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EVALUATION OF IMPACT MODELS FOR BODY-TO-BARRIER COLLISIONS

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Abstract. *Impact is a complex phenomenon present in a great number of applications, and in many of them the prediction of the properties of the colliding bodies after the phenomenon is of vital importance. In order to perform that, impact models need to be created to represent the phenomenon, so that one can predict the properties of the bodies in contact. However, the modeling is challenging due to the complexity of the phenomenon in question. Therefore, this work presents the evaluation of two impact models of a body-to-barrier collision: a classic Kelvin-Voigt model with an elasto-plastic spring and a nonlinear force-deflection model. As it was shown, the latter model is more suitable for the impact modeling, as the coefficient of restitution changes with the increase of the impact velocity, which is not seen in the Kelvin-Voigt model. In order to evaluate the models proposed their responses were compared with experimental signals.*

Keywords: *Impact model, body-to-barrier collision, nonlinear model, experimental signal.*

1. INTRODUCTION

Impact is defined as the collision between two bodies that occur over a short time interval, during which the bodies exert large forces on each other (Gharib and Hurmuzlu, 2012; Brach, 1991; Brogliato, 1999; Stronge, 2004), which in some cases can generate faults in mechanical parts. During impact, the material properties of the bodies may change due to the high contact forces. Hence, the prediction of the properties of the bodies after the impact is crucial for determine the lifetime of structures and mechanical parts. Thus, the main objective of developing the impact model is to obtain the response of the bodies during the post-impact, based on the known parameter during the pre-impact (Gilardi and Sharf, 2002). However, due to the complexity of the phenomenon, there is not a model that fulfill all the requirements to describe impact, they need to be continuously improved as different results are obtained experimentally.

In the majority of cases, the bodies that are subjected to impact have two phases: a compression phase, which starts when the bodies are in contact and compressed against each other; and a restitution phase that acts after the latter and goes till the bodies separate completely (Gharib and Hurmuzlu, 2012). Depending on the features of these two impact phases, the collision can be classified as: perfectly elastic (no energy loss), partially elastic (energy loss with no permanent deformation), perfectly plastic (all energy loss with permanent deformation) and partially plastic (some energy loss with permanent deformation) (Ahmad *et al.*, 2006).

A parameter that is commonly used to express the energy loss during an impact is the coefficient of restitution (COR). The COR may depend on pre-impact velocities, geometry, and material properties of the colliding bodies, contact time, and friction (Gharib and Hurmuzlu, 2012). Although many models are based mainly on the COR, the impact phenomenon depends on lots of parameters such as particle size, initial velocity, particle densities, Poisson ratio, Young's moduli, yield pressure, stress-strain parameters, strain rate, and Tabor's constant for each of the impacting particles (Weir and Tallon, 2005). To cite some applications of impact models and COR, they are widely used to solve the problems in sports

engineering (Cross, 2010, 2014; Goodwill and Haake, 2001), geology (Ashayer, 2007; Imre and Springman, 2008), coal gasification industry (Gibson *et al.*, 2013), automotive (Batista, 2006; Pawlus *et al.*, 2014; Varanis and Mereles, 2016, 2017), and others. In addition, in Varanis *et al.* (2017) some models of a vehicle crash are presented and compared with experimental results found in the literature.

In this paper two impact models are presented with the purpose of representing a body-to-barrier collision: a Kelvin-Voigt model and a nonlinear force-deflection model. First, it was made a comparison between the two models, and then a discussion of the features that an impact model must have to represent the phenomenon correctly based on results found in the literature. After that, the displacement, velocity and acceleration obtained by solving the equations of motion of each model are compared with an experimental signal.

2. IMPACT MODELS

Most of the impact models between two bodies can be treated using discrete or continuous models. In the former, the impulse-momentum method is applied and the velocity is instantaneous changed right after the collision. Thus, this kind of model does not give information of the contact time (Ahmad *et al.*, 2006). As for the continuous impact model, a force-displacement relationship allows one to predict the colliding bodies behavior during the impact itself. These force-deflection relationships can be given in several ways and evaluated by means of experimental procedures.

The impact model that will be investigated in this paper, which is depicted in Fig. 1, is a continuous single-domain of freedom system of a body impacting a rigid barrier, and its equation of motion is given by,

$$m\ddot{x} + F(x, \dot{x}) = 0 \quad (1)$$

where m is the mass of the system, x is the displacement, F is the impact force and the dots represent time differentiation. In this model, the impact force F can take any desired form, which will influence the system displacement. In the following subsections, the two impact force relations considered are presented.

2.1 Kelvin-Voigt

One impact force model is the Kelvin-Voigt, which is a linear viscoelastic model consisting of a spring and a damper connected in parallel. The impact force for this model is given as:

$$F = kx + c\dot{x} \quad (2)$$

where k is the stiffness parameter and c the damping coefficient. Although the model given by Eq. (2) is simple and represent a nonzero contact time collision, it does not correspond to the real behavior at the beginning and at the end of impact (Püst and Peterka, 2003; Püst, 1998). Nonetheless, this model results in COR that is independent of impact velocity (Ahmad *et al.*, 2006).

However, the Kelvin-Voigt model can be modified to represent a plastic collisions and a velocity dependent COR, by considering different stiffness at the compression and restitution phases. In this way, Equation (2) can be written as,

$$F = \begin{cases} k_c x + c_c \dot{x}; & \text{for } x < x_m \\ k_r x + c_r \dot{x}; & \text{for } x \geq x_m \end{cases} \quad (3)$$

where the subscripts c and r represent the properties at the compression and restitution phases, respectively; and x_m is the maximum displacement or the position where the compression phases ceases. On substituting the impact force given by Eq. (3) in the equation of motion of the system, Equation (1), one may have,

$$\begin{aligned} m\ddot{x} + c_c \dot{x} + k_c x &= 0; & \text{for } x < x_m \\ m\ddot{x} + c_r \dot{x} + k_r x &= 0; & \text{for } x \geq x_m \end{aligned} \quad (4)$$

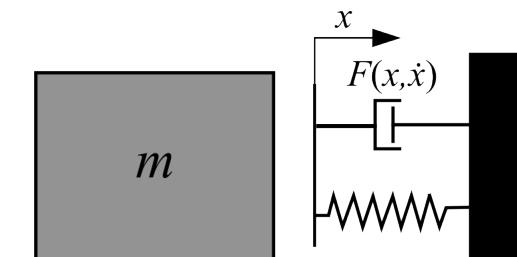


Figure 1. Impact model.

In order to simplify the analysis, the damping coefficient will not be considered at the compression phase, thus one can write Eq. (4) as,

$$\begin{aligned} m\ddot{x} + k_c x &= 0; & \text{for } x < x_m \\ m\ddot{x} + c\dot{x} + k_r x &= 0; & \text{for } x \geq x_m \end{aligned} \quad (5)$$

being $c = c_r$. The damper is added only in the restitution phase because the use of only a spring in the compression phase facilitates the obtainment of the parameters of the system and this phase is well represented by this use (Varanis *et al.*, 2017). In order to solve Eq. (5), some parameters need to be obtained experimentally, they are the maximum displacement (x_m), the time of dynamic crash (t_m) and the initial velocity ($\dot{x}_0$). In addition, it's worth noting that after the system reach the maximum deformation it will no longer oscillate around the origin, but around the permanent deformation (x_p). Thus, the initial condition for the second differential equation of Eq. (5) will be $x_i = x_m - x_p$, which represent the elastic recovery of the system, and a zero initial velocity.

2.2 Nonlinear Force-deflection Model

The other impact force that was considered is a nonlinear model, and has a general form of

$$F = f(x)(1 + g(\dot{x})). \quad (6)$$

According to Púst (1998) and Púst and Peterka (2003), such a type of nonlinear function is sufficiently general to match well the physical properties not only during the deformation but also at the beginning of compressive period and at the end of return one of the impact duration. In Eq. (6), the functions $f(x)$ and $g(\dot{x})$ can take any form; here it is considered the simplest case, which is,

$$F = kx(1 + c\dot{x}) \quad (7)$$

This force model has the advantage that the impact force, F , is always positive during the impact and nonzero at the beginning and at the ending of the compression and restitution phases (Púst and Peterka, 2003). On substituting the impact force given by Eq. (7) in Eq. (1), one may have,

$$m\ddot{x} + kx(1 + c\dot{x}) = 0 \quad (8)$$

The stiffness and damping coefficients, k and c , must be chosen to fit the experimental data, since there is no method to find their values analytically.

3. EXPERIMENTAL SETUP

To perform the experiments an impact bench was built, which is shown in Fig. 2. The bench was made with steel plates of 3 mm of thickness. The impact body used was a cubic aluminum object of 0.06 kg of mass, which was launched using a spring to a rigid steal plate as the barrier. The latter was mounted on the bench by means of slots made in solid square tubes welded perpendicular to each other. This slots allowed the barrier of the impacting object to be positioned, as well as to be changed if necessary. In order to control the path taken by the object, a hole was drilled at its center which passes a shaft that goes till the rigid barrier.

The experimental signals were obtained using an impact accelerometer, which was mounted at the opposite face of the impacting one. The signal acquisition was done using the USB-6251 from National Instruments and a Brüel & Kjær signal

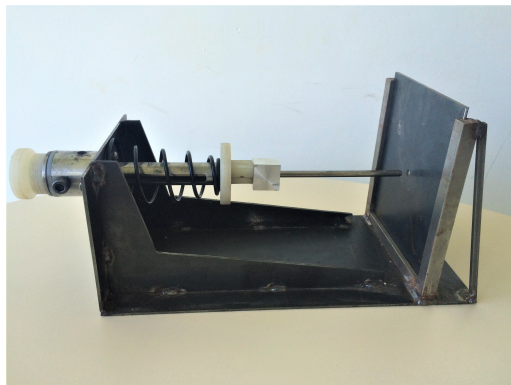


Figure 2. Impact bench.

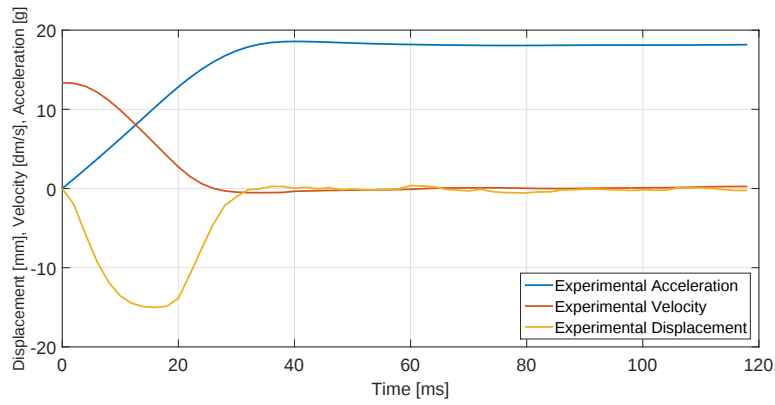


Figure 3. Experimental Signal.

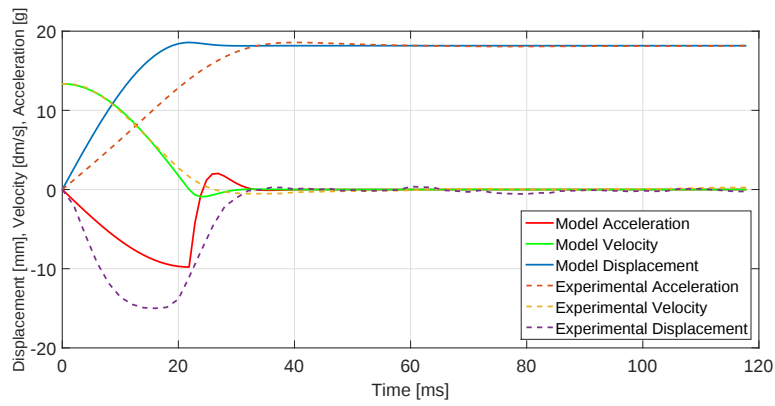


Figure 4. Comparison of the experimental signal with the Kelvin-Voigt model.

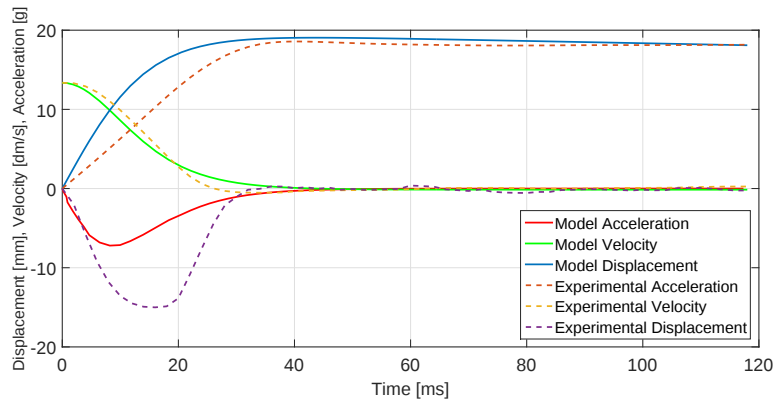


Figure 5. Comparison of the experimental signal with the nonlinear force model

conditioning. The experiments were carried out at the Vibration and Control laboratories in the Mechanical Engineering Faculty of the University of Campinas. Figure 3 shows the experimental signal obtained, where the units (mm, dm/s and g) were chosen so that all the kinematic quantities could be represented in one figure. The measurement was performed with a sample frequency of 0.5 MHz and two-hundred thousand points. Also, the velocity and displacement were obtained by numeric integration.

4. RESULTS AND DISCUSSION

The initial conditions used to solve Eqs. (5) and (8) were $x_0 = 0$ and $\dot{x}_0 = 1.3357$ m/s. The maximum and permanent deformation were obtained experimentally as $x_m = 0.0186$ m and $x_p = 0.0182$ m, respectively. Thus, the initial condition for the second differential equation of Eq. (5) was $x_i = 0.4$ mm and $\dot{x}_i = 0$, as discussed in Section 2.1. Therefore, the responses of the models could be evaluated and they are shown in Figs. 4 and 5. By analyzing the figures, one can note that the responses of both systems were close to the experimental signal. However, Figure 5 shows that the responses of the nonlinear force model represent better the measurement. This model has the disadvantage that the stiffness and

Table 1. Comparison of experimental and numeric values of each model

	Maximum Deformation (x_m)[cm]	Permanent Deformation (x_p)	Rebound Velocity (v')[dm/s]	Time of Dynamic Crash (t_m)[ms]
Experimental Value	1.858	1.816	5.273	26.00
Kelvin-Voigt Model	1.858	1.816	7.586	21.84
Nonlinear Force Model	1.820	1.661	2.00	35.29

damping coefficient can only be obtained experimentally, but it can be improved by a methodology for choosing them, which is not presented in this work. Table 1 summarizes the results obtained with each model and compares them with the experimental data. The Kelvin-Voigt model, despite giving closer values of displacement and time of dynamic crash with the experimental values than the other model, as one can note by Tab. 1, gave a response with less similarity than the nonlinear force model when compared with the experimental signal.

5. CONCLUSION

In this paper an evaluation of two impact models were done by comparing the responses given by them with an experimental signal. The models proposed were a Kelvin-Voigt and a nonlinear force model. The former was modified to have different stiffness parameter in the compression and restitution phases, which also give a velocity dependent COR. The other model proposed has a nonlinear force-deflection characteristic, and represent more accurate the impact phenomenon because the contact force is always positive and a have a nonzero value at the beginning of the compression and at the end of the restitution phase, a feature that is not seen in the Kelving-Voigt system. However, the stiffness and damping parameters of this model can only be obtained experimentally. In addition, the results showed that the model which best fitted the signal was the one with the nonlinear force.

6. ACKNOWLEDGEMENTS

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