



24th COBEM - 2017



24th ABCM International Congress of Mechanical Engineering  
December 3-8, 2017, Curitiba, PR, Brazil

COBEM-2017-1246

## MATHEMATICAL MODEL FOR HEAT TRANSFER IN STEEL BILLETS SUBJECTED TO SLOW COOLING

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**Abstract.** For steel billets cooling after the continuous casting process, structures called steel mill bags are used for store the billets. The billets are stacked on XX to YY sets up billets. These bags are to enable the slow cooling of the billets and thus improve the metallurgical quality of the material. The study aimed to create a computational tool able to simulate the cooling profile of these billets, helping decision making in the industrial environment. The model allows simulation of various load situations, various sizes and types of billets. The result was numerically validated using the existing model, already validated experimentally. The difference for the existing model was approximately 2%, which validates the results and let the tool able to be used in the industrial environment. The model allowed better planning of production, representing a potential gain of 12 hours in 72 hours of production

**Keywords:** Cooling; Billets; Computational Mathematical Model; Heat Transfer; Cost reduction.

### 1. INTRODUCTION

Steel Mill bags are structures where steel billets are stored after being slugged and cut. These bags are intended to promote slow cooling of stored billets. The billets are positioned at different moments of time, and at approximately the same initial temperature. (Silva, 2014) developed a computational code capable of representing the thermal profile during the cooling of the set of billets, calculating the energy balance for each billet. With the help of the Engineering Equation Solver (EES) were developed a code capable of simulating the temperature x cooling time. Initial temperature of billet, temperature environment, diameter and length of the billets, as input parameters. The results obtained by (Silva, 2014) were validated with a small-scale experiment in the UFMG laboratory.

The model developed by (SILVA 2014) presented good results despite the limitations for use in the industrial environment. Among these limitations we can mention: The number and diameter of the billets were fixed, limiting their use in most activities; The EES software presented a graphical interface with little appeal and low resources, requiring the user to prior knowledge of the model and software; The EES is academic software and did not allow the simulation of XX (how much) of billets and also of a period XX (how much) of time.

The objective of this work was to improve the model developed by (Silva, 2014) creating an easy-to-use mathematical tool without the limitations mentioned above. The new model used the C++ programming language for code development and graphical interface. To solve the energetic balances contained in the mathematical model, the technique of numerical methods of successive substitutions was used (Chapra, 2004).

The proposed computational code was numerically validated through the results obtained with the code developed in the EES.

### 2. MATERIALS AND METHODS

#### 2.1 Mathematical Model

The mathematical model of cooling considers that the billets are positioned in triangular arrangements inside the bag, as in Fig.1.

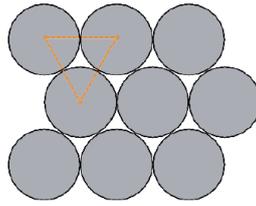


Figure 1. Billets Positions Triangular Arrangements

It is considered that the billets exchange heat only by natural convection and radiation. Boundary conditions vary for each billet depending on the position occupied by the billet within the arrangement. In this way the billets of the left side of the arrangement, for example, exchange heat with the environment on the left side and with the neighboring billets on the upper, lower and right sides. The model considers the loading position of each billet, individually determining the boundary conditions of the cooling. Like the boundary conditions the cooling time is different for each billet depending when it was loaded in the bag. Diameter, length and material are considered the same for all billets.

## 2.2 Energy Balance

According (Incropera, 2003) The energy balance was solved in a discrete way for each billet, approaching  $dT/dt$  by  $\Delta T/\Delta t$ . A time step,  $\Delta t$ , is equal to 1 minute, temperature and dependent properties were updated every iteration. The equation of the problem as in Eq. (1)

$$m \cdot C_p \cdot \frac{\Delta T}{\Delta t} = h_{conv} \cdot A \cdot (T - T_{\infty}) + F \cdot h_{rad} \cdot A \cdot (T - T_{\infty}) \quad (1)$$

Where:  $m$  is the mass of a billet,  $C_p$  is the specific heat of the steel,  $T$  is the billet temperature,  $t$  is time,  $h_{conv}$  is the convective heat transfer coefficient,  $A$  is the surface area of the billet,  $F$  is the form factor,  $h_{rad}$  is the coefficient of heat transfer by radiation. The coefficient of heat transfer by radiation as in Eq. (2).

$$h_{rad} = \varepsilon \cdot \sigma \cdot (T + T_{\infty}) + (T^2 + T_{\infty}^2) \quad (2)$$

Where:  $\varepsilon$  is the emissivity of the surface of the steel billets (equal to 0.91),  $\sigma$  is the constant of Stefan Boltzmann whose value is  $5,67 \cdot 10^{-8} \frac{W}{m^2 \cdot K^4}$ . The coefficient of convective heat transfer as in Eq. (3).

$$h_{conv} = \frac{\overline{NU} \cdot K}{D} \quad (3)$$

Where:  $\overline{NU}$  is the mean value of the Nusselt number of the billet,  $K$  is the thermal conductivity,  $D$  is the diameter.

For infinite horizontal cylinder, length  $\gg$  diameter, the mean number of Nusselt along the surface, can be estimated for numbers of Rayleigh,  $Ra$ , less than  $10^{12}$  as in Eq. (4).

$$\overline{NU} = \left\{ 0.60 + \frac{0.387 \cdot Ra^{\frac{1}{4}}}{\left[ 1 + \left( \frac{0.559}{Pr} \right)^{\frac{9}{16}} \right]^{\frac{8}{27}}} \right\}^2 \quad (4)$$

Where:  $Ra$  is numbers of Rayleigh and  $Pr$  is number of Prandtl.

## 2.3 Form Factor

The triangular arrangement is configured when the billets are positioned so that each three billets an equilateral triangle is observed. This triangle is formed by the union of the centers of the cylinders as in Fig.2.

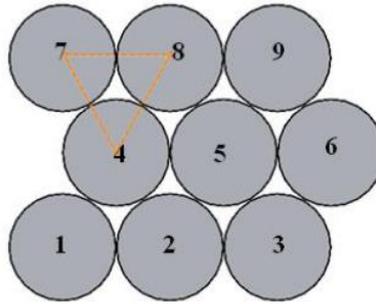


Figure 2. Typical configuration for Triangular Arrangements

For two parallel billets with equal radius  $r$ , separated by a distance  $s$ , as in Fig.3, the form factor  $F_{ab}$  can be calculated as in Eq. (5). This equation was used for billets that were side by side.

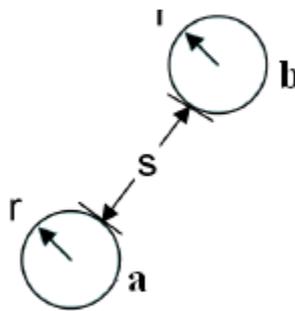


Figure 3. Equal billets separated by distance  $s$

$$F_{ab} = \frac{1}{\pi} \left[ \sqrt{(1 + s/2r)^2 - 1} + \text{SIN}^{-1} \left( \frac{1}{(1+s/2r)} \right) - (1 + s/2r) \right] \quad (5)$$

For billets that were diagonally, the form factor  $F_d$  as in Eq. (6). For such application assumed that no radiation escaped in the axial direction of the billet and that all thermal radiation was exchanged between the billets.

$$4 F_{ij} + 4 F_d = 1 \quad (6)$$

It is observed that billet 5 (FIG.2) is surrounded by six billets (billets 2, 3, 4, 6, 8 and 9). So, the factor between billet five and each neighbor billet is  $1/6$ . However, there is also the form factor found between billets 1 and 4 ( $F_{1-4}$ ) which there is only the interference of one billet (billet 2). Billets with two interferences have form factor of  $1/6$ . Half of the difference between the form factor of the two configurations, with two interferences and without interference, is the value that each interference contributes to the form factor as in Eq. (7).

$$\text{Interference} = \frac{F_{2int} - F_{0int}}{2} \quad (7)$$

Where:  $F_{2int}$  represents the form factor between two billets that have interference with two other billets, for example  $F_{5-1}$ ;  $F_{0int}$  represents the form factor between two billets that have no interference from any other billet and Interference represents the portion of the billet that interferes between the billets  $i$  and  $j$

## 2.4 Numerical Method

In the energy balance equation Eq. (1),  $T$  is function of  $h_{rad}$  and  $h_{conv}$  which are also function of temperature. In order to solve the equation was necessary an iterative method, in this case the technique of successive substitutions where temperature is calculated in function of temperature, as in Eq. (8) (Chapra, 2004). To implement this method and solve the equations was written a code in c ++

$$T = f(T) \quad (8)$$

To accelerate the convergence of the method, the value found in the previous iteration was arbitrated as the initial temperature value. The criterion of convergence of the method as in Eq. (9).

$$\left| \frac{T_p - T_{p-1}}{T_p} \right| \leq E \quad (9)$$

Where: E is equal to 0,001 and p is iteration index

Figure 4 illustrates the iteration process:

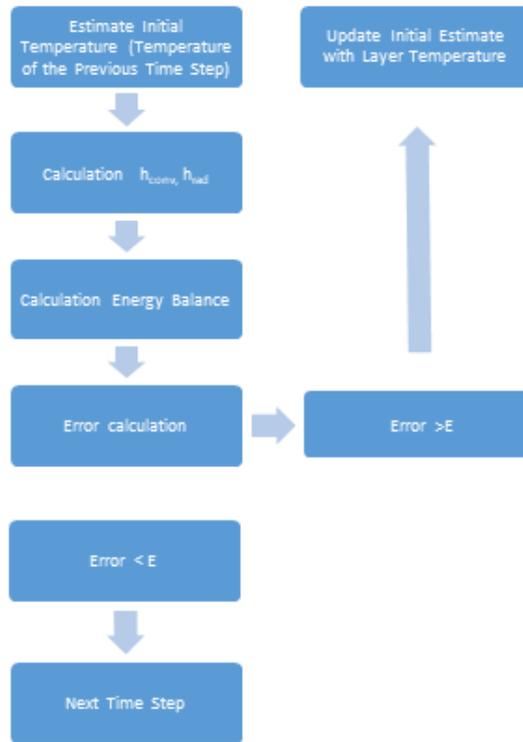


Figure 4. Iteration Process

### 3. RESULTS

The proposed model was numerically validated using the existing model, already validated experimentally. For the same initial conditions, the cooling profile was simulated in both models. In the simulation the time  $t = 0$  h represents the start of the cooling. The initial conditions are the same. The results of the simulation can be seen in Tab. 1.

Table 1. Average Billet Temperature

	0 h	20 h	40 h	60 h	80 h	100 h	120 h
Average Temperature proposed model [°C] <sup>(1)</sup>	260	100	73	60	52	47	43
Average Temperature EES Model [°C]	260	100	72	58	51	45	42

<sup>(1)</sup> The simulation conditions are the same as those used by (SILVA 2014)

The cooling profile calculated by the two models as in Fig. 3.

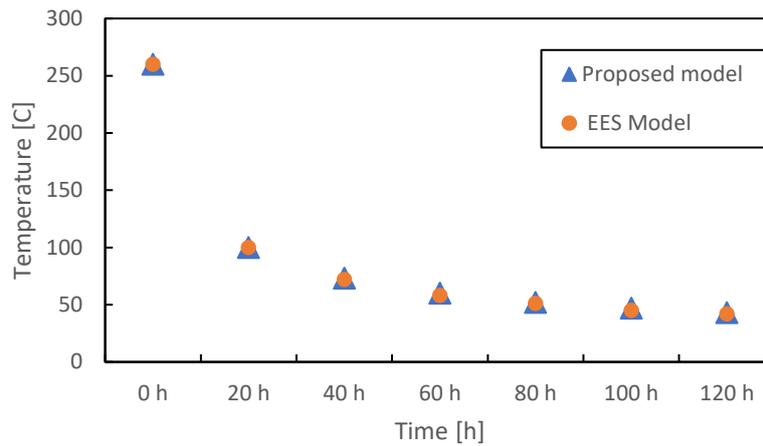


Figure 5. Cooling Profile

The relative error of the proposed model in relation to the model developed in the EES was calculated as in Eq. (10).

$$Erro = \frac{T_{EES} - T_{Modelo}}{T_{EES}} \cdot 100 \quad (10)$$

The relative error of the proposed model in relation to the EES model can be seen in Tab. 2.

Table 2. Relative Error

	0 h	20 h	40 h	60 h	80 h	100 h	120 h
Relative Error	-	0%	-2%	-3%	-2%	-4%	-2%

The mean relative error of the model was approximately 2%.

#### 4. CONCLUSION

The result obtained with the new model generated an error related to the previous model of approximately 2%. The error is mainly due to the difference between materials properties libraries in both models and also the different technique of numerical methods.

The error found is small, taking into account the industrial needs to which this work is proposed, in this way the computational tool is apt to be used in the industrial environment.

The cooling model allowed a better programming of the unloading of the billets contained in the bag. Featuring a potential gain of approximately 12 hours in 72 hours of production.

#### 5. ACKNOWLEDGEMENTS

To Vallourec Soluções Tubulares do Brasil for promoting research and development work.

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