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ISOTROPIC WRENCH CAPABILITY MAPS AND ISOTROPIC VELOCITY CAPABILITY MAPS FOR A 3RRR PLANAR PARALLEL MANIPULATOR.

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Abstract. *This paper presents a method to obtain analytically the maximum pure isotropic force and the maximum pure isotropic velocity in a 3RRR symmetrical planar parallel manipulator (SPPM). The proposed method is obtained from a general kineto-static model of the studied manipulator. Maximum pure isotropic force capability maps and maximum pure isotropic velocity capability maps are obtained for the studied manipulator by using different values for the desired moment and the desired angular velocity.*

Keywords: *Isotropic force capability, Isotropic velocity capability, Davies method, Screw theory*

1. INTRODUCTION

Industrial robots can be lead to their force/velocity capability limit once some specific condition tasks are given. When performing a task, the force/velocity required to accomplish it could exceed the own capabilities of the robot, and there may be damage or injury (Weihmann, 2011). In robotics, the force/velocity capability of a manipulator is defined as the maximum force/velocity that can be applied (or sustained) by a manipulator for a given direction of the applied force/velocity based on the limits of the actuators (Mejia *et al.*, 2014a,b). This property can change its magnitude in function of the chosen angle of application of the force. In order to deal with these variations, another property known as the maximum isotropic force capability can be studied. The maximum isotropic force/velocity capability in a manipulator can be defined as the maximum force that a manipulator can apply (or sustain) in all directions (Weihmann *et al.*, 2011).

One of the first works related with the force/velocity capabilities was presentes by Yoshikawa *et. al* in Ref. (Yoshikawa, 1985) defining the velocity manipulability ellipsoid and the force manipulability ellipsoid. Other studies related to this research field can be found in the literature. Frantz *et. al* presented a analysis of the wrench capability for cooperative robotic systems in industrial applications (Frantz *et al.*, 2015). Muraro *et. al* presented a complete kinematic and static analysis of a cable driven spatial mechanism for bedridden patients showing the related forces in this kind of systems (Muraro *et al.*, 2015). Mejia *et. al* presented an analysis of the influence of the assembly modes on the force capability polygons and polytopes in parallel manipulators in Ref. (Mejia *et al.*, 2015a,c) and posteriorly proposed methodologies to evaluate the force capability of planar parallel manipulators with different net degree of constraint in Ref. (Mejia *et al.*, 2015d, 2016)

2. 3RRR SYMMETRICAL PARALLEL MANIPULATOR

Parallel manipulators are mechanical structures that consist in a mobile platform connected to a fixed platform by several branches in order to transmit the movement. Generally, the number of branches of parallel manipulators is equal to their degree of freedom (DoF), and the motors are usually located near the fixed base (Tsai, 1999).

This paper studies a *"3RRR symmetrical planar parallel manipulator"*. This manipulator has its fixed and mobile platforms joined by using three limbs and each limb has three rotational joints which axes are perpendicular to the $(x-y)$ plane. The first of the three joints in each limb is actuated.

The studied manipulator basically is a mechanism with nine joints and eight links, and its mobility can be calculated using the Chebychev-Grubler-Kutzbach criterion, as shown in Eq. (1).

$$M = \lambda(n - j - 1) + \sum_{i=1}^{j} f_i = 3(8 - 9 - 1) + 9 = 3 \tag{1}$$

In the studied manipulator, the fixed base is considered as the frame of the system, the l_1 links belonging to the three limbs are considered as the inputs links and the mobile platform is considered as the end effector link. The primary actuation is applied to the active joints A_1 , A_2 and A_3 of the fixed platform.

The studied manipulator has the mobile and fixed platforms formed by equilateral triangles with sides l_m and l_f respectively. The limbs are formed by two links with longitudes l_1 and l_2 respectively. A *"3RRR symmetrical planar parallel manipulator (SPPM)"* was represented schematically in "Fig. 1". In this study we used the length values: $l_f = 0.5$ [m], $l_m = l_1 = l_2 = 0.2$ [m].

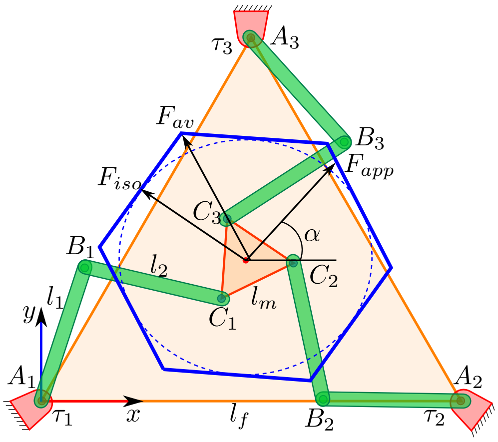

Figure 1. A 3RRR symmetrical planar parallel manipulator with a graphical representation of three of its main force capability indices

3. KINEMATIC AND STATIC ANALYSIS OF PARALLEL MANIPULATORS

The instantaneous analysis of forces and velocities are essential for the performance and design evaluation of parallel and serial manipulators Firmani *et al.* (2008). An instantaneous twist is a screw quantity that contains both angular and translational velocities of the end-effector, i.e., $V = \{\omega^T; v^T\}^T$. Whereas, a wrench is a screw quantity that contains the forces and moments acting on the end-effector, i.e., $F = \{f^T; m^T\}^T$. Velocities and forces at the end effector of a manipulator are obtained through a direct relation with the joint velocities ($\dot{q}$) and joint torques (τ) respectively. For serial manipulators, the relationship between the task and joint spaces is defined by the well known linear transformations shown in Eqs. (2) and (3) below (Tsai, 1999).

$$\dot{q} = J^{-1}\dot{x} \tag{2}$$

$$\tau = J^T F \tag{3}$$

In the other hand, For parallel manipulators, the relationship between the task and joint spaces is defined by the well known linear transformations shown in Eqs. (4) and (5) below. In Eqs. (2) to (5) the term J is referred to as the Jacobian matrix (Tsai, 1999).

$$\dot{q} = J\dot{x} \tag{4}$$

$$\tau = (J^T)^{-1}F \tag{5}$$

Several methodologies which allow us to obtain a complete analysis of velocities and forces in a robotic mechanisms can be found in the literature; however, in this paper the formalism presented in Davies (1983c) is used as the primary mathematical tool to analyze the mechanisms. The Davies method appears in many publications in the literature and further explanations regarding its use can be found in Davies (1983a,b,c); Weihmann *et al.* (2011, 2012); Weihmann (2011); Cazangi (2008). The Davies method provides a systematic way to relate the joint forces/moments and linear/angular velocities in closed kinematic chains Cazangi (2008). This method is based on ***graph theory***, ***screw theory*** and the ***Kirchhoff cut-set law*** and it can be used to obtain the statics of a robotic mechanism as a matricial expression Cazangi (2008).

The Davies' method, together with the relationships shown in Eqs. (2) to (5), can be used in order to simplify the necessary procedures to obtain the forces and moments of the manipulator from the differential kinematic model and viceversa.

4. METHOD TO OBTAIN THE MAXIMUM ISOTROPIC FORCE/VELOCITY (F_{iso}/V_{iso})

Recently, Mejia *et. al* presented in Ref. (Mejia *et al.*, 2015b) a modified scaling factor method to obtain the maximum force with a prescribed moment in cooperative planar manipulators with a net degree of constraint equal to three ($C_N = 3$). Based in such a method, Mejia *et. al* and JCH Pineda *et. al* presented in Refs. (Mejia *et al.*, 2017) and (Pineda *et al.*, 2017) a modification of the method that allow to obtain the Maximum Isotropic Force Capability and the Maximum Isotropic Force Capability Maps in planar manipulators. Below, the Scaling Factor Method to obtain the maximum isotropic force with a prescribed moment (F_{iso}) presented in (Mejia *et al.*, 2017) is summarized in Section 4.1 Section 4.2 presents an adaptation of the Scaling Factor Method to obtain the maximum isotropic force in order to obtain the maximum isotropic velocity with a prescribed angular velocity (V_{iso}).

4.1 Scaling Factor Method to obtain the maximum isotropic force with a prescribed moment (F_{iso})

Formally, the maximum isotropic force can be defined as the maximum force that a manipulator can apply (or sustain) in all directions Weihmann *et al.* (2011). The method reviewed in this section solves the main problem identified in the literature for another maximum force capabilities methods, related with the variations in the forces' magnitudes as a function of the direction of the applied wrenches.

Initially, the proposed method requires a unit wrench $\$_F$ to be imposed at the end effector of the manipulator, such unit wrench must include an associated value μ as the component of the moment in z. The unit wrench is represented as $\$_F = [\cos(\theta), \sin(\theta), \mu]^T$, and the backward statics equation shown in Eq. (5) can be rewritten as:

$$\begin{bmatrix} \tau_{A_1} \\ \tau_{A_2} \\ \tau_{A_3} \end{bmatrix} = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{bmatrix} \cdot \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \\ \mu \end{bmatrix} \tag{6}$$

In order to solve the maximum isotropic force capability problem (F_{iso}), first, an expansion for Eq. (6) can be obtained as:

$$\tau_i(\theta, \mu) = a_{i,1} \cdot \cos(\theta) + a_{i,2} \cdot \sin(\theta) + a_{i,3} \cdot \mu \qquad i = 1, 2, 3 \tag{7}$$

then, assuming the existence of an scaling factor (Ψ) saturating the actuated joints and the knowledge of the maximum value of wrench accepted at each actuated joint ($\tau_{i_{max}}$), two conditions need to be respected simultaneously, these conditions are shown in Eqs. (8) and (9).

$$\tau_i \cdot \Psi = \tau_{i_{max}} \qquad i = 1, 2, 3 \tag{8}$$

$$\mu \cdot \Psi = M_d \qquad i = 1, 2, 3 \tag{9}$$

multiplying Eq. (7) by Ψ and substituting Eqs. (8) and (9) in this equation, we can obtain a generalized equation that describes the scaling factor Ψ as a function of the direction for the application of the force (θ) as shown in Eq. (10).

$$\Psi(\theta) = \frac{\tau_{i_{max}} - a_{3i} \cdot M_d}{a_{3i-2} \cdot \cos(\theta) + a_{3i-1} \cdot \sin(\theta)} \tag{10}$$

Due that the numerator in Eq. (10) is assumed as a constant, the maximum value that the scaling factor $\Psi(\theta)$ can assume is obtained when the denominator in Eq. (10) is minimum. In this way, by deriving the denominator in Eq. (10), equating to zero and solving, it is possible to obtain the critical values for the direction θ which minimize the values of the scaling factor Ψ as shown in Eq. (11).

$$\theta_{crit} = \tan^{-1}\left(\frac{a_{3i-2}}{a_{3i-1}}\right) \tag{11}$$

The values obtained for $\theta_{i_{crit}}$ are the possible directions in which the maximum isotropic force can be found. By evaluating those values obtained from Eq. (11) into the Eq. (10) and manipulating algebraically, we obtain a generalized expression that defines the maximum isotropic force as shown in Eq. (12).

$$\psi_i = \frac{(\tau_{i_{max}} - a_{i,3} \cdot M_d)}{\sqrt{(a_{i,1})^2 + (a_{i,2})^2}} \qquad i = 1, 2, 3 \tag{12}$$

The scaling factors of Eq. (12) can be placed in a set. The scaling factor (Ψ) in this set with the minimum value is the maximum factor which all joint torques/forces can be scaled by and still remain at or below their corresponding maximum values, and represents the maximum isotropic force of the robotic manipulator i.e.:

$$\Psi = \min(\psi_i) \qquad i = 1, 2, 3 \tag{13}$$

4.2 Scaling Factor Method to obtain the maximum isotropic velocity with a prescribed angular velocity (V_{iso})

In this section, and in a similar way as previously defined for the maximum isotropic force, the maximum isotropic velocity can be defined as the maximum linear velocity that a manipulator can follow in all directions.

The proposed method to obtain the maximum isotropic velocity with a prescribed angular velocity (V_{iso}) requires a unit twist $\$_Q$ to be imposed at the end effector of the manipulator, such unit twist must include an associated value σ as the component of the angular velocity in z. The unit twist is represented as $\$_Q = [\cos(\theta), \sin(\theta), \sigma]^T$, and the backward differential kinematic equation shown in Eq. (4) can be rewritten as:

$$\begin{bmatrix} \omega_{A_1} \\ \omega_{A_2} \\ \omega_{A_3} \end{bmatrix} = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{bmatrix} \cdot \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \\ \sigma \end{bmatrix} \tag{14}$$

In order to solve the maximum isotropic velocity capability problem (V_{iso}), first, an expansion for Eq. (14) can be obtained as:

$$\omega_i(\theta, \sigma) = a_{i,1} \cdot \cos(\theta) + a_{i,2} \cdot \sin(\theta) + a_{i,3} \cdot \sigma \qquad i = 1, 2, 3 \tag{15}$$

then, assuming the existence of an scaling factor (Φ) saturating the actuated joints and knowing the maximum value of angular velocity in each actuated joint ($\omega_{i_{max}}$), two conditions need to be respected simultaneously, these conditions are shown in Eqs. (16) and (17).

$$\omega_i \cdot \Phi = \omega_{i_{max}} \qquad i = 1, 2, 3 \tag{16}$$

$$\sigma \cdot \Phi = \Omega_d \qquad i = 1, 2, 3 \tag{17}$$

multiplying Eq. (15) by Φ and substituting Eqs. (16) and (17) in this equation, we can obtain a generalized equation that describes the scaling factor Φ as a function of the direction of the velocity (θ) as shown in Eq. (18).

$$\Phi(\theta) = \frac{\omega_{i_{max}} - a_{3i} \cdot \Omega_d}{a_{3i-2} \cdot \cos(\theta) + a_{3i-1} \cdot \sin(\theta)} \tag{18}$$

Due that the numerator in Eq. (18) is assumed as a constant, the maximum value that the scaling factor $\Phi(\theta)$ can assume is obtained when the denominator in Eq. (18) is minimum. In this way, by deriving the denominator in Eq. (18), equating to zero and solving, it is possible to obtain the critical values for the direction θ which minimize the values of the scaling factor Φ as shown in Eq. (19).

$$\theta_{crit} = \tan^{-1}\left(\frac{a_{3i-2}}{a_{3i-1}}\right) \tag{19}$$

The values obtained for $\theta_{i_{crit}}$ are the possible directions in which the maximum isotropic velocity can be found. By evaluating those values obtained from Eq. (19) into the Eq. (18) and manipulating algebraically, we obtain a generalized expression that defines the maximum isotropic velocity as shown in Eq. (20).

$$\phi_i = \frac{(\omega_{i_{max}} - a_{i,3} \cdot \Omega_d)}{\sqrt{(a_{i,1})^2 + (a_{i,2})^2}} \qquad i = 1, 2, 3 \tag{20}$$

The scaling factors of Eq. (20) can be placed in a set. The scaling factor (Φ) in this set with the minimum value is the maximum factor which all joint angular/linear velocities can be scaled by and still remain at or below their corresponding maximum values, and represents the maximum isotropic velocity of the robotic manipulator i.e.:

$$\Phi = \min(\phi_i) \qquad i = 1, 2, 3 \tag{21}$$

5. RESULTS

To validate the Scaling Factor Method to obtain the maximum isotropic force with a prescribed moment (F_{iso}) and the maximum isotropic velocity with a prescribed angular velocity (V_{iso}) shown in Sections 4.1 and 4.2 respectively, the planar parallel manipulator shown in “Fig. 1” was used as an study case.

By applying the proposed Scaling Factor Methods to obtain the maximum isotropic force with a prescribed moment (F_{iso}) and the maximum isotropic velocity with a prescribed angular velocity (V_{iso}), for all reachable positions in the manipulator’s workspace, it is possible to obtain a complete map representing the behavior of the F_{iso} and V_{iso} along the complete the workspace. It allow us to evaluate the places where the manipulator could execute a determined task in a more efficient way.

“Figure. 2” shows different maximum isotropic force capability maps for the studied SPPM by using the topological information shown in Section 2. and the functional parameter $\tau_{i_{max}} = \pm 4.2$ [Nm], but the desired moment M_d was varied. In “Figure. 2.a)” was used $M_d = -1$ [Nm]. In “Figure. 2.b)” was used $M_d = 0$ [Nm]. In “Figure. 2.c)” was used $M_d = 1$ [Nm] and In “Figure. 2.d)” was used $M_d = 2$ [Nm].

In the other hand, “Figure. 3” shows different maximum isotropic velocity capability maps for the studied SPPM by using the topological information shown in Section 2. and the functional parameter $\omega_{i_{max}} = 5$ [rpm], but the desired angular velocity Ω_d was varied. In “Figure. 3.a)” was used $\Omega_d = -1$ [rpm]. In “Figure. 3.b)” was used $\Omega_d = 0$ [rpm]. In “Figure. 3.c)” was used $\Omega_d = 1$ [rpm] and In “Figure. 2.d)” was used $\Omega_d = 3$ [rpm].

The obtained force/velocity capability maps allow us to estimate the regions within the work space where the manipulator has the best performance in terms of force/velocity.

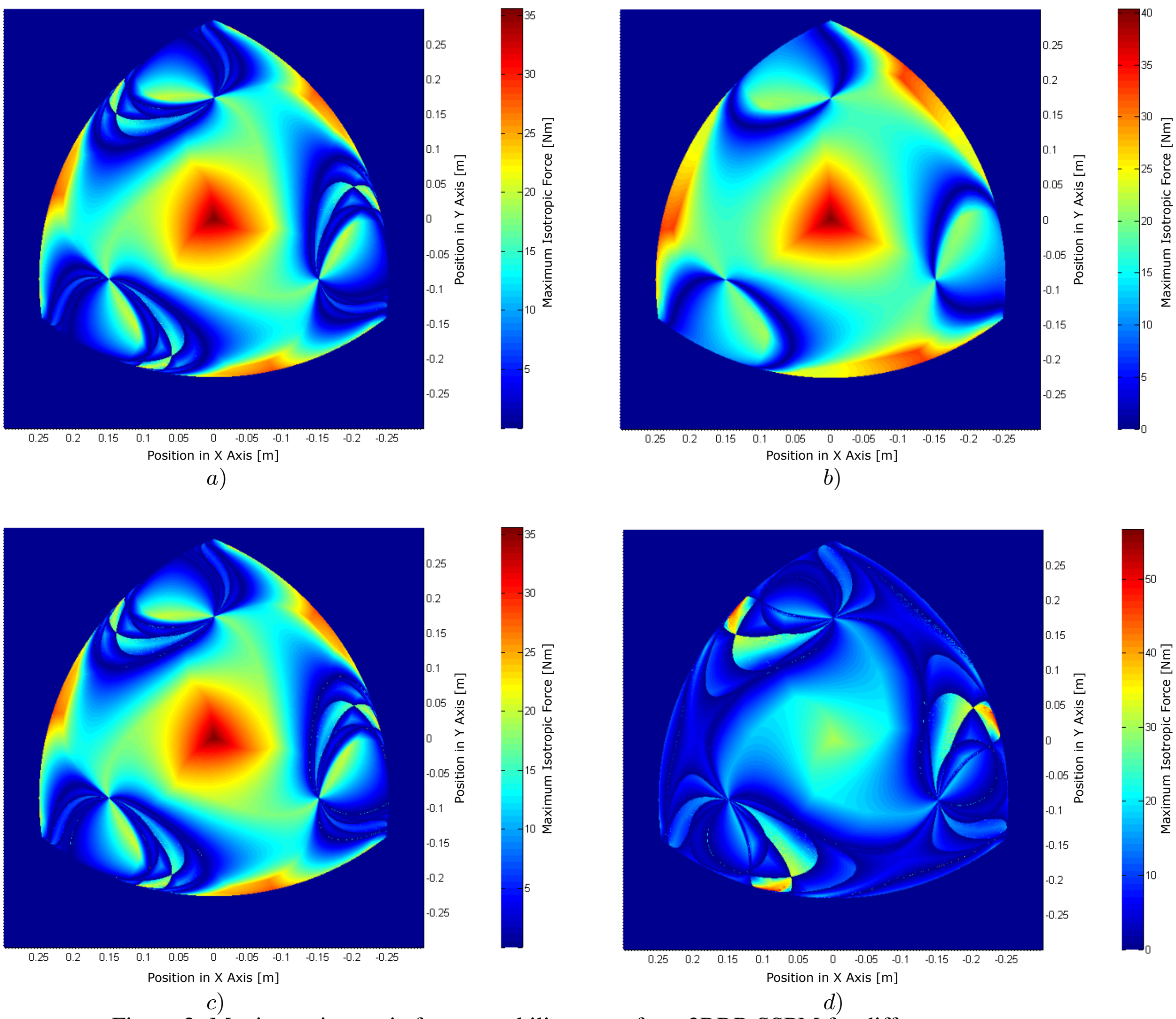

Figure 2. Maximum isotropic force capability maps for a 3$\underline{R}$RR SSPM for different parameters

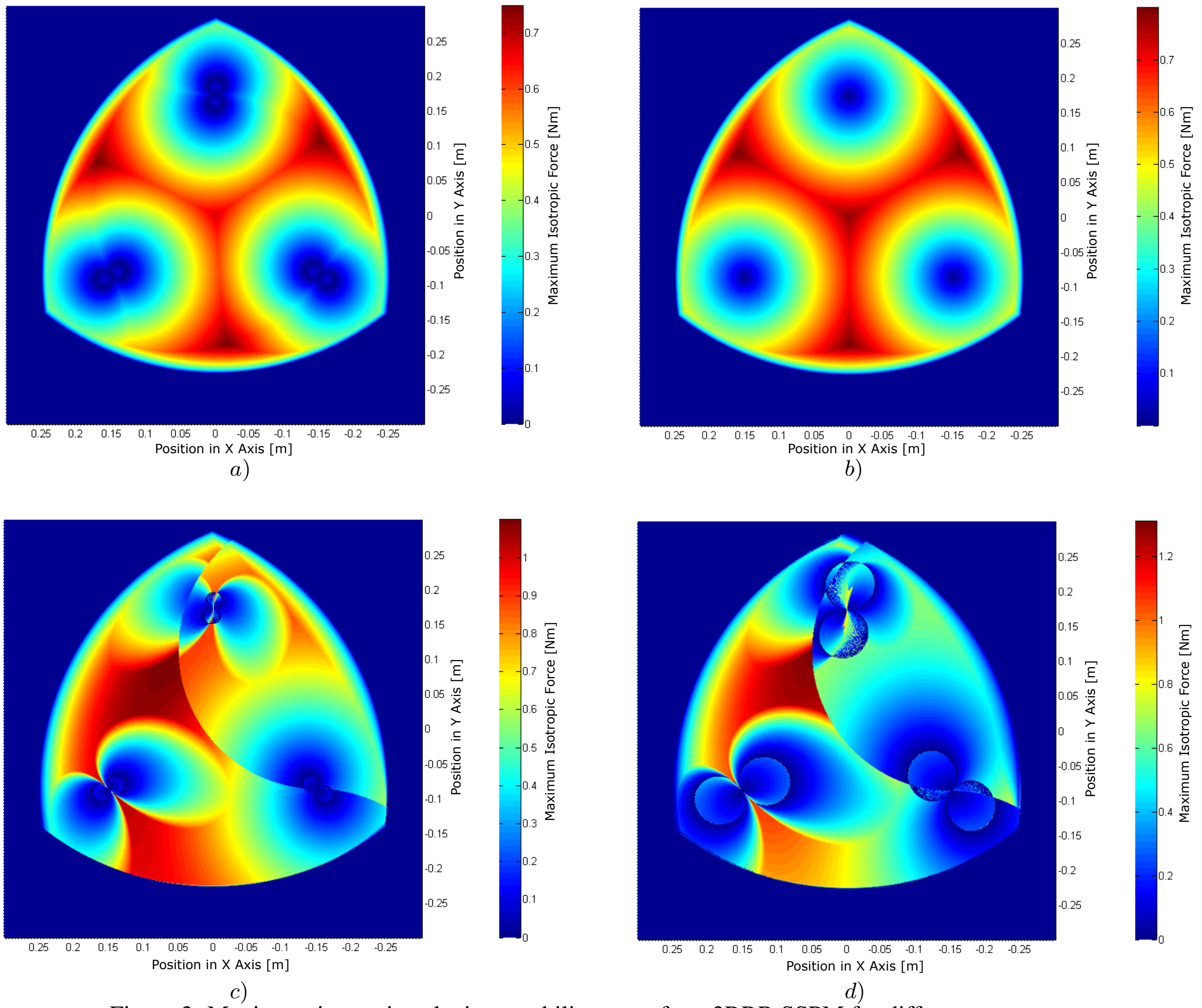

Figure 3. Maximum isotropic velocity capability maps for a 3RRR SSPM for different parameters

6. CONCLUSIONS

This paper studied how the maximum isotropic force with a prescribed moment (F_{iso}) and the maximum isotropic velocity with a prescribed angular velocity (V_{iso}) changes along the working space of a symmetrical parallel manipulator, opening the possibility to visualize the best regions where the manipulator can act in order to improve its efficiency.

A modification of the Scaling Factor Method presented by Mejia *et. al* was proposed in order to obtain analytically the maximum isotropic velocity with a prescribed angular velocity. The proposed method in this paper uses a generalized model of the statics or differential kinematics of the manipulator as an starting point to solve the problem of the maximum isotropic force/velocity. Authors used the formalism presented by Davies as the primary mathematical tool to analyze the mechanisms statically and after that, the relations shown in Eqs. (4) and (5) were used in order to obtain the differential kinematics of the studied manipulator.

The present study may be extended in various ways. Manipulators with different DOFs, kinematic chains and including dynamic behavior may be studied, and variations on the imposed moment may be considered in future researches.

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