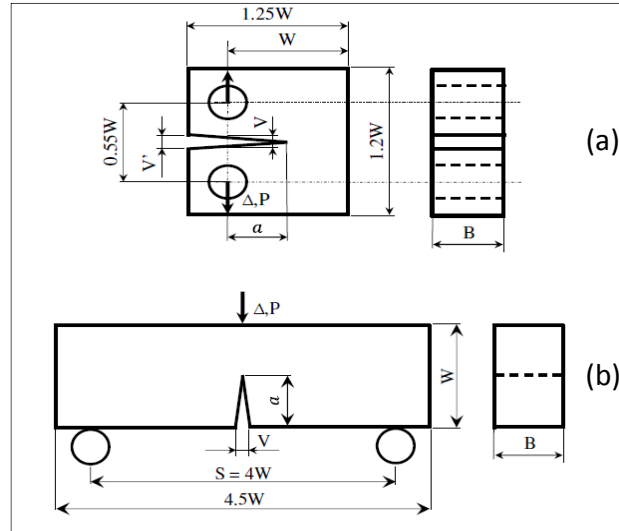


Figure 1. (a) C(T) and (b) SE(B) specimens for fatigue and fracture toughness tests.

C(T) specimens are widely employed, have its testing techniques well documented and demand less material than SE(B) samples. However, the occurrence of large deflections and friction between the sample and the loading pins represents disadvantages of this geometry. As an alternative, SE(B) specimens (which are also widely employed for laboratory testing) present less errors caused by large deflections and the friction is comparatively lower, which allow the control systems of servo-hydraulic testing machines to work better. On the other hand, the 3-point bending causes the central upper roll to establish a contact resulting in an indentation on the crack propagation plane. It may cause changes on the stress fields, imprecision in crack-size estimation and instability of the experimental results; in addition, the crack plane presents shear stresses and a bending moment that varies in the horizontal vicinity (see fig. 2(a)). To minimize such contact effects, SE(B) specimens loaded under 4-point bending (Fig. 2(b)) are gaining relevance. This loading scheme is favorable since imposes to the crack plane a uniform bending moment without shear or contact stresses. The main challenge for employing the 4-point loading scheme is that solutions for elastic compliance (C) and K_I are not widely available or detailed in the literature; in addition, there is no consensus about the ideal upper rollers' span; in fact, the conducted review of the literature revealed very limited results and comprehensive applications of this geometry, which calls the attention for the opportunity to investigate good practices regarding this specimen.

In this scenario, the objectives of this work are: i) present a review of the techniques and solutions available in the literature for testing 4-point bending SE(B), such as the work of Schwalbe et al. (2002); ii) define the most appropriate specimen geometry (in special rollers' span) based on Hertzian contact stresses and desired bending moments; iii) develop K_I solutions considering varying geometrical features; iv) develop respective elastic compliance solutions that enable the prediction of the instantaneous crack size during tests on this geometry. This will favor practical applications of the studied geometry for accurate fatigue/fracture testing and structural integrity assessments.

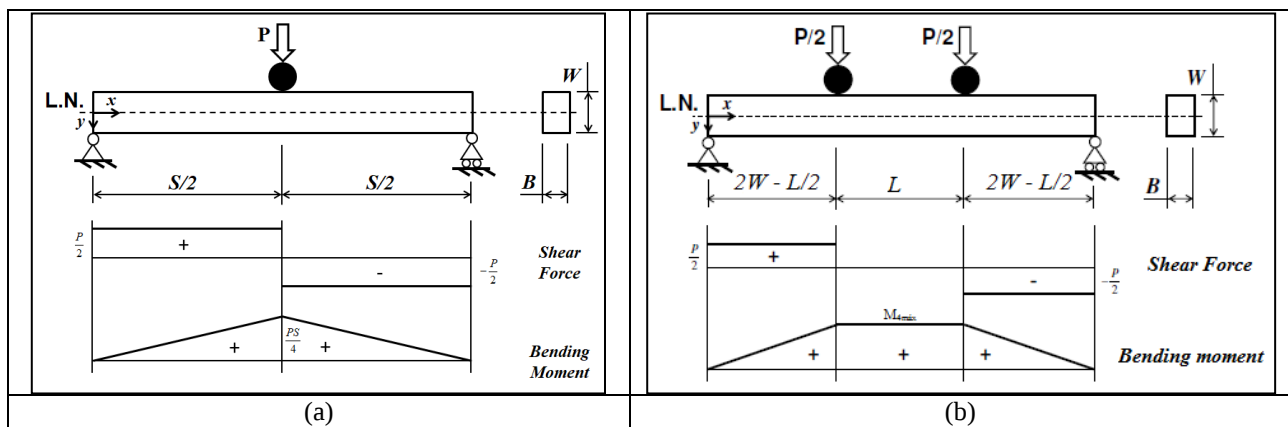


Figure 2. Bending moment and Shear force for (a) 3-point bending SE(B) and (b) 4-point bending SE(B).

2. THEORETHICAL BACKGROUND

In this section, SE(B) specimens and available solutions are detailed both for 3-point and 4-point bending loading schemes. In addition, Hertzian stresses are detailed and a brief discussion regarding the literature is conducted.

2.1 3-point bending SE(B) specimens and available solutions

As displayed in Fig. 1(b), 3-point bending SE(B) specimens present a contact on the crack propagation plane. This loading strategy generates the shear and bending moment distributions presented by Fig. 2(a). In terms of the determination of the stress intensity factor, most of the K_I solutions found in the literature for bending geometries have the following form:

$$K_I = \frac{P \cdot S}{\sqrt{B \cdot B_N} \cdot W^{3/2}} \cdot f\left(\frac{a}{W}\right), \quad (1)$$

where P is the applied load and $f(a/w)$ is a dimensionless geometry function. The specimen dimensions B , W , and a are defined in figure 1(b) and further details can be found in Anderson (2017). For SE(B) specimens under 3-point bending, the solution of $f(a/w)$ extracted from ASTM E1820 (2016) and Tada et al. (1973) is:

$$f\left(\frac{a}{W}\right) = \frac{3 \cdot \sqrt{\frac{a}{W}}}{2 \cdot \left(1 + 2 \cdot \frac{a}{W}\right) \cdot \left(1 - \frac{a}{W}\right)^{3/2}} \cdot \left\{ 1,99 - \frac{a}{W} \cdot \left(1 - \frac{a}{W}\right) \cdot \left[2,15 - 3,93 \cdot \left(\frac{a}{W}\right) + 2,7 \cdot \left(\frac{a}{W}\right)^2 \right] \right\}. \quad (2)$$

To evaluate instantaneous crack size during testing, elastic unloading compliance solutions are also necessary. For 3-point bending SE(B) specimens, several formulae are available and based on a normalized compliance, defined as:

$$\mu = \frac{1}{\sqrt{\frac{B_{eff} f W E C}{S/4} + 1}}, \quad (3)$$

where the effective thickness (B_{eff}) is given by:

$$B_{eff} = B - \frac{(B - B_N)^2}{B} \quad (4)$$

and B_N means the net thickness in the crack plane if side-grooves are included. Polynomial solutions for the crack size (a/W) as a function of normalized compliance (μ) could be found in ASTM E 1820 (2016 – Eq. 5) and in Donato & Moreira (2013 – Eq. 6) including 3D effects.

$$\frac{a}{W} = 0.999748 - 3.9504\mu + 2.9821\mu^2 - 3.21408\mu^3 + 51.51564\mu^4 - 113.031\mu^5, \quad (0.0 \leq a/W \leq 1.0) \quad (5)$$

$$\frac{a}{W} = 0.988 - 3.529\mu - 1.404\mu^2 + 21.068\mu^3 - 31.779\mu^4 + 18.947\mu^5 \quad (0.1 \leq a/W \leq 0.7) \quad (6)$$

2.2 4-point bending SE(B) specimens and available solutions

The 4-point bending SE(B) specimen is illustrated in Fig. 3. The dimension L is an object of study of this work, since no results were found in the literature regarding its effects on testing and optimum values. The other dimensions are based on previously mentioned ASTM standards E399 (2012) and E1820 (2016). Comparing Fig. 3 to Fig. 1(b) and Fig. 2(b) to 2(a) it can be realized that the 4-point loading: i) eliminates the contact and indentation in the crack plane; ii) also eliminates shear forces on the crack plane; iii) guarantees a uniform bending moment regardless of the precision on crack location and; iv) moreover, this loading method allow access to the top and bottom faces for additional instrumentation if necessary. Several results regarding the effects of L on loads and contacts stresses will be presented next.

In terms of K_I solutions for 4-point bending SE(B) specimens, references are limited but the work of Calomino et al. (1994) proposes:

$$K_I = \frac{P \cdot (S-L)}{\sqrt{B \cdot B_N} \cdot W^{3/2}} \cdot f\left(\frac{a}{W}\right), \quad (7)$$

$$f\left(\frac{a}{W}\right) = \frac{3}{2} \sqrt{\frac{\frac{a}{W}}{\left(1 - \frac{a}{W}\right)^3}} \left\{ 1.9887 - 1.326 \frac{a}{W} - \frac{\left(3.49 - 0.68 \frac{a}{W} + 1.35 \left(\frac{a}{W}\right)^2\right) \frac{a}{W} \left(1 - \frac{a}{W}\right)}{\left(1 + \frac{a}{W}\right)^2} \right\}. \quad (0.0 \leq a/W \leq 0.95) \quad (8)$$

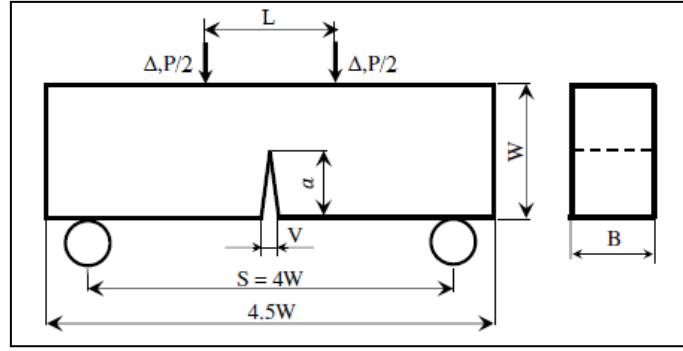


Figure 3. SE(B) specimen subjected to 4-point bending.

Considering elastic unloading compliance solutions based on CMOD, the works from Tarafder (1994) and Ruggieri & Souza (2017) are relevant and state that the normalized compliance (for $L = 2W$ in their cases) is given by:

$$\mu = \frac{1}{\sqrt{\frac{4EB_{eff}CW}{S} + 1}} \quad , \quad (9)$$

where the relationships between μ and a/W were respectively proposed in polynomial forms by Tarafder (1994 – Eq. 10) and Ruggieri & Souza (2017 – Eq. 11) as:

$$\frac{a}{W} = 1.01562 - 3.06851\mu + 4.53990\mu^2 - 23.32393\mu^3 + 71.16366\mu^4 - 69.78915\mu^5 \quad \text{and} \quad (10)$$

$$\frac{a}{W} = 1.3177 - 10.2567\mu + 65.6950\mu^2 - 271.2962\mu^3 + 549.9747\mu^4 - 420.7141\mu^5 \quad (0.10 \leq a/W \leq 0.80) \quad (11)$$

Other interesting results can be found in Huh & Song (2000) and Yuan (2007). Despite the presented solutions for 4-point bending are available in the literature, they are in some cases based on analytical solutions that present limitations (due to the solid mechanics simplifying assumptions) or on relatively simplified numerical FE models. Consequently, it is considered relevant to validate such proposals and include 3d effects, contact, varying spans, among other studies.

2.3 Contact Hertzian stresses

For SE(B) specimens, contact stresses are of large relevance and should be taken into account in order to avoid deleterious effects to the mechanical response or to the singular stress fields being characterized. Fig (4) illustrates the contact region between a cylinder and a plane, which can be adopted as representative of the contact between SE(B) specimens and the loading rollers. According to Hertz theory, the formulae that characterize the stress tensor in the vicinity of the contact can be determined using Eqs. (12-18) for plane conditions, whose details can be found in the work of Johnson (1985).

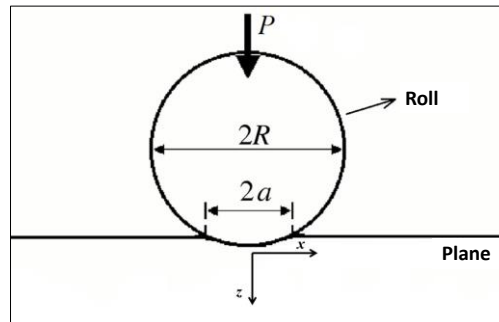


Figure 4. Reference and main features of a cylinder-plane contact.

According to Johnson (1985), the pressure between the contact surfaces can be defined as

$$P = \frac{\pi a^2 E'}{4R} \quad , \quad (12)$$

where

$$a^2 = 4PR/\pi E' \quad (13)$$

and R and E' respectively means the roll radius and the effective Young's modulus. The stresses in the x-z space can be quantified as (Johnson, 1985):

$$m^2 = \frac{1}{2} [\{(a^2 - x^2 + z^2)^2 + 4x^2z^2\}^{\frac{1}{2}} + (a^2 - x^2 + z^2)] \quad (14)$$

$$n^2 = \frac{1}{2} [\{(a^2 - x^2 + z^2)^2 + 4x^2z^2\}^{\frac{1}{2}} - (a^2 - x^2 + z^2)] \quad (15)$$

$$\sigma_x = -\frac{P_0}{a} \left\{ m \left(1 + \frac{z^2 + n^2}{m^2 + n^2} \right) + 2z \right\} \quad (16)$$

$$\sigma_z = -\frac{P_0}{a} m \left(1 - \frac{z^2 + n^2}{m^2 + n^2} \right) \quad (17)$$

$$\tau_{xz} = -\frac{P_0}{a} n \left(\frac{m^2 - z^2}{m^2 + n^2} \right) \quad (18)$$

P_0 is the maximum contact pressure and is given by:

$$P_0 = \sqrt{\frac{PE'}{\pi R}} \quad (19)$$

3. MATERIALS AND CONSTITUTIVE LAWS

In order to simulate Hertzian stresses along the crack plane, four materials were evaluated, including two steels (AISI-1010 and AISI-4340) and two aluminium alloys (1050 and 7075) to take into account high-strength and low-strength steels and aluminums and the different stiffness of such material classes. The main mechanical properties of the adopted materials were obtained from Matweb database (2015) and are presented by Table 1.

Table 1 – Main mechanical properties of the adopted materials

Material	ASTM nomenclature	HB	σ_{ys} (MPa)	σ_{uts} (MPa)	E (Gpa)	ν
Steel	1010	95	180	325	210	0,30
	4340	363	862	1282	210	0,30
Aluminium	1050	35	124	131	70	0,33
	7075	150	503	572	70	0,33

4. NUMERICAL PROCEDURES

Initially, shear and bending moment in SE(B) specimens were investigated for $L=0.5W$, $L=W$ and $L=1.5W$ (Fig. 3), in order to determinate the most suitable value for the span L . It is important to highlight that the applied load increases as the distance between the upper load points of application (L) increases, because the bending moment (M), which is the responsible for crack propagation in this scenario, is directly proportional to the distance between the bottom roll and the closest upper roll, creating a binary. Therefore, to apply a constant bending moment with greater L distance, the load applied by the machine P has to increase, which on the other hand causes greater contact stresses. A good solution means maximizing bending (trying to get close to 3-point bending levels) with the minimum load (P – considering that now it will be divided in two rolls), and at the same time the desired shielding of the crack plane regarding contact stresses.

4.1 Prediction of Hertzian stresses

To evaluate the effects of contact stresses along the crack plane, Hertzian stresses were determined using a special Matlab code based on Eqs. (12-19). The algorithm was developed by the authors and it, basically, describes the whole stress tensor in each selected discrete point in terms of σ_x , σ_y , σ_z , τ_{xz} and σ_{vM} (von Mises equivalent stress).

4.2 Finite element models for K_I and elastic unloading compliance assessment

Thereafter, multiple finite element models were created using MSC Patran as pre-processor. To obtain accurate loads- P vs. $CMOD$ - V evolutions and the respective stress-strain fields, a highly refined mesh was built with the crack incorporated in all models (for details see Donato & Moreira, 2013). Plane strain, plane stress and 3D conditions were investigated (Fig. 5) with crack depths between $a/W=0.1$ and $a/W=0.7$. Both 3-point and 4-point bending configurations were evaluated. The plane strain symmetric models, for example, have ~24000 elements and ~28000 nodes.

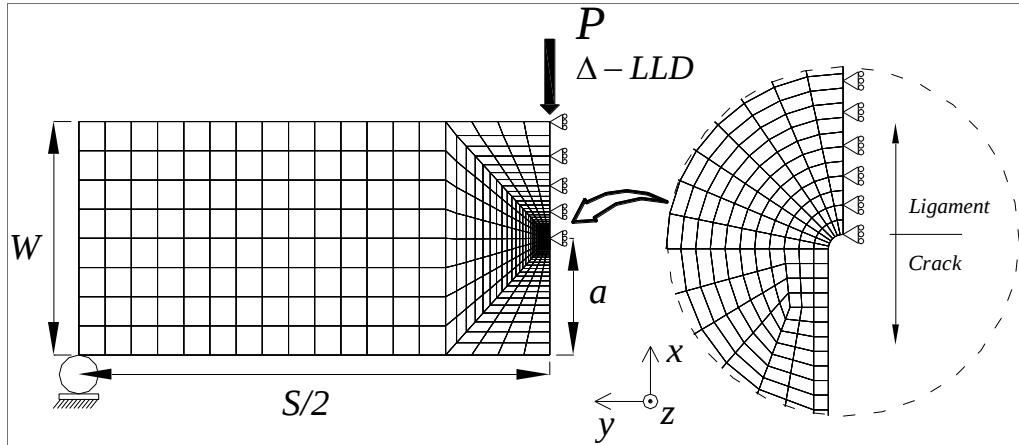


Figure 5. Example of symmetrical 3-point bending SE(B) model. 4-point bending is analogous.

The models were processed using WARP3D research code considering linear-elastic material. K_I solutions derived from numerical J -integral computations in accordance to Rice's (1968) original proposal adapted by WARP3D and recommended practices found in Courtin et al. (2005). Compliance (C – inverse of stiffness) derived from load-displacement records and generated normalized compliances (μ) for each sample using the previously presented appropriate solutions from ASTM and the literature both for 3-point and 4-point bending. The results could be compared to the available ones to clarify its contribution and the limitation of existing practices.

5. RESULTS

5.1 Effects of span L on applied bending

Fig. (6) demonstrates how the bending moment (M) decreases as the span L increases (see Fig. 3 for dimensions), keeping load P unaltered. For example, $L = W$ provides 75% of the original 3-point bending moment with the same total load P , but now divides in $P/2$ for each roll. One can increase such loads in each roll in order to reach 100% of the original bending moment. However, assessing contact stresses is also interesting before taking this simple solution.

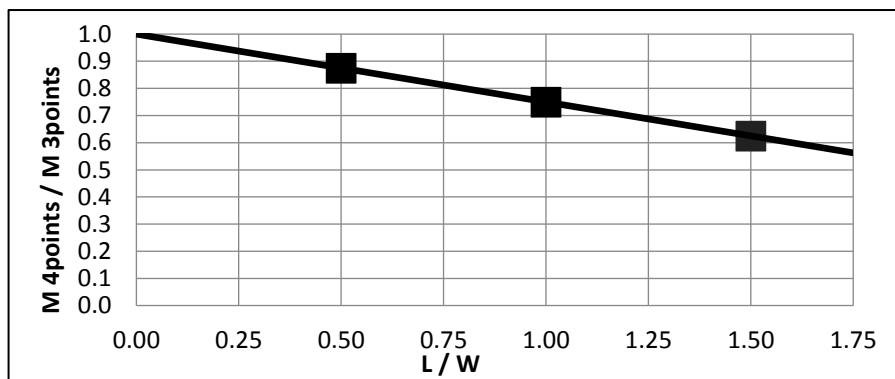


Figure 6. Graphic of the ratio between bending moment ($M_{4points}$) and L/W , for the same applied load P . The maximum bending moment (for $L/W = 0$) is $M_{3points}$. The points that include FE models were highlighted.

For the considered loading modes and materials, were estimated the loads that could bring each specimen to its σ_{uts} in the crack vicinity, but neglecting the singularity as a first macroscopic approximation. Such loads are presented by Table 2 and are considered overestimated since linear-elastic assumptions were employed – they are useful as the maximum possible loads to be considered for studying the contact phenomenon, as follows.

Table 2. Estimated force on each upper roll to bring the sample to its σ_{uts} .

Load on each roll (N)			Distance L			
			$L = 0$ (3-point bending)	$L = 0.5 \cdot W$	$L = W$	$L = 1.5 \cdot W$
Material	steel	1010	44.731	25.561	29.821	35.785
		4340	176.447	100.827	117.631	141.158
	aluminum	1050	18.030	10.303	12.020	14.424
		7075	78.727	44.987	52.484	62.981

5.2 Recommended geometry for 4-point bending considering Hertzian stresses

Each sample of different material was simulated with the loads displayed on table 2. The obtained results in terms of stress distributions are very similar for all materials, so only the results for 1010 steel are displayed on Table 3. It is remarkable the reduction of contact effects on the crack plane for the 4-point bending when compared to the 3-point bending loading. First, the load is reduced to half the original; second, as L increases, the contact effect on the crack plane is reduced, which can be noticed by the color codes in Table 3. The evaluation of resulting Hertzian from Table 3 stresses along each crack plane conducted to Table 4. As previously mentioned, in this study, it was imposed as a criterion that the von Mises stress in the propagation region did not exceed 10% of the material yield strength. Consequently, Table 4 reveals that this requirement is met by $L = W$. Consequently, despite the simplifications, Tables 3-4 indicate that it is coherent to conclude that the most suitable span of those studied is $L=W$. In this case, the contact stress did not exceed 10% of the material yield strength on the crack propagation plane and the load applied in each roll could be reduced compared to $L=1,5W$. An increase in 33% on the load applied to each roll will make feasible to reach 100% of the original 3-point bending moment, while at the same time each roll will operate with $\sim 67\%$ of the original 3-point bending load. It means less local contact stresses, combined to the same bending moment and thus crack driving forces.

Table 3. von Mises stresses for 1010 steel under varying span conditions. [MPa]

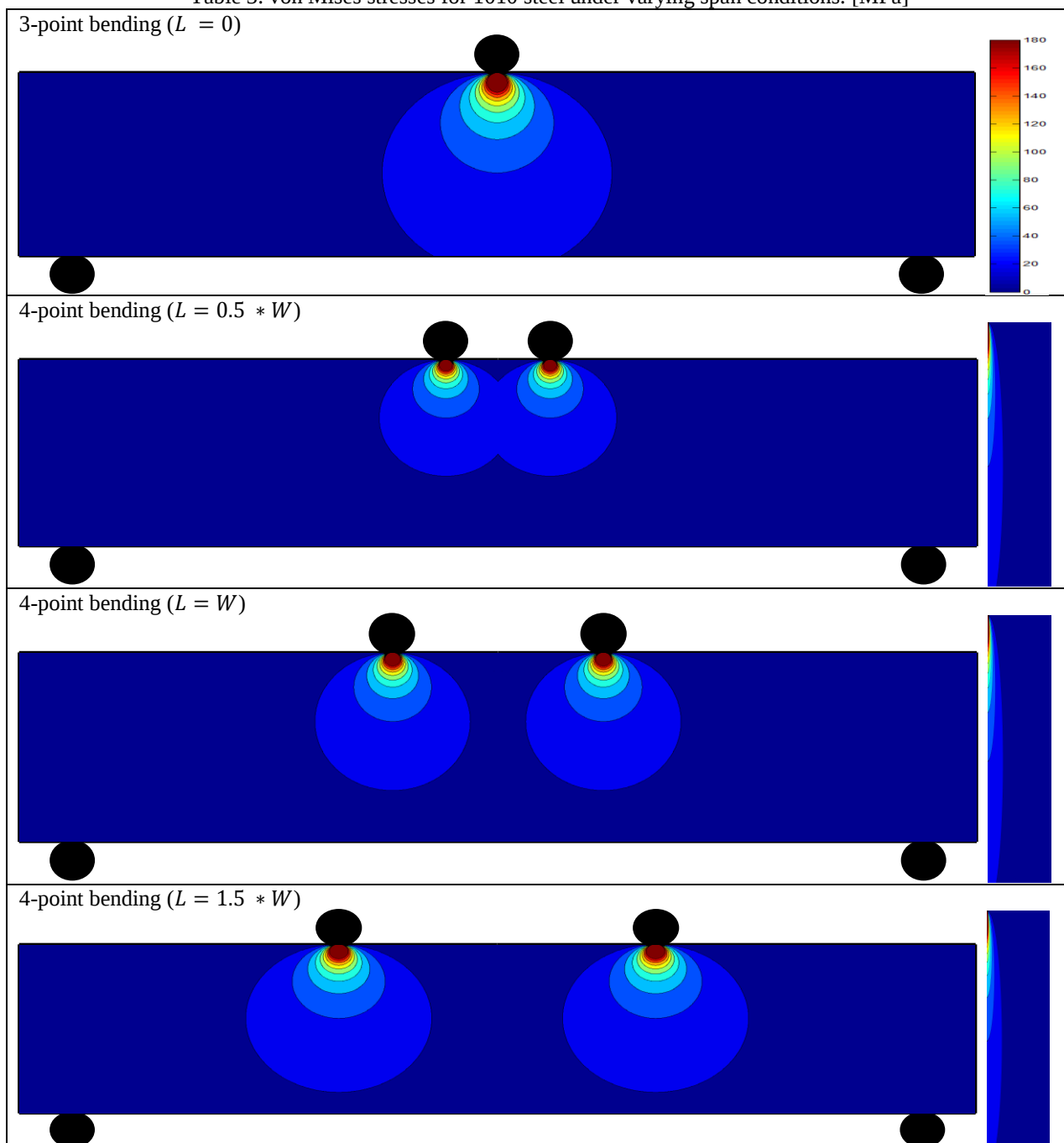


Table 4. von Mises stresses on the crack propagation plane for different materials and spans.

	$L = 0.5W$			$L = W$			$L = 1.5W$		
	σ_{vm} (Mpa)	%Sus	%Sys	σ_{vm} (Mpa)	%Sus	%Sys	σ_{vm} (Mpa)	%Sus	%Sys
Steel AISI 1010	34.5	7%	17%	20.11	4%	10%	16.1	3%	8%
Steel AISI 4340	69.0	7%	14%	40.24	4%	8%	32.2	3%	6%
Aluminum 1050	10.3	7%	10%	6.00	4%	6%	4.8	3%	5%
Aluminum 7075	41.4	7%	10%	24.00	4%	6%	19.18	3%	5%

5.3 K_I solutions for SE(B) specimens under 4-point bending

Before presenting the K_I solutions developed for 4-point bending, it is important to inform that the technique employed to determine numeric K_I based on WARP3D J -integral computations was first verified for 3-point bending conventional SE(B) specimens and presented deviations below $\pm 1\%$ both for plane strain and plane stress conditions when compared to respective ASTM solutions. This result provided confidence on the developed method and 4-point bending solutions could be obtained. Considering Eq. (7) as the reference, the dimensionless functions from Eqs. (20-22) can be employed to determine K_I respectively for 3D 1T specimens, plane strain and plane stress conditions. Figure 7 shows that plane conditions are in good agreement to Calomino et al. (1994), which is consistent to other sources found by the authors. On the other hand, no results including 3D-effects had been found in the literature by the authors and Eq. (20) represents a contribution in this aspect – deviations between 6 and 8% were found if compared to plane conditions.

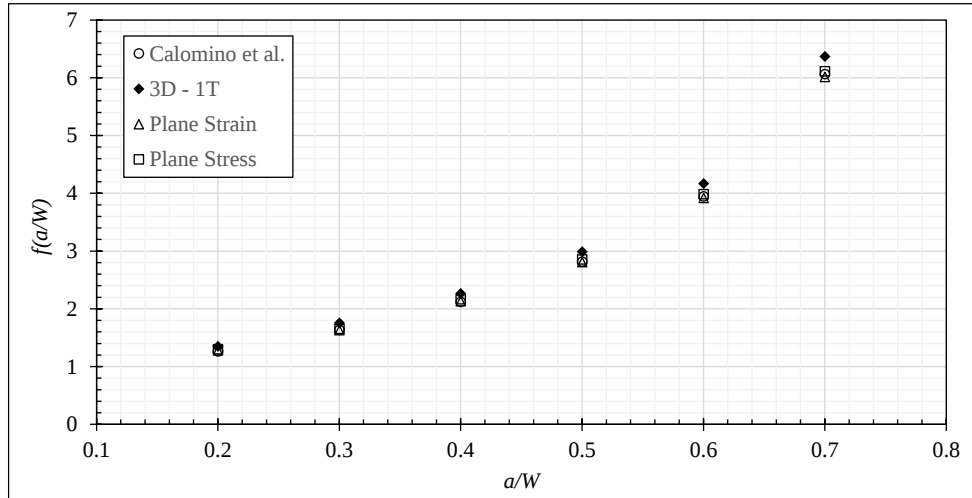


Figure 7. K_I solutions for 3D 1T specimens, plane strain and plane stress conditions.

$$f\left(\frac{a}{W}\right) = 1.75105 - 10.42333\left(\frac{a}{W}\right) + 62.34948\left(\frac{a}{W}\right)^2 - 120.30718\left(\frac{a}{W}\right)^3 + 94.23958\left(\frac{a}{W}\right)^4, \quad \text{3D - 1T} \quad (0.20 \leq a/W \leq 0.70) \quad (20)$$

$$f\left(\frac{a}{W}\right) = 2.37404 - 17.35907\left(\frac{a}{W}\right) + 85.83547\left(\frac{a}{W}\right)^2 - 154.23600\left(\frac{a}{W}\right)^3 + 110.92188\left(\frac{a}{W}\right)^4, \quad \text{Plane Strain} \quad (0.20 \leq a/W \leq 0.70) \quad (21)$$

$$f\left(\frac{a}{W}\right) = 2.40348 - 17.52007\left(\frac{a}{W}\right) + 86.69469\left(\frac{a}{W}\right)^2 - 155.79838\left(\frac{a}{W}\right)^3 + 112.15625\left(\frac{a}{W}\right)^4, \quad \text{Plane Stress} \quad (0.20 \leq a/W \leq 0.70) \quad (22)$$

5.4 Elastic Compliance solutions for 4-point bending

For determining elastic unloading compliances, the loads (P) and displacements at the edge of crack ($CMOD - V$) were obtained from FE models. With these data, compliance C could be estimated, since $C = \Delta V / \Delta P$ and each crack size from the FE models could be related to the normalized compliance (μ). Considering Eq. (9), which is only applicable for 4-point bending SE(B) specimens with $L = 2W$, Eq. (23) was derived to allow the use of other spans (for example $L = W$ as recommended here). Eq. (24) presents the obtained solution for 4-point bending specimens. Combining Eqs. (24) to (23), Fig. (8) shows that current results are in good agreement with Ruggieri & Souza (2017) even considering that their developments were based on $L = 2W$; this is fully accommodated by the normalization process. If the solution proposed by Tarafder (1994) is taken into account, a slight difference can be noticed (maximum around 3%).

$$\mu = \frac{1}{\sqrt{\frac{EB_{eff}CW}{S-L} + 1}} \quad , \quad (23)$$

$$\frac{a}{W} = 1.0079 - 5.0001\mu + 6.1646\mu^2 - 13.512\mu^3 + 111.1\mu^4 - 206.11\mu^5 \quad , \quad (0.10 \leq a/W \leq 0.70) \quad (24)$$

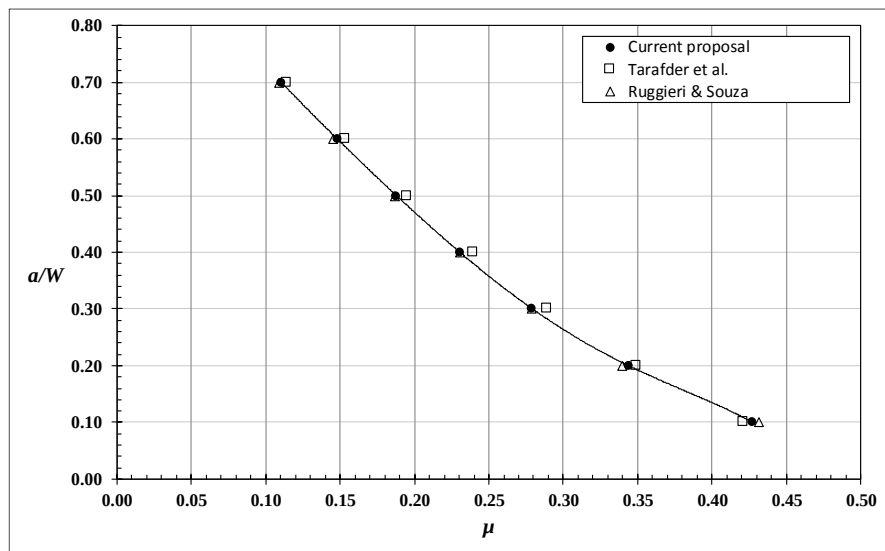


Figure 8. Normalized elastic unloading compliance solution for 4-point bending specimens.

Such compliance results mean that obtained solutions are in accordance to the available literature and, combined to proposed K_I formulae and the recommended roller span $L = W$, current research effort represents one step in the direction of enhancing the accuracy and applicability of 4-point bending SE(B) specimens.

6. CONCLUDING REMARKS

Considering the studied cases, the recommended span between upper loading rollers is $L = W$ (50.8 mm for 1-T specimens), which minimizes local indentation and stress disturbance in the near-tip crack region.

The proposition of $L = W$ for the 4-point bending provides 100% of the original 3-point bending moment, while at the same time imposes to each upper roll $\sim 67\%$ of the original 3-point bending load.

The solutions for K_I and elastic compliance could be obtained based on refined FE models for the desired conditions, are in accordance with the literature and were presented in the paper in order to support real applications.

While the well-known 3-point bending solutions are widely available in the literature, the results for 4-point bending demand alternative formulae and deserve attention.

The obtained results increase the applicability and utility of 4-point bending SE(B) geometries. However, further nonlinear studies and experimental validations are recommended and are being conducted.

7. ACKNOWLEDGEMENTS

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