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MODAL PARAMETERS IDENTIFICATION METHODS BASED ON OUTPUT-ONLY

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Abstract. *Due to the necessity of achieving very accurate estimates of the dynamic properties of the structures, the modal analysis have been an attractive method. Different methods of signal analysis were developed to obtain the modal parameters. The Operational Modal Analysis (OMA) takes into account only the system response without knowledge of system input system, while the Experimental Modal Analysis (EMA) is developed in laboratory, with all input loads being controlled. The OMA catches signals that can be on time domain or frequency domain and, depending of the signal origin, different modal parameters identification methods are used. The modal parameters are: resonance frequency, damping ratio and modal shapes. In this work were used two time domain methods: SSI-Cov and SSI-Data. There were used data of three systems: transmission lines cable, free-free beam and injector body. To each experiment were applied the two OMA methods and, then, the results were compared with conventional modal analysis method RFP (Rational Fraction Polynomial).*

Keywords: *Modal Analysis, modal parameters, identification methods, OMA.*

1. INTRODUCTION

Over time, the structural analysis has become increasingly necessary, because the increasing requirements with respect to the safety and the useful life of the mechanical systems. Dynamic analysis through experimental vibrational data is a useful tool. Therefore, for the analysis of these systems, the development of simpler and faster methods has made necessary.

Modal properties have been used for various applications in the field of structural health monitoring, damage detection, design, dynamic model for structural controls, etc.

The measurement of a system is realized through sensors such as accelerometers and microphones, and the signals obtained are analyzed with the help of signals analysis software and/or developing of specific software.

In cases known as deterministic, that is, those experiments performed in laboratories, the input data are previously known and remain unchanged over time. By knowing and controlling the system input signals, the results of the dynamic analysis are more accurate.

However, it is necessary to have in mind that in the practical systems, such as those approached in this work, there are many other inputs, such as those of the environment that cannot be controlled or measured. In these cases, where it is not possible to obtain the input data accurately, the stochastic approach is used.

In the stochastic realization, environmental vibration measurements are performed, where the excitation forces are variable and it is not possible to measure them in a deterministic way. Therefore, for the purposes of a modal identification, it is necessary to assume a hypothesis regarding its characteristics, using the output data to obtain the modal parameters.

The modal parameters are: resonance frequency, damping factor and vibration modes. Methods of system identification aim to estimate the structural properties of a system by studying the output data of an experimental measurement and knowing the input (excitation) of the same.

Therefore, the great challenge is to develop methods that go the opposite way of the current one, that is, to use only the data of response of the system to obtain the modal parameters. In this work will be used methods of modal analysis, already developed, to analyze the dynamic behavior of different mechanical systems. After obtaining the answers, the modal data will be compared with conventional methods of modal analysis.

2. MODAL ANALYSIS

Modal Analysis is an experimental technique to characterize the dynamic properties of a vibrating system. The most important modal parameters are presented below, according to González (2007):

- Natural frequency: Frequencies at which the system tends to vibrate at maximum amplitude.
- Mode shapes: Relation between the amplitudes of the freedom degrees.
- Damping ratio: Damping ratio present in the system, it is related with the critical damping. It depends of the adopted model.

Modal analysis is used to obtain global characteristics of a structure. It has been widely applied to vibration trouble shooting, structural optimal design, model updating, and structural health monitoring in aerospace, mechanical and civil engineering. Based on the vibration testing technique, a large variety of algorithms for signal processing and data analysis are presented. (Wang *et al.*, 2016)

The modal analysis can be classified as:

- Experimental Modal Analysis (Conventional);
- Operational Modal Analysis.

According to Simensen (2013), system identification methods, which are used for EMA and OMA, aim to predict the structural properties of a system by studying the output of experimental measurement. The practice of EMA and OMA is based on the assumption that the dynamic behavior of any system can be expressed in terms of modal characteristics. The distinction between OMA and EMA lies in the requirements of the input data. While EMA requires that the input load must be known or at least estimated, the OMA techniques require response measurements only.

. Modal parameters are computed from the identified system model so a good identification method influences the quality and the quantity of modal parameters that can be estimated. This explains the increasing interest in accurate system identification methods for modal analysis. (Cara *et al.*, 2012)

Conventionally, a modal test is conducted with some special excitation devices, such as shakers or impact hammers, which exert excitation forces on the test subjects. The excitation force and the resulting response are simultaneously recorded by various transducers and data acquisition systems. To obtain the modal parameters, frequency response functions or impulse response functions are generally estimated from the input and output time histories. Due to the requirement of a noise-free environment and the need for complex artificial excitation devices, such testing, usually, can only be performed in the laboratory. As a result, the modal analysis that makes use of both input and output data is named Experimental Modal Analysis (EMA). (Wang *et al.*, 2016)

Structural health monitoring of civil structures is increasingly becoming popular due to the technological advances and developments in sensors and data acquisition systems as well as the significant developments in robust system identification techniques. (Yun *et al.*, 2011)

2.1 EMA

Experimental modal analysis (EMA) identifies the dynamic response from measurements of the applied force and the vibration response. According to Schanke (2015), it is a classic input-output method where input need to be applied, controlled and measured. The output, vibration response, also need to be measured.

Argoul and Erlicher (2005) explain that modal analysis and identification involve the theory of linear time-invariant conservative and non-conservative dynamical systems. In this theory, the normal modes are of fundamental importance because they allow to uncouple the governing equations of motion. Also, they can be used to evaluate the free or forced dynamic responses for arbitrary sets of initial conditions. Modal analysis of a structure is performed by making use of the principle of linear superposition that expresses the system response as a sum of modal responses.

According to Ghalishooyan and Shooshtari (2015)EMA method estimates the modal parameters of structures based on the known artificial input force and recorded output responses. The input force is applied to the structures by shakers or impact hammers and the output responses are generally measured by accelerometers sensors. Consequently, EMA is performed in laboratory condition and the experimental instruments and data signal processing algorithms play a pivotal role in modal parameter estimation.

2.2 OMA

Operational Modal Analysis aims to predict modal parameters identification of a structure using only vibration output under ambient conditions.

According to Ghalishoonyan e Shooshtari (2015), primary studies about OMA were established in 1990s. The algorithms estimating the dynamic parameters of structures just based on the output responses. According Simensen (2013) other reasons for choosing to apply OMA is that it does not interfere with operational use of a structure, and the input loads do not need to be known.

Operational Modal Analysis (also known as output-only modal analysis or OMA) techniques are one of the newer methods of performing modal analysis. OMA techniques aim at obtaining modal parameters characterizing the dynamics of the structure/system based only on the knowledge of response (i.e. output) of the structure to various ambient excitations, which are not measured. (Tcherniak *et al.*, 2010)

According Au (2016), the input excitation is not measured (often impractical to do so) but assumed to be ‘broadband random’ so that the statistical characteristics of measured response reflect primarily the properties of vibration modes rather than excitation. High economy and feasibility in data collection is a major advantage.

The basic equations of OMA algorithms, according Ghalishooyan e Shooshtari (2015), are mathematically similar to EMA methods and most of OMA techniques are the extension of EMA algorithms. According to the domain of implementation, OMA methods can be categorized into time domain and frequency domain approaches.

Two drawbacks of classical Operational Modal Analysis (OMA) are that the scaling factors of the mode shapes can not be determined and that the number of modes that can be identified is usually rather low because the frequency content of the ambient excitation tends to be narrow-banded. (Reynders *et al.*, [s.d.])

OMA takes in measured data in form of a signal. According to Rainieri (2014, apud Schanke (2015), a signal is a physical quantity varying with respect to one or more independent variables and associated to information of interest. The signal can be in different domains (time or frequency) and can be converted from one to another. A system converts an input signal into an output signal. Finding response to a known system and given input is called a forward problem. While an inverse problem is where output is known, but either input or system characteristics are unknown. Noise is undesired signal superimposed on the signal of interest. Since input is not controlled for OMA some assumptions need to be made. If a structure is excited by white noise, a Gaussian distributed, statistically independent value with a constant input spectrum, then all the modes are equally excited and the output spectrum contains full information about the structure. However, according to Schanke (2015), the naturally occurring loads (wind, traffic etc.) are uncontrollable and immeasurable and noise is likely to occur during a measurement. Therefore in OMA the structure is assumed to be excited by unknown forces, which are the output of the excitation system loaded by white noise.

According to Simensen (2013), assumptions made when using OMA are:

- Linearity: There is a constant link between an input and the corresponding output.
- Stationarity: The systems modal characteristics do not change in time.
- Observability: Sensors are located so that the modes of interest are possible to extract from the data. It is important to avoid placing sensors at nodal points, and to have an adequate number of sensors.

The success of any operational modal analysis (OMA) depends on the ability to accurately estimate the modal parameters from a set of output only time domain data streams with no quantitative knowledge about the input forces, which generally are assumed to be normally distributed incoherent excitations. Consequently, the natural response of the excited systems recorded by the response sensors is expected to consist of a linear superposition of resonant incoherent vibrations mixed with a normally distributed background noise. (Nita *et al.*, 2017)

According to Ghalishooyan e Shooshtari (2015) OMA techniques based on the analysis of response time histories or correlation functions are referred to as time domain methods. Frequency domain techniques for OMA purposes are not very popular due to the numerical conditioning issues. On the other hand, Time domain methods are usually more suitable to handle noisy data, and they can avoid some signal processing errors, such as leakage.

Two drawbacks of classical Operational Modal Analysis (OMA), according Reynders e Roeck [s.d.], are that the scaling factors of the mode shapes can not be determined and that the number of modes that can be identified is usually rather low because the frequency content of the ambient excitation tends to be narrow-banded.

2.3. Stabilization diagram

When the modal parameters are found the stabilization diagram can be constructed. According to Rainieri (2014), apud Schanke (2015), the aim of the stabilization diagram is to separate the physical poles from the mathematical poles. The mathematical poles tends to be more scattered and typically do not stabilize. Therefore, physical modes can be determined from an alignment of stable poles. To find these alignments you need to separate the stable poles from the unstable ones. This is based on the comparison of the poles associated to a given model order with those obtained from a one-order lower model.

The natural frequencies and damping ratio of poles from two orders are compared:

$$\frac{|f(n-1)-f(n)|}{f(n-1)} < x \tag{1}$$

$$\frac{|\varsigma(n-1)-\varsigma(n)|}{\varsigma(n-1)} < y \tag{2}$$

Where x is the limit of frequency specified by the user and y is the limit to the damping ratio specified by the user.

Schanke (2015) explain that, only the poles that fulfill a stabilization criteria defined by the user (x and y) are labeled as stable. The size of these depends on several factors, among them the structure complexity and the measurements accuracy. For natural frequency the values should coincide well and a low stability requirement should be used. However for damping ratios the values can vary more. Especially for lightly damped modes where their percentage variation could be relatively large. The value should initially be chosen relatively small and then increased if needed.

Traditionally, modal parameters are extracted for a series of increasing model orders up to an over-estimation of the system and the parameters obtained at each order are compared with the ones obtained at the previous order in a stabilization diagram.

The tool used for discriminating numerical poles from the physical poles is the stability diagram. The stabilization diagram shows the poles of a system at different model orders. The frequency is plotted on the abcis and the model order is plotted on the ordinate of a stabilization diagram. The poles that correspond to an order are compared with the poles of one order lower system. Physical poles occur at the same frequency at increasing model orders forming a vertical column of poles. In other words, they tend to stabilize, hence the term ‘‘stabilization diagram’’. An expert engineer then selects an unknown number of poles at different frequencies. (Bakir, 2011)

3. STOCHASTIC MODAL IDENTIFICATION METHODS

Identifying modal properties without input information is theoretically more involved. Results have much higher variability/uncertainty and sensitivity to algorithmic parameters compared to their counterparts identified with free or forced (known input) vibration data. Frequency domain methods make use of spectral quantities in a selected frequency band for identifying the modes within it. In doing so the identification (ID) model only needs to account for the modes dominating the band and so can be significantly simplified. For well-separated modes the band can be selected to cover only one mode. In general the number of close modes rarely exceeds three. ID results are insensitive to activities in other bands because their spectral data (e.g., FFT) do not enter into the calculation process (e.g., likelihood function in Bayesian ap- proach). This is especially attractive for OMA since ambient data contains a variety of activities in different bands which are irrelevant to identifying the mode(s) of interest or difficult to model. (Au, 2016)

According Peeters (2000), the estimation of the modal parameters is the particular type of identification and stochastic means that the structure is excited by an unmeasurable input force and that only output measurements (e.g. accelerations) are available. In these methods the deterministic knowledge of the input is replaced by the assumption that the input is a realization of a stochastic process (white noise).

Stochastic Subspace Identification (SSI) method is thought to be one of the most effective identification algorithms in the time domain operational modal analysis. The covariance-driven type SSI requires the estimation of the covariance matrix at first, whereas the data-driven type SSI makes direct use of stochastic response data to identify modal parameters. (Wang *et al.*, 2016)

3.1. Covariance-driven stochastic subspace identification (SSI-Cov)

The Covariance-driven Stochastic Subspace Identification method (SSI-COV) is addressing the so-called stochastic realization problem, the problem of identifying a stochastic state-space model from output-only data. (Peeters, 2000)

According to Schanke (2015), a system of order n is possible to observe only if the so called observability matrix and controllability matrix is of rank n. However, as the system order is usually unknown for complicated problems, a conservative approach is to overestimate the order of the system. Due to the fact that the system order is overestimated additional nonphysical poles, mathematical poles, will occur next to the physical poles. To be able to distinguish them and find the correct poles a stabilization diagram has to be used. This overestimated max order, n_{max} ,, is used as an input for the problem and therefore some experience is needed to choose an appropriate value. If the max order is chosen to be smaller than the correct system order you will not get any correct results. If you however assume this value too high you will get too many nonphysical modes and it will be harder to derive which modes that are the actual correct physical modes, in addition the computational time greatly increases. Another input required for Cov-SSI is the data matrix. It can either be measured deformation, velocity or acceleration under the influence of environmental loads. The data matrix Y have dimensions $l * N$ where l is the number of measurement channels and N is the number of measurements. The time between each measurement is the time step. The last input is the magnitude of block rows which will be explained shortly.

The first step of this method is to calculate the output correlations. $[R_i]$ denotes the unbiased estimate of the correlation matrix at time lag i based on a finite number of data:

$$[R_i] = \frac{1}{N-i}\left[Y_{(1:N-1)}\right][Y_{(i:N)}]^T \tag{3}$$

Where $[Y_{(1:N-1)}]$ is the data matrix Y with the last block rows i removed. And $[Y_{(i:N)}]^T$ is the transpose data matrix with the first block rows i removed. Therefore each $[R_i]$ matrix get dimensions $l * l$. The estimated correlations at different time lags)are then gathered into a matrix called the block Toeplitz matrix:

$$[T_{1|i}] = \begin{bmatrix} [R_i] & [R_{i-1}] & \cdots & [R_1] \\ [R_{i+1}] & [R_i] & \cdots & [R_2] \\ \vdots & \vdots & \ddots & \vdots \\ [R_{2i-1}] & [R_{2i-2}] & \cdots & [R_i] \end{bmatrix} \tag{4}$$

The Toeplitz matrix contains $i \times i$ $[R_i]$ matrices and is therefore of dimensions $li \times li$. For the identification of the modal parameters of a system of order n,, the Toeplitz matrix need to be $n \times n$. Therefore the following need to be true for the number of block rows i:

$$li \geq n \tag{5}$$

Where n is the system order. However as mentioned earlier the system order is usually unknown and the following should be fulfilled:

$$i_{min} = \frac{n_{max}}{l} \tag{6}$$

For complicated structures the number of block rows should be higher than this minimum criteria for better results. The magnitude x of this depends on the problem and should be chosen as an input by the user:

$$i_{min} = x\frac{n_{max}}{l} \tag{7}$$

3.2. Data-driven stochastic subspace identification (SSI-Data)

The Data-driven Stochastic Subspace Identification method (SSI-DATA), as well as the previous method, SSI-COV, identifies a stochastic model using output-only.

Subspace methods identify state-space models from (input and) output data by applying robust numerical techniques such as QR factorization, SVD and least squares. As opposed to SSI-COV, the SSI-DATA avoid the computation of covariances between the outputs. (Peeters, 2000)

Firstly the number of block rows i is set equal to x times the maximum system order (n_{max}) divided by the number of measurement channels l:

$$i = x\frac{n_{max}}{l} \tag{8}$$

Where x is the magnitude of block rows as for SSI-Cov.

Then the Hankel matrix is constructed. The number of columns j of the Hankel matrix is assumed to be ∞ for the statistical proof of the method, and therefore j needs to be large. In practical applications it is set as $N - 2i + 1$ so that all given data samples are used under the construction of the Hankel matrix.

$$j = N - 2i + 1 \tag{9}$$

The Hankel matrix is constructed directly from the measured data $Y(1{:}N)$:

$$\left[H_{0|2i-1}\right] = \frac{1}{\sqrt{j}}\left[\begin{array}{cccc} [y_0] & [y_1] & \cdots & [y_{j-1}] \\ [y_1] & [y_2] & \cdots & [y_j] \\ \vdots & \vdots & \ddots & \vdots \\ [y_{i-1}] & [y_i] & \cdots & [y_{i+j-2}] \\ \hline [y_i] & [y_{i+1}] & \cdots & [y_{i+j-1}] \\ [y_{i+1}] & [y_{i+2}] & \cdots & [y_{i+j}] \\ \vdots & \vdots & \ddots & \vdots \\ [y_{2i-1}] & [y_{2i}] & \cdots & [y_{2i+j-2}] \end{array}\right] \quad (10)$$

Then the LQ factorization of the Hankel matrix is done:

$$\left[H_{0|2i-1}\right] = [L][Q] \quad (11)$$

4. RESULTS

The SSI-Cov and SSI-Data methods were applied for modal analysis of three systems. For each system, results were obtained for the two SSI methods and then, compared to the traditional modal FRP analysis, the analysis of these results was done by Souza (2017).

The first system is a 54m transmission line cable subjected to a mechanical load of 14% of rated tensile strength. These data were originally used by Calado (2016). The system was excited by an impact hammer and the vibration data were obtained by five accelerometers disposed along the sample.

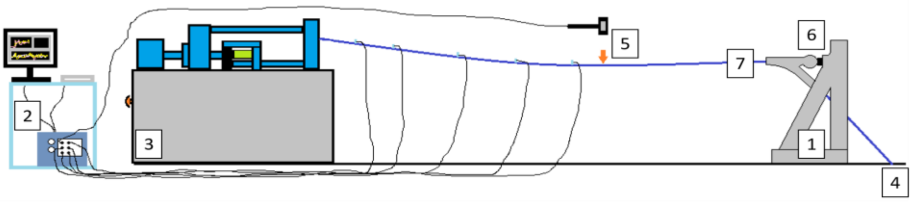

Figure 1. Test bench of conductive cables

The components of the bench showed at the Figure 1 are: 1. Rigid cables support; 2. Acquisition and control; 3. Traction system and load cell; 4. Anchorage and counterweights; 5. Impact hammer and accelerometers; 6. Sample anchorage; 7. Cable sample

Table 1 shows the results obtained with two OMA methods and with conventional RFP modal analysis method. The natural frequencies of the first five vibration modes are similar but there are discrepancies in the values of the damping ratio.

Table 1. Modal parameters (excitation with impact hammer)

	RFP		SSICov		SSIData	
Mode	Frequency (Hz)	Damping ratio	Frequency (Hz)	Damping ratio	Frequency (Hz)	Damping ratio
1	1.182	0.00045	1.182	0.0000902	1.183	0.00049
2	2.187	0.00080	2.187	0.0002666	2.187	0.00077
3	3.281	0.0021	3.281	0.002141	3.281	0.0020
4	4.372	0.0011	4.372	0.0009047	4.372	0.0012
5	5.459	0.00074	5.459	0.0005068	5.459	0.00050

The second system analysed is a free-free steel beam whose geometrical properties are: length=85cm, height=1,27 cm and width=2,54cm. There were analysed a system without damage and another with damage.

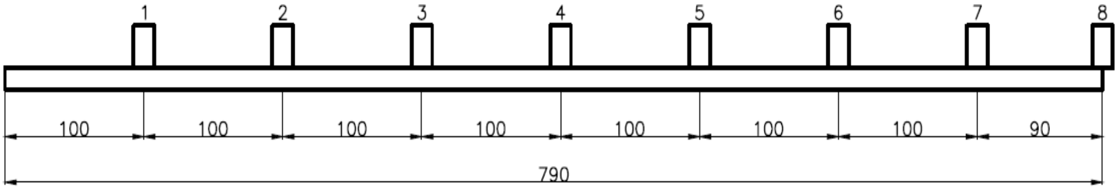

Figure 2. Accelerometers locations on beam

Table 2 shows the modal parameters of fourth modes of the system without damage. Table 3 shows the results of the system with damage. The data obtained with SSI-Data method presented values of the damping ratio near to the conventional RFP method. The natural frequencies presented good results for the two methods.

Table 2. Modal parameters (system without damagee)

	RFP		SSICov		SSIDat	
Mode	Frequency (Hz)	Damping ratio	Frequency (Hz)	Damping ratio	Frequency (Hz)	Damping ratio
1	97.046	0.0042	97.179	0.0024	97.149	0.0029
2	256.287	0.00061	256.312	0.00038	256.284	0.00053
3	498.631	0.00091	498.677	0.00079	498.689	0.00087
4	823.77	0.0028	823.797	0.0027	823.752	0.0027

Table 3. Modal parameters (system with damage)

	RFP		SSICov		SSIDat	
Mode	Frequency (Hz)	Damping ratio	Frequency (Hz)	Damping ratio	Frequency (Hz)	Damping ratio
1	97.457	0.0047	97.70	0.0018	97.685	0.0022
2	255.73	0.00060	255.76	0.00034	255.76	0.00052
3	496.240	0.0031	496.32	0.0030	496.296	0.0031
4	818.467	0.0041	818.49	0.0043	818.454	0.0046

The third system is an injector body without damage and the with damage. The system was excited with an impact hammer and the signals were captured with one accelerometer and one microphone. In this case, it was considered only the two first vibration modes.

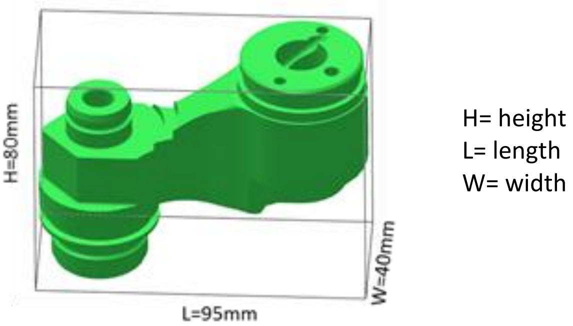

Figure 3. Injector body

It was noticed in Table 4 and Table 5, lower differences between the modal parameters using different methods.

Table 4. Modal parameters (accelerometer)

System without damage						
	RFP		SSICov		SSIData	
Mode	Frequency (Hz)	Damping ratio	Frequency (Hz)	Damping ratio	Frequency (Hz)	Damping ratio
1	6371.04	0.00122	6371.20	0.00114	6371.30	0.00120
2	7057.82	0.00031	7057.90	0.00027	7057.90	0.00031
System with damage						
1	6292.24	0.00092	6291.90	0.00078	6292.04	0.00084
2	6860.22	0.00086	6865.90	0.00041	6867.42	0.00042

Table 5. Modal parameters (microphone)

System without damage						
	RFP		SSICov		SSIData	
Mode	Frequency (Hz)	Damping ratio	Frequency (Hz)	Damping ratio	Frequency (Hz)	Damping ratio
1	6370.83	0.00114	6371.80	0.00053	6372.10	0.00118
2	7057.60	0.00042	7058.80	0.00034	7058.70	0.00047
System with damage						
1	6291.22	0.00078	6291.60	0.00075	6291.70	0.00083
2	6859.42	0.00104	6867.80		6865.20	0.00063

5. CONCLUSIONS

Small differences in modal values were observed considering the conventional and operational methods of modal analysis. The natural frequencies were well adjusted, but the damping factor values showed greater differences. The SSI-Data method presented better results comparing to the SSI-Cov method.

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