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COMPUTIONAL INTERFACE TO SIZE A BI SUPPORTED POWER TRANSMISSION SHAFT WITH UP TO FOUR CYLINDRICAL GEARS

Layse Freitas Boere de Moraes
Bruno da Cunha Diniz
Universidade Federal da Bahia, Mechanical Engineering Department, Rua Aristides Novis 2, Federação, Salvador, Brazil.
layseboere@gmail.com, bruno.diniz@ufba.br

Leonardo Lima Gusmão
Universidade Federal da Bahia, Mechanical Engineering Department, Rua Aristides Novis 2, Federação, Salvador, Brazil.
leonardolimagusmao@gmail.com.

Abstract. *The sizing of a power transmission shaft exhibits an interactive nature, which presents an intrinsic interconnection between the shaft and the design of its individual elements. Consequently, the sizing process is longstanding, irksome and can accumulate many calculation errors. Seeking an instrument that provides greater reliability, reduces the time spent on a project and allows a quick design comparison for different materials. A simple and intelligible mathematical algorithm was developed, tested and validated in MATLAB® environment, that automates the calculations of a bi supported power transmission sizing with up to four cylindrical gears. As a result, outputting the shaft rebound diameters based on fatigue strength and strength of materials.*

Keywords: *Power-transmission-shaft, MatLab®, Algorithm, Mechanical-Projects*

1. INTRODUCTION

There is not a standard methodology to perform a machine project sizing, once this is an interdependent and interactive process. The engineer makes decisions in order to obtain an initial design delineation, and these parameters must be repeatedly recalculated aiming at the best configuration that meets the engineering criteria. Consequently, is longstanding and irksome process, that can accumulate many calculation errors. (Norton, 2013).

Grounding on the undeniable relevance of computational systems in the engineering field and seeking an instrument that provides greater reliability, reduces the time spent on a project and allows a quick design comparison for different materials. A mathematical algorithm was developed, tested and validated in MATLAB® environment, that automates the calculations of a bi supported power transmission sizing with up to four cylindrical gears.

In this point, it is important to emphasize that the algorithm was not developed to be a specialist system for power shafts calculations – this program was implemented in Microsoft Visual Basics 6® language by Tolfo, Araújo and Marco Filho in 2002 (Tolfo *et al.*, 2002) –. The intention is to make available a simple and intelligible mathematical algorithm to assist project decisions, extinguishing calculation errors and reducing the required time in the shaft design execution.

Through a friendly interface, the program requires project data inputs such as power, rotation and shaft length, operating temperature and rotation speed. The material data (flow stress, ultimate stress, modulus of elasticity and surface treatment), the axial arrangement of the elements on the shaft, the safety and reliability factor are also required. Using criteria based on fatigue strength and strength of materials, the algorithm, as a final result, outputs the shaft rebound diameters. Presenting itself as an instrument to subsidize design decisions as it assigns greater calculations reliability, minimizes the time required in its execution, and provides a rapid design comparison for different materials.

2. ALGORITHM INTERFACE AND DATA INPUT

2.1 General Data Input

The initial design delineation adopts the solution for axial accommodation of rotating components via shaft rebound diameters suggested by Shigley *et al.* in 2005. It is important to note that all gears rotate with the shaft as a programming consideration.

The algorithm data input follows the flowchart in Fig. 1.

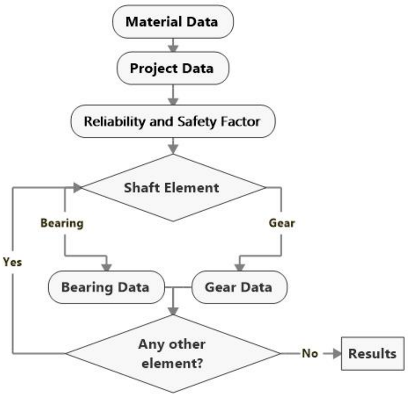

Figure 1. Algorithm Data Input Flowchart

Through the input boxes, user must insert the project data as indicated in Fig. 2.a. Parameters regarding the operation setting, Fig. 2.b, and the information regarding reliability and safety factor are also required, Fig. 2.c.

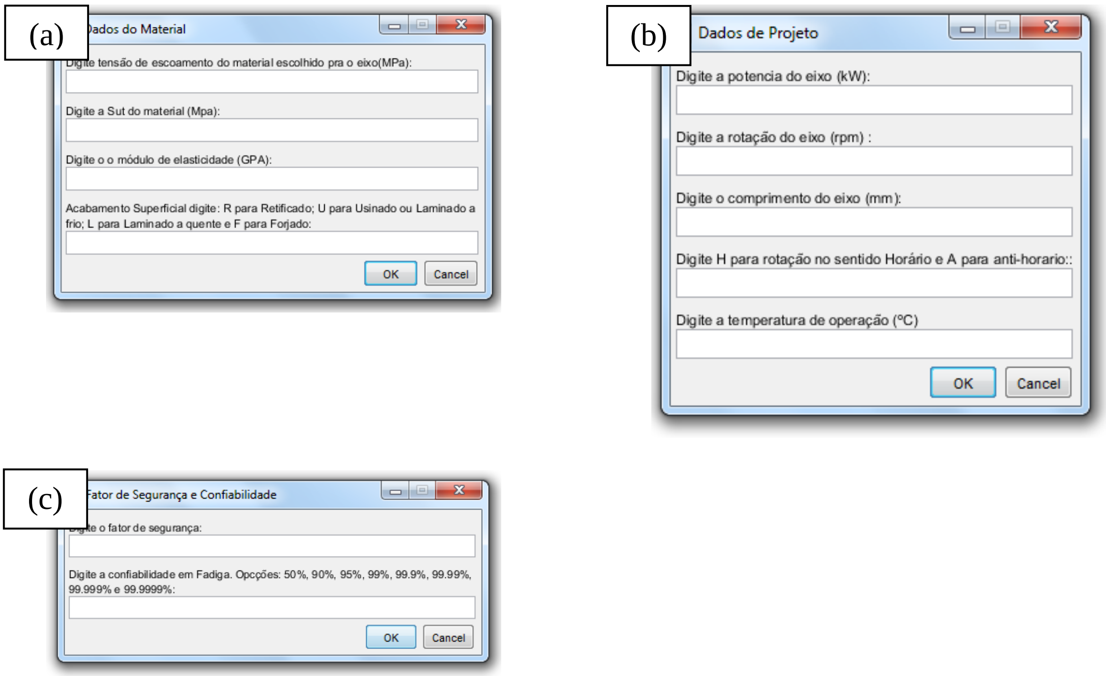

Figure 2. Input boxes. (a) Material Data; (b) Project Data; (c) Reliability and Safety Factor

2.2 Axis Component Data Input

Following the flowchart, the user must choose which component, bearing or gear, is the first shaft element, Fig. 3.a. For gears, subsequent boxes, as demonstrated in Fig.3.b, should be filled with the image support, as indicated in Fig. 3.c. It is important to notice that the power percentages sum of all gears should be 100 %.

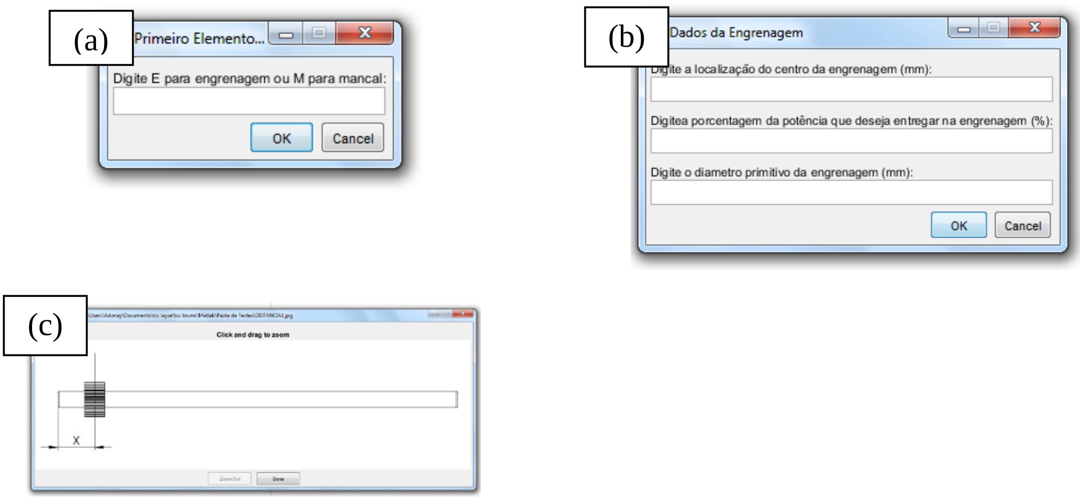

Figure 3. Input Boxes. (a) Shaft Element; (b) Gear Data; (c) Gear Support Image

The procedure continues as the user inputs the gear quadrant data and the engagement gear angle data, Fig. 4 a. and Fig. 4.b., using the support image, Fig. 4.c.

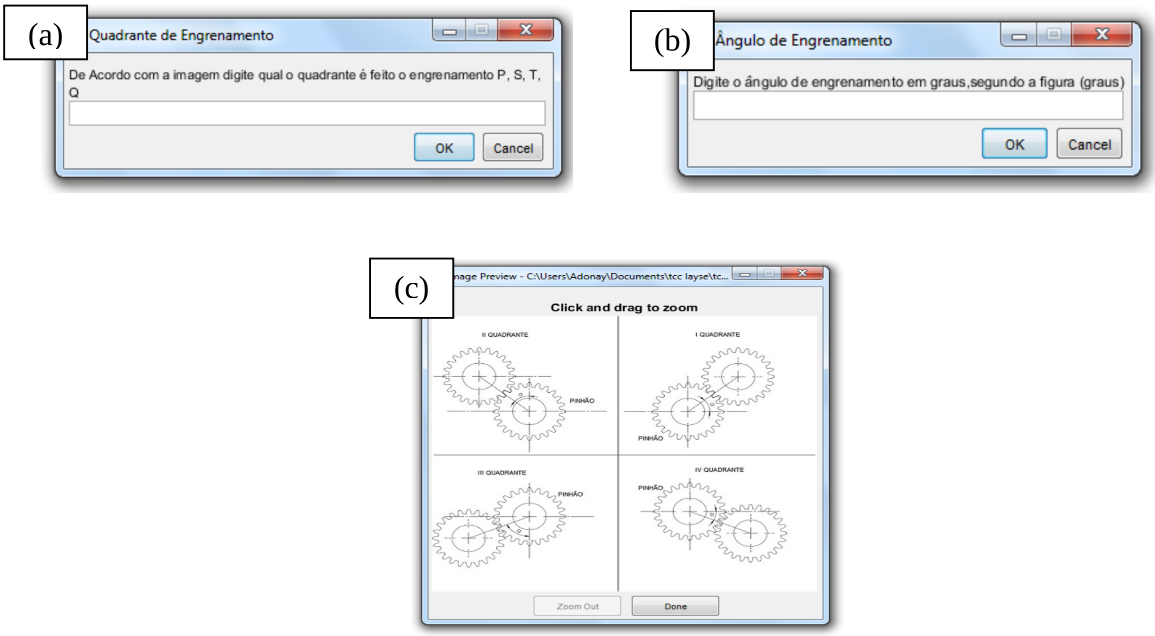

Figure 4. Input Boxes. (a) Gear Quadrant Data; (b) Engagement Gear Angle Data; (c) Gear Angle Support Image

This subsequence is requested to each gear, and similarly solicited for bearings (Fig. 5.a and Fig. 5.b). The algorithm versatility allows shafts configuration with up to four cylindrical gears, thus, it is perfectly capable of sizing shafts with three, two, or even one gear.

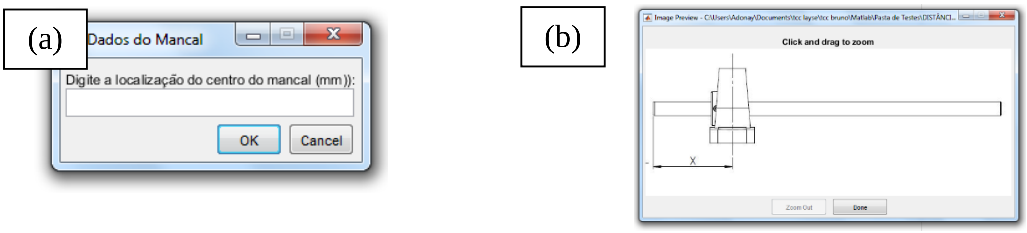

Figure 5. Input Boxes. (a) Bearing Data; (b) Bearing Support Image

In axes configurations with less than 6 elements, once completed the data input the user must type "N" in the remaining boxes Fig. 6.a. A warning box as in Fig. 6.b will appear at the end of processing.

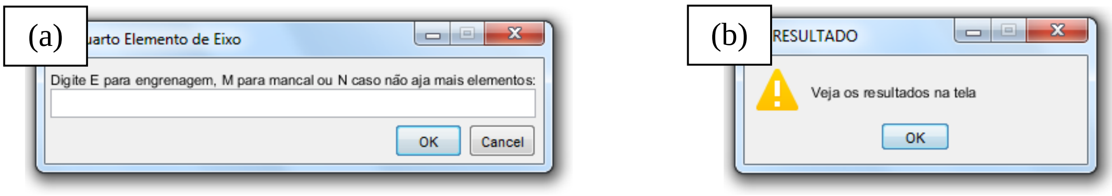

Figure 6. Input Boxes. (a) Remaning Box; (b) Results

3. CALCULATION PARAMETERS

3.1 Torque and Force

The mean torque is the axis power divided by rotation in rad/s. Reorganizing Eq. (1).

$$T_{mean} = \frac{P_{mean}}{\omega_{mean}} \tag{1}$$

The force exerted on the shaft by cylindrical gear W and its components W_t (tangential) and W_r (radial) are calculated by Eq. (2); Eq. (3) and Eq. (4) considering the standard pressure angle $\phi = 20°$.

$$W = \frac{W_t}{\cos\phi} \tag{2}$$

$$W_t = \frac{T_p}{r_p} \tag{3}$$

$$W_r = W_t \tan\phi \tag{4}$$

For inclined vectors, the forces on Z and Y axes are decomposed using the engagement gear angle required in Fig. 4.b. A previous load study enabled the programming of the algorithm in order to consider the forces directions. Thus, the program solves the free-body diagrams to determine the reactions in the bearings. And it automatically calculates the bending moments and torques at each critical point on the axis.

4. MATERIAL AND FATIGUE FAILURE CRITERIA

4.1 Strenght of Materials

In a multiaxial stress state, it is impossible to accurately predict when the maximum normal stress exceeds the uniaxial yield strength since other two normal stress components may influence the yield behavior at the critical locations. In addition, the designer generally does not have the yield stress multiaxial limits, since they require costly and time-consuming experiments (Collins, 2006).

Failures prevention in a component subjected to multiaxial stress, theories are validated experimentally. All these theories are based on loading severity parameters, such as stress, strain or strain energy density, which can be easily determined for the multiaxial stress states, called combined stress theories of failure (Collins, 2006).

In order to prevent failures resulting from static loading, the preliminary shaft rebound diameters dimensioning is performed using the Von Mises (Distortion Energy Theory) yield criterion, once this theory agrees well with the real data for ductile materials. (Shigley *et al.*,2005). For a rotating shaft with constant bending and torsion, the bending stress is completely reversed and the torsion is steady: $M_m = 0$ e $T_a = 0$ (Budynas, 2011). So, solving a Von Mises Equation for the diameter, Eq. (5):

$$d = \sqrt[3]{\frac{32\sqrt{M_a^2 + \frac{3}{4}T_m^2}}{\pi S_y}} \tag{5}$$

4.2 Fatigue Failure Criteria

Combined loadings involving axial, bending or torsion introduce complications because they involve normal and shear stresses, each with alternating and mean components, and several determinants of fatigue strength are dependent on these combinations. In addition, there are stress-concentration factors associated with each loading mode (Shigley *et al.*, 2005). Aiming to prevent fatigue failure and considering the combined loading, results obtained by the Distortion Energy Theory (Von Misses Criterion) is adjusted by the ASME-Elliptic Fatigue Failure Criterion, once this criterion is used in ANSI / ASME B106.1M-1985 for design of transmission shafting (Collins, 2006). The shaft rebound diameters at critical locations are obtained by Eq. (6), in which $M_m = 0$ e $T_a = 0$.

$$d=\left\{\frac{16n}{\pi}\left[4\left(\frac{K_f M_a}{S_e}\right)^2+3\left(\frac{K_{fs}T_m}{S_y}\right)^2\right]^{\frac{1}{2}}\right\}^{\frac{1}{3}} \quad (6)$$

5. ENDURANCE LIMIT

Although it is known that the determination of endurance limits by fatigue test has a stochastic nature, such an approach was not considered because it would introduce unnecessary complexity in the calculations. Therefore, the algorithm estimates the endurance limits using Eq. (7) for steels, in which S_{ut} is the minimum tensile strength and S'_e refers to the rotating-beam specimen endurance limit.

$$S'_e \cong \begin{cases} 0,5S_{ut} & S_{ut} \leq 1400\ MPa \\ 700\ MPa & S_{ut} > 1400\ MPa \end{cases} \quad (7)$$

Then, Marin's factors are applied to adjust the endurance limit (Eq. 8) (SHIGLEY et al.,2005):

$$S_e = k_a k_b k_c k_d k_e k_f S'_e \quad (8)$$

The user determines the surface finish when inputs it at the Material Data box, see Fig.2.a. With this information, through Tab.1 and Eq. 9 it is possible to obtain the surface condition modification factor (k_a) value (Budynas, 2011).

$$k_a = aS_{ut}^b \quad (9)$$

Table 1. Surface Condition Modification Factor

Surface Finish	*S_{ut}, Mpa*	*Expoent b*
Ground	1.58	-0.085
Machined or cold-drawn	4.51	-0.265
Hot-rolled	57.7	-0.718
As-forged	272.	-0.995

The Size Modification Factor k_b and the Load Modification Factor k_c are specified by Eq. 10 and Eq. 11 respectively. The shaft has combined torsion and bending loads, however $k_c=1$, since this combination of stresses is already managed by Von Misses stress (Budynas *et al.*, 2011).

$$k_b = \begin{cases} \left(\frac{d}{7,62}\right)^{-0,107} = 1,24d^{-0,107} & 2,67 \leq d \leq 51mm \\ 1,51d^{-0,157} & 51 \leq d \leq 254mm \end{cases} \quad (10)$$

$$k_c = \begin{cases} 1 & bending \\ 0,85 & axial \\ 0,59 & torsion \end{cases} \quad (11)$$

The Temperature Factor k_d is given by the fourth order polynomial curve expressed in Eq. 12, valid only for $37 \leq Tc \leq 540°C$.

$$k_d = 0,9877 + 0,6507(10^{-3})Tc - 0,3414(10^{-5})Tc^2 + 0,5621(10^{-8})Tc^3 - 6,246(10^{-12})Tc^4 \quad (12)$$

The Reliability Factor k_e uses parameters inputted by the user, see Fig. 2.c, and is established using Tab. 2. The Miscellaneous-Effects Factor k_f is disregarded in this approach.

Table 2. Reliability Modification Factor Corresponding to 8% Standard Deviation of the Endurance Limit

Reliability, %	*Reliability Factor* k_e,
50	1.000
90	0.897
95	0.868
99	0.814
99.9	0.753
99.99	0.702
99.999	0.659
99.9999	0.620

6. STRESS CONCENTRATION

Stress concentration effects occur in cases of bending and torsion loads. Thus, it is interesting to evaluate the stress concentration associated with various geometric configurations in order to ensure that the maximum stresses acting on these elements are supported by the specified material (Juvinall and Marshek, 2008). Stress-concentration factors values (K_{ts} and K_t) are determined by the empirical equations at Fig. 7.a. and Fig.7.b.

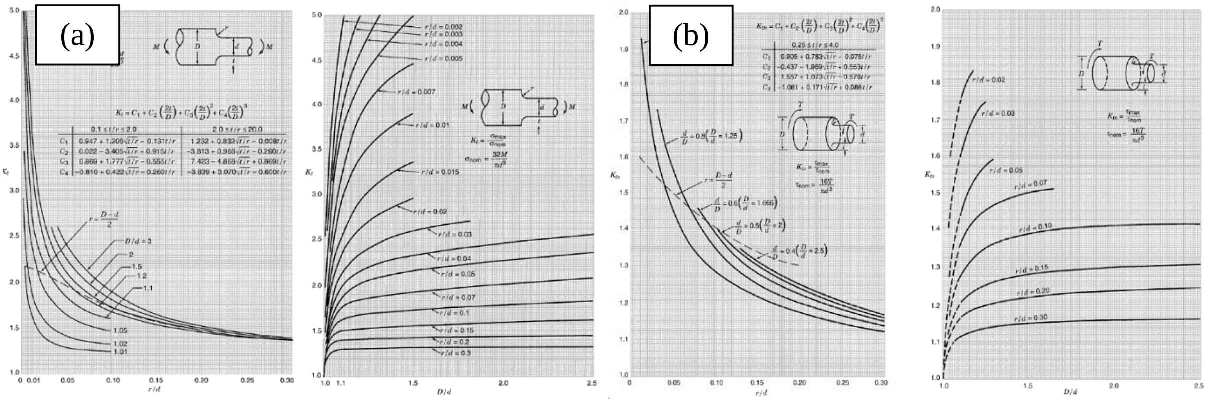

Figure 7. Stress Concentration Factors. (a) For bending of a stepped bar of circular cross section with a shoulder fillet. (b) For torsion of a shaft with a shoulder fillet. From: Pilkey and Pilkey, 2008.

Through Neuber's Rule equation (Eq. 13) associated with the Neuber constant $\sqrt{a}$, obtained with the third-degree polynomial expressed in Eq. 14 was possible to obtain fatigue stress-concentration (K_f) (Shigley *et al.*, 2005).

$$K_f=1+\frac{Kt-1}{1+\sqrt{a}/r} \quad (13)$$

$$\sqrt{a}=0,245799-0,307794\left(10^{-2}\right)S_{ut}+0,150874\left(10^{-4}\right)S_{ut}^2-0,266978(10^{-7})S_{ut}^3 \quad (14)$$

For fatigue concentration factor related to shear stress, $K_{fs} = K_{ts}$ was considered generating a small oversize, since it was not possible to obtain equations for the notch sensitivity curves (q_{shear}).

7. RESTRICTIONS

The developed algorithm presents some restrictions that require the user's attention:

- The user cannot choose the failure criteria for ductile materials or fatigue criteria, since the standard criteria are Distortional Strain Energy and ASME-Elliptical.
- Stress-concentration factor equations are empirical and therefore limited to a values range of $0.1 \leq t/r \leq 2.0$ and $2.0 \leq t/r \leq 20$ for K_t and $0.25 \leq t/r \leq 4.0$ for K_{ts} (see Fig. 7.a and 7.b).
- The pressure angle is set at $\varphi = 20$ °, making it impossible to use the algorithm to calculate shafts that contain gears with different pressure angles.

- The elements weight was not considered.
- Set screws, keys, pins, retaining rings were not considered.
- There is no critical speed for shaft calculation.

8. VALIDATION

Aiming to validate the developed algorithm, a bi supported shaft with a central gear was sized through two different methodologies. The first, calculating in Excel® Spreadsheets together with resistance analyzes in Ftool® program, and the second, inputting the project parameters in the previously presented algorithm. The two methodologies use the same theoretical foundation to calculate the project parameters, which initially consists of sizing the minimum diameter to support static loads at the shaft critical points through Distortion Energy Theory (Von Mises Criterion). Then, to achieve fatigue failures prevention and considering the combined loading profile, the results obtained previously are adjusted by the conjunction with the ASME-Elliptic Fatigue Failure Criteria.

Finally, a new shoulder to support the central element is inserted, adopting the Shigley et al. 2005 suggestion, in which good design practices indicates an estimate for shoulders which $D/_d$ is between 1.2 and 1.5.

9. RESULTS

9.1 Results in Excel® Spreadsheets and Resistance Analyzes in Ftool® Program

The project parameters are shown in Tab.3 and Tab. 4:

Table 3. Material and Project Data

Material Data		Project Data	
Material	AISI 1025	Power	50 kW
Tensile Strength, Yield	370 Mpa	Rotation	1350 rpm
Tensile Strength, Ultimate	440 Mpa	Shaft Length	500 mm
Modulus of Elasticity	205 Gpa	Rotation	Clockwise
Surface Finish	Cold-drawn	Operating Temperature	25 ºC

Table 4. Gear Data, Security and Reliability

Gear Data		Security and Reliability	
Pinch diameter	350 mm	Safaty Factor	2
Power Percentage	100%	Reliability	99.99%

The calculation of forces exerted on the shaft by the gear was performed using Excel® spreadsheets, and the reactions in the bearings and bending moments for both planes were obtained through the use of the Ftool® program (Fig. 8). The calculation of material strength and minimum diameters to support static loads at the shaft critical points through Distortion Energy Theory are shown in Tab. 5.

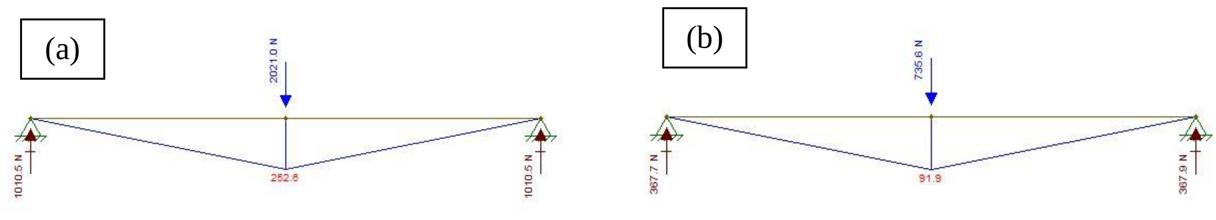

Figure 8. Ftool® Results. (a) Bending reactions and moments in the XZ plane.; (b) Bending reactions and moments in the XY plane

Table 5. Static Analysis at the Critical Points of the Shaft Results.

	Bearing 1	Gear	Bearing 2
Location in x	0 mm	250 mm	500 mm
Force (X-Z)	1010,5 N	2021,02 N	1010,5 N

Force (X-Y)	367,9 N	735,59 N	367,9 N
Bending Moment (X-Z)	0 Nm	252,6 Nm	0 Nm
Bending Moment (X-Y)	0 Nm	91,91 Nm	0 Nm
Total Bending Moment	0 Nm	268,8 Nm	0 Nm
Torque	353,68 Nm	353,68 Nm	353,68 Nm
Minimum Diameter	0,020354 m	0,223864 m	0,020354m
Final Minimum Diameter	20 mm	22 mm	20 mm

The fatigue data used in the calculations are shown in Tab. 3 and Tab. 4.

Table 6. Marin's Factors and Endurance Limit

Marin’s factors		Endurance Limit	
Ka	0,898796935	Sut	440 MPa
Kb	0,8891452	Se’	221,73 MPa
Kc	1	Se	124,409801 MPa
Kd	1	Sut	440 MPa
Ke	0,702		
Kf	1		
Ka	0,898796935		
Kb	0,8891452		
Kc	1		

Table 7. Stress Concentration in the Elements

Stress Concentration	Bearing 1	Gear	Bearing 2
Kt	1,55908	1,55908	1,545972
Kf	1,826664	1,826664	1,277282
Kts	1,285072	1,285072	1,277089
Kfs	1,285072	1,285072	1,277089

The minimum diameters adjusted by the ASME-Elliptic Theory, including the gear accommodating shoulder, in which D / d = 1.2 (same as in the algorithm) are shown in Tab. 8.

Table 8. Final Results

Bearing 1	36 mm
Gear	43 mm
Shoulder	52 mm
Bearing 2	43 mm

9.2 Results in Excel® Spreadsheets and Resistance Analyzes in Ftool® Program

The same project parameters, Tab.3 and Tab.4, were used in the data input of the developed algorithm, so the output data are expressed in Fig.9.

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MATLAB Command Window                                         Page 1

Resultados valores para os elementos na ordem de entrada de dados

Plano XY
Forças(N)
F=3.678e+02
F=7.356e+02
F=3.678e+02
Momentos Fletores (Nm)
M=0.00
M=-9.194867e+01
M=0.00


Plano XZ
Forças(N)
F=1.011e+03
F=2.021e+03
F=1.011e+03
Momentos Fletores (Nm)
M=0.00
M=-2.526269e+02
M=0.00


 Momentos Fletores e Torçores na ordem de entrada de dados
Resultante dos Momentos Fletores (Nm)
M= 00000
M= 2.68840e+02
M= 00000
Torçores (Nm)
T= 3.53678e+02
T= 3.53678e+02
T= 3.53678e+02


MATLAB Command Window                                         Page 2


 Diâmetros de escalonamento pela Energia de Distorção
D (mm)=20
D (mm)=22
D (mm)=20


 Diâmetros de escalonamento pela combinação DE-ASME-Elíptico
D (mm)=36
D (mm)=43
D (mm)=52
D (mm)=43
```

Figure 9. Algorithm Results

10. CONCLUSIONS

Analyzing the results presented, it's confirmed that the proposed algorithm presents good compliance with the reaction forces and bending moments with Ftool® program. In addition, the results obtained through the two calculation methodologies have good agreement and, therefore, the algorithm can be validated.

The case study presented with elastic curve analysis was also developed. The critical points were analyzed by singular functions and the spherical roller bearings restrictions were referenced at maximum typical slopes intervals and transverse deflections (Budynas *et al.*, 2011). However, the algorithm presented inconsistent results regarding deflections and slopes, generating errors around 25% in the final diameters.

11. REFERENCES

Budynas, R. G., and Nisbett, J. K. (2011). **Elementos de Máquinas de Shigley: Projeto de Engenharia Mecânica.** Porto Alegre: Bookman.

Collins , J. A. (2006). **Projeto Mecânico de Elementos de Máquinas: Uma Perspectiva de Prevenção da Falha**. Rio de Janeiro: LTC.

Norton, R. L. (2013). **Projeto de Máquinas: Uma Abordagem Integrada.** Porto Alegre: Bookman.

Pilkey, W. D., and Pilkey, D. F. (2008). **Peterson's Stress Concentration Factors**. John Wiley & Sons. 3th edition.

Shigley, J. E., Mishke, C. R., and Budynas, R. G. (2005). **Projeto de Engenharia Mecânica.** Porto Alegre: Bookman.

12. RESPONSIBILITY NOTICE

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