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NONLINEAR EDDY VISCOSITY MODELS FOR VISCOELASTIC TURBULENCE

Angela O. Nieckele

Pontifícia Universidade Católica do Rio de Janeiro, Department of Mechanical Engineering, 22451-900, Rio de Janeiro, RJ, Brazil
nieckele@puc-rio.br

Roney Thompson

Universidade Federal do Rio de Janeiro, Department of Mechanical Engineering (COPPE), 21941-450, Rio de Janeiro, RJ, Brazil
rthompson@mecanica.coppe.ufrj.br

Gilmar Mompean

Université de Lille 1, Department of Mechanical Engineering, F-59655 Villeneuve d'Ascq, Lille, France
gilmar.mompean@polytech-lille.fr

Abstract. *Viscoelastic turbulence is an important area of research that has phenomenological, as well as practical motivations for the study. In the present work we use tensorial projections of a DNS database of the turbulent flow of a viscoelastic fluid in order to find the corresponding coefficients of a nonlinear eddy viscosity model, i.e. a model that consists of a tensor basis that encompasses a set of mean kinematic tensors beyond the mean rate-of-strain tensor. One important finding is that, as elasticity increases, the Boussinesq hypothesis decreases its capability to capture the Reynolds stress tensor.*

Keywords: *Viscoelastic turbulence, Nonlinear eddy viscosity models, DNS, Reynolds average, Tensorial decompositions.*

1. INTRODUCTION

Since the discovery that few amounts of polymer added to a Newtonian turbulent flow was able to decrease the drag in a flow (Toms, 1948) viscoelastic turbulence has become an increasing area of research. The first investigations were mainly restricted to experiments and important findings, such as the maximum drag reduction (MDR) asymptote, known as Virk's asymptote (Virk *et al.*, 1970). What happens is that drag reduction (DR) saturates in a certain point, i.e. MDR cannot be crossed by increasing concentration, molecular weight, elasticity of the fluid or the Reynolds number of the flow.

This field is very challenging and the phenomenon of DR is not yet fully understood, with many open questions. At the end of the nineties, the first Direct Numerical Simulation (DNS) of a viscoelastic turbulent flow was conducted (Sureshkumar *et al.*, 1997). Since then, other DNS investigations were significant to enhance our understanding of this phenomenon.

One important area that is weakly explored in the literature is how to model the Reynolds stress tensor that arises from a Reynolds average filter of the momentum equation considering the viscoelastic nature of the material. In this connection, we can stress the works of Pinho and co-workers on the subject (Resend *et al.*, 2013; Masoudian *et al.*, 2013).

In the present work we extend the procedure employed by Nieckele *et al.* (2016) in the context of Newtonian turbulence to the case of a viscoelastic turbulence using the DNS database available at (Thais *et al.*, 2012).

2. METHODOLOGY

We consider here two types of tensorial projections in order to decompose a symmetric tensor, \mathbf{G} , with respect to a second one, \mathbf{F} . The first decomposition splits \mathbf{G} into a part that is proportional to \mathbf{F} and another that is orthogonal to \mathbf{F} . It is given by

$$\mathbf{G} = \mathbf{G}_F + \tilde{\mathbf{G}}_F = \frac{\mathbf{G} : \mathbf{F}}{\mathbf{F} : \mathbf{F}} \mathbf{F} + \frac{(\mathbf{F} : \mathbf{F})\mathbf{G} - (\mathbf{G} : \mathbf{F})\mathbf{F}}{\mathbf{F} : \mathbf{F}} \quad (1)$$

The second decomposition is composed by the projection of \mathbf{G} onto the subspace of the tensors that have the same eigenvectors as \mathbf{F} , and the complement of this projection

$$\mathbf{G} = \Phi_G^F + \tilde{\Phi}_G^F = \mathbf{1}^{FF} : \mathbf{G} + (\mathbf{1}^{\delta\delta} - \mathbf{1}^{FF}) : \mathbf{G} \quad (2)$$

$$\mathbf{1}^{FF} = \sum_{i=1}^3 \mathbf{e}_i^F \mathbf{e}_i^F \mathbf{e}_i^F \mathbf{e}_i^F \quad (3)$$

i.e. it is a fourth order tensor associated to the unit eigenvectors of \mathbf{F} . These decompositions are employed decomposing the deviatoric part of the Reynolds stress with respect to a tensorial basis of mean kinematic quantities.

With these kinds of decomposition, we are able to create different backbones for models. At the present work, four models for the traceless Reynolds stress, \mathbf{a}

$$\mathbf{a} = -\overline{\mathbf{u}'\mathbf{u}'} - 1/3 \text{tr}(-\overline{\mathbf{u}'\mathbf{u}'}) \quad (4)$$

were investigated, and are defined as

$$M_I : \mathbf{a}_I = \alpha \mathbf{D} \quad (5)$$

$$M_{II} : \mathbf{a}_{II} = \alpha_o \mathbf{I} + \alpha_D \mathbf{D} + \alpha_{D2} \mathbf{D}^2 \quad (6)$$

$$M_{III} : \mathbf{a}_{III} = \alpha_o \mathbf{I} + \alpha_D \mathbf{D} + \alpha_{D2} \mathbf{D}^2 + \beta_P \mathbf{P} \quad (7)$$

$$M_{IV} : \mathbf{a}_{IV} = \alpha_D \mathbf{D} + \beta_P \mathbf{P} \quad (8)$$

where \mathbf{D} is the symmetric part of the velocity gradient

$$\mathbf{D} \equiv \frac{1}{2} (\nabla \mathbf{U} + \nabla^T \mathbf{U}) \quad (9)$$

and $\mathbf{P} = \mathbf{D} \mathbf{W}^* - \mathbf{W}^* \mathbf{D}$ is the non-persistence-of-straining tensor, with \mathbf{W}^* as the relative vorticity tensor, defined as the vorticity \mathbf{W} measured with respect to the rate of rotation of the eigenvectors of \mathbf{D}

$$\mathbf{W}^* \equiv \mathbf{W} - \boldsymbol{\Omega}^D \quad (10)$$

$$\mathbf{W} \equiv \frac{1}{2} (\nabla \mathbf{U} - \nabla^T \mathbf{U}) \quad (11)$$

where $\boldsymbol{\Omega}^D \equiv \dot{\mathbf{e}}_k^D \mathbf{e}_k^D$, with \mathbf{e}_k^D as the unit eigenvectors of \mathbf{D} , and $\dot{\mathbf{e}}_k^D$ the material time derivative of \mathbf{e}_k^D .

To measure the quality of the models approximations with regard to DNS data, the indexes

$$Idx_i = 1 - \frac{2}{\pi} \cos^{-1} \sqrt{\frac{\text{tr} \mathbf{a}_i^2}{\text{tr} \mathbf{a}^2}} \quad i \in \{ I; II; III; IV \} \quad (12)$$

can be employed, where 1 is excellent adherence with the DNS data e zero is the opposite.

3. RESULTS AND DISCUSSION

The channel flow of a FENE-P material is governed by four parameters: (i) Reynolds number Re_τ , which is based on the friction velocity u_τ , half channel height H , and the solution viscosity at zero shear rate, ν_o ,

$$Re_\tau = \frac{u_\tau H}{\nu_o} \quad ; \quad u_\tau = \sqrt{\frac{\tau_w}{\rho}} \quad ; \quad \nu_o = \nu_s + \nu_p \quad (13)$$

where τ_w is the wall shear, and ρ is de density.(ii) β_o , the ratio of the solvent Newtonian viscosity ν_s over the solution viscosity at zero shear rate, $\nu_o = \nu_s + \nu_p$

$$\beta_o = \frac{v_s}{v_s + v_p} \quad (14)$$

(iii) the maximum molecular extensibility L^2 and (iv) friction Weissenberg number, We

$$We = \frac{\lambda u_\tau^2}{v_o} \quad (15)$$

where λ is the elastic relaxation time of the polymer.

The four models were applied to the DNS data of a FENE-P material for a channel flow (Thais *et al.*, 2012), for $Re_\tau=180$ and 1000, Weissenberg number $We=50$ and 115, and maximum dimensionless extensibility of the model dumbbell, $L=30$ and 100 and $\beta_o=0.9$.

The correlation index for $Re_\tau=180$ is presented in Fig. 1 for all cases, while Fig. 2 corresponds to $Re_\tau=1000$. Note that similar results were obtained for both Reynolds numbers. Further, note that for both Reynolds numbers, for both types of fluids, i.e., Newtonian and FENE-P material, the correlation index is equal to one only for model III.

The smallest correlation index corresponds to model I, followed by model II. Both models present similar behaviour, with an increase of the index as y^+ increases. Model IV presents an approximately constant correlation index, along the boundary layer, approximately equal to 0.8.

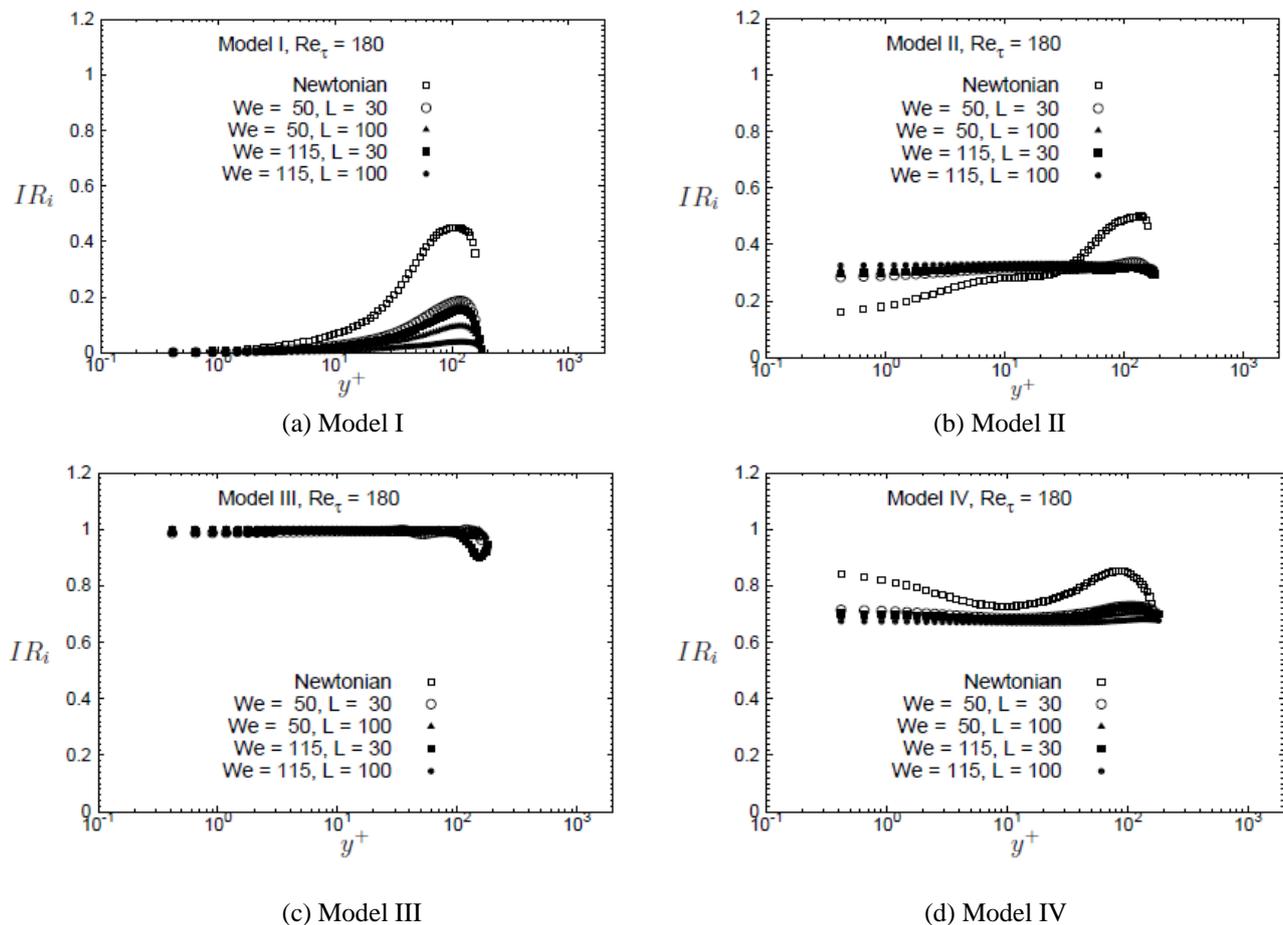


Figure 1. Index of four models. $Re_\tau=180$

4. CONCLUSIONS

The Boussinesq hypothesis continually decreases its ability to capture the Reynolds stress tensor obtained from the DNS of a FENE-P material. This fact shows the importance of considering nonlinear eddy viscosity models in viscoelastic turbulence. Adding nonlinear terms in the symmetric part of the velocity gradient and considering the mean non-persistence-of-straining tensor enhances substantially the adherence of the model to DNS data.

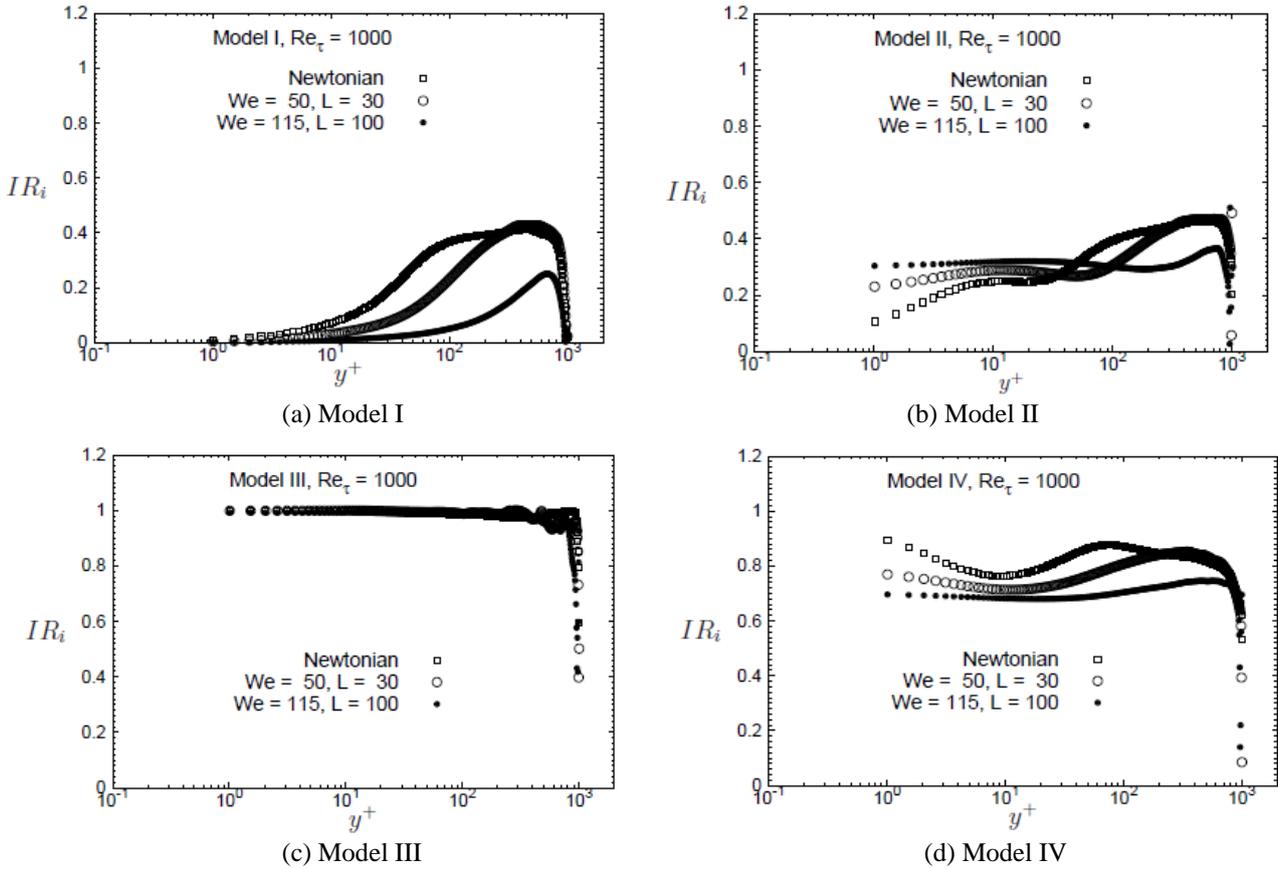


Figure 2. Index of four models. $Re_\tau=1000$.

5. ACKNOWLEDGEMENTS

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7. RESPONSIBILITY NOTICE

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