24th COBEM - 2017

24th ABCM International Congress of Mechanical Engineering
December 3-8, 2017, Curitiba, PR, Brazil

,

COBEM-2017-5805

MODELLING AND CONTROL OF A WHEELED MOBILE ROBOT

Allan Carlos Ferreira de Oliveira
Magno Enrique Mendoza Meza
Federal University of ABC Engineering, Modeling and Applied Social Sciences Center (CECS), Santo André, Brazil.
allancfo2012@gmail.com; magno.meza@ufabc.edu.br

Diego Paolo Ferruzzo Correa
Federal University of ABC Engineering, Modeling and Applied Social Sciences Center (CECS), São Bernardo dos Campos, Brazil.
diego.ferruzzo@ufabc.edu.br

Abstract. *This paper proposes the representation of the dynamic model of a wheeled mobile robot (WMR), considering the center of mass and the center of rotation separated by a distance 'a'. This system has three degrees of freedom, presents nonholonomic restrictions, two motorized wheels and a free wheel which ensures its stability. The dynamic model of WMR was performed through the Euler-Lagrange formalism and model obtained in this work will be used in the design of a control strategy to trajectory tracking. Designed controllers were based on the PID and Backstepping technique and the results of the simulations demonstrate the convergence of robot velocity to controller velocity, consequently trajectory tracking. However, some divergences were verified, where the linear control presents a settling time of 10s and maximum overshoot at the control input (torque) of 0.3 Nm, unlike the nonlinear control where settling time is close to 0 and maximum overshoot in the torque of 0.8 Nm*

Keywords: *Backstepping control; PID control; wheeled mobile robot.*

1. INTRODUCTION

Control of wheeled mobile robots (WMR) has attracted the attention of many researchers in robotic control because of their difficulties in control design and implementation. WMR is known by its nonholonomic constraint system and during the last decade, many control methods have been proposed to address the problem of tracking control of a mobile robot under nonholonomic constraints (Zidani, *et al.*, 2015).

It is defined as nonholonomic, finite-dimensional system, where some kind of constraint is imposed on one or more states of system. These limitations can be caused by the conservation of angular moment, conditions imposed by the impossibility of moving in one or more directions, as a result of the imposition of restrictions during the design of the control system, by the fact that the system does not have actuators in all directions of the space of the problem, and in several other situations (Figueredo and Jota, 2014).

The problem of trajectory tracking consists in making the robot to reach and follow a determined trajectory in Cartesian space, starting from a certain initial configuration, inside or outside the trajectory (Vieira, 2005). The base structure is composed of three wheels (a front passive wheel and two active rear wheels) and its movement is directly related to the velocity of the wheels. For a rectilinear displacement, the wheel velocities must be equal; To turn left or right it is necessary that the velocities of the wheel (on the opposite side that you wish to turn) is higher than the other, and if there is a need to rotate around the center, the velocities must have the same module, however opposite directions.

The appropriate design of a control requires knowledge about the kinematic and dynamic model of the robot. In relation to kinematics, it is fundamental to understand the behavior of the mechanical system that begins with the process of describing the movement's contribution of each wheel. In the dynamics is made a study of the movement in which the forces are considered, using the formulation of Lagrange.

The purpose of the controller for the kinematic model is to produce velocity outputs for the robot to make the tracking error between the actual and reference trajectories converges to zero. A torque controller is designed based on the system dynamics such that the velocities of the mobile robot converges to the generated desired velocities (Mohareri, *et al.*, 2012).

The objective of this paper is the development of a dynamic controller (torque controller) for wheeled mobile robot which has its center of rotation and center of mass located at different points. It will be designed a conventional controller (PID) and a nonlinear control based on the Backstepping technique, comparing its results. In addition, it is fundamental to determine the parameters of the controllers.

2. DYNAMIC MODEL OF WMR

Consider a WMR shown in Fig. 1. It can be modeled by a platform of mass *m* and width *L*, operated by two independent wheels of radius *r* and a freewheel ensuring its stability (Zidani, *et al.*, 2015). The center of rotation (A) is located on the axis (Y ') which connects the two wheels and moment of inertia (I) is at a distance *a* from the axis Y' on the axis X '.

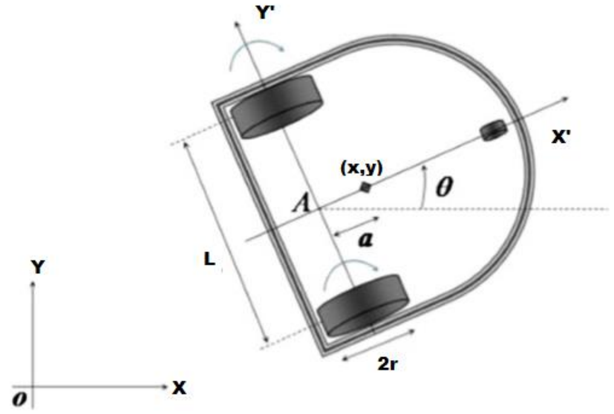

Figure 1. Representation of WMR

The robot configuration, also called posture (**q**), can be written by the three degrees of freedom.

$$q(t) = [x(t) \quad y(t) \quad \theta(t)] \tag{1}$$

where θ is the orientation of the robot in relation to the fixed reference in space (YOX) and (x,y) are the coordinates of center of mass.

The well-known dynamic equation of the mobile robot system with n-generalized coordinates $\mathbf{q} \in \mathbf{R}^{nx1}$, and inputs r = n-m, can be described as (Chih-Yang, *et al.*, 2012):

$$M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + F(\dot{q}) + G(q) + \tau_d = B(q)\tau - A^T(q)\lambda \tag{2}$$

where $\mathbf{M(q)} \in \mathbf{R}^{nxn}$ is a symmetric positive definite inertia matrix, $\mathbf{V}_m(\mathbf{q},\dot{\mathbf{q}}) \in \mathbf{R}^{nxn}$ is the centripetal and Coriolis matrix, $F(\dot{q}) \in \mathbf{R}^{nx1}$ denotes the surface friction, $\mathbf{G(q)} \in \mathbf{R}^{nx1}$ is the gravitational vector, $\tau_d \in \mathbf{R}^{nx1}$ denotes bounded unknown disturbances including unstructured unmodeled dynamics, $\mathbf{B(q)} \in \mathbf{R}^{nx(n-m)}$ is the input transformation matrix, , $\boldsymbol{\tau} \in \mathbf{R}^{(n-m)x1}$ is a control input vector, $\mathbf{A} \in \mathbf{R}^{mxn}$ is a matrix associated with the nonholonomic constraints, $\lambda \in R^{mx1}$ is a Lagrange multiplier associated with the constraints, and $\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}$ denote velocity and acceleration vectors, respectively.

The variables in the Eq. (2) are defined as:

$$M(q) = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & ma^2 + \ I \end{bmatrix} \quad A^T = \begin{bmatrix} -\sin\theta \\ \cos\theta \\ -a \end{bmatrix} \quad B(q) = \frac{1}{r}\begin{bmatrix} \cos\theta & \cos\theta \\ \sin\theta & \sin\theta \\ \frac{L}{2} & \frac{-L}{2} \end{bmatrix} \quad \tau = \begin{bmatrix} \tau_D \\ \tau_E \end{bmatrix} \tag{3}$$

It is considered a planar motion of the robot, in this way, there is no slope in the path to be covered, then the term of gravity is eliminated in the dynamic equation. In addition, suppose that friction and disturbances are neglilible in the system (Mohareri, *et al*,2012).

$$G(q) = \ F(\dot{q}) = \ \tau_d = 0 \tag{4}$$

Differentiating the kinematic model, which is derived from the position vector , we have:

$$\ddot{q}(t) = \dot{J}(q)v(t) + J(q)\dot{v}(t) \quad (5)$$

Substituting Eq. (4) and (5) into (2), we have:

$$M(q)(\dot{J}(q)v(t) + J(q)\dot{v}(t)) + V_m(q,\dot{q})(J(q)v(t) = B(q)\tau - A^T(q)\lambda \quad (6)$$

In order to eliminate the Lagrange coefficients (λ) it is necessary to multiply the above equation by J^T, resulting in:

$$J^T M(q)\dot{J}(q)v(t) + J^T M(q)J(q)\dot{v}(t) + J^T V_m(q,\dot{q})J(q)v(t) = J^T B(q)\tau \quad (7)$$

can be rewritten as:

$$\bar{M}\dot{v}(t) + \overline{V_m}v(t) = \bar{B}(q)\tau \quad (8)$$

where: $\bar{M} = J^T(q)M(q)J(q)$, $\overline{V_m} = J^T(q)M(q)\dot{J}(q) + J^T(q)V_m(q,\dot{q})J(q)$ and $\bar{B}(q) = J^T(q)B(q)$.

After performing the calculation of the matrices and replacing them in (8), we arrive at the following dynamic model:

$$\begin{bmatrix} m & 0 \\ 0 & ma^2 + I \end{bmatrix}\begin{bmatrix} \dot{v} \\ \dot{\omega} \end{bmatrix} + \begin{bmatrix} 0 & -ma\dot{\theta} \\ ma\dot{\theta} & 0 \end{bmatrix}\begin{bmatrix} v \\ \omega \end{bmatrix} = \frac{1}{r}\begin{bmatrix} 1 & 1 \\ \frac{L}{2} & \frac{-L}{2} \end{bmatrix}\begin{bmatrix} \tau_D \\ \tau_E \end{bmatrix} \quad (9)$$

3. PID CONTROLLER

The solution to design a PID is to linearize the system around the equilibrium point. To find the equilibrium points (v_{equ} e ω_{equ}), we consider $\dot{v} = \dot{\omega} = 0$ and $\tau_{D\ equ} = \tau_{E\ equ} = 0$. Thus, we have:

$$0 = 0.03\omega_{equ}{}^2 \rightarrow \omega_{equ} = 0 \quad (10)$$

$$0 = -16.67 v_{equ}\omega_{equ} \rightarrow v_{equ} \in R^+ \quad (11)$$

There are several equilibrium points for the system, since v_{equ} can assume any positive value. It will be taken into account the maximum velocity that the system reaches, which in this case is 1.4 m / s. Thus, the linearized model is given by equation below, and it is verified that the system is controllable, since the rank of the controllability matrix is equal to 2.

$$A' = \begin{bmatrix} 0 & 0 \\ 0 & -23 \end{bmatrix} \; ; \; B' = \begin{bmatrix} 103 & 103 \\ 7159 & -7159 \end{bmatrix} \; ; \; C' = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (12)$$

In order to apply a monovariable control technique, which is usually simpler, it is possible to use uncouplers in the system or to design the PID controllers to meet their respective loops, loop 1 and loop 2 (Fig. 2), and then check the influence on the other output.

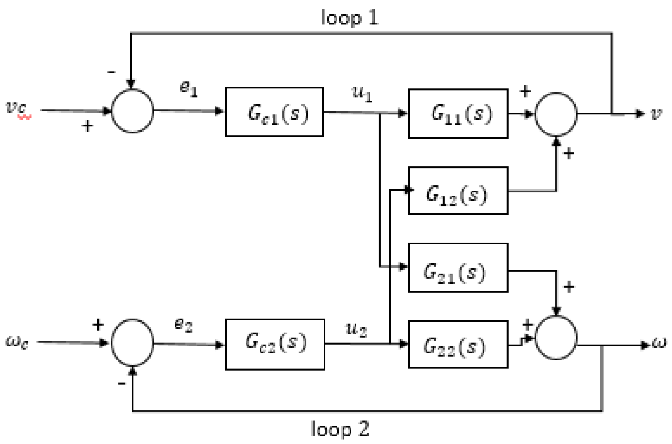

Figure 2. Dynamic of the WMR

The closed-loop transfer functions are given by:

$$G_{mf1} = \frac{103(K_{d1}s^2 + K_{p1}s + K_{I1})}{(103K_{d1}+1)s^2 + 103K_{p1}s + 103K_{I1}} \quad (13)$$

$$G_{mf2} = \frac{-7159(K_{d2}s^2 + K_{p2}s + K_{I2})}{(1-7159K_{d2})s^2 + (23-7159K_{p2})s - 7159K_{I2}} \quad (14)$$

To obtain settling time (criterion 2%), $t_a = 4$s, similar to the reference signal, considering a damping factor of $\zeta = 0.8$, the poles should be allocated to $s_{1,2} = -1 \pm j0.75$. Consequently, characteristic equation $= s^2 + 2s + 1.6526$.

Finally, equating the characteristic equation of the system with the characteristic equation above, we have the following parameters:

$$K_{p1} = \frac{2}{103}\ ;\ K_{I1} = \frac{1.5626}{103}\ ;\ K_{d1} = 0 \quad (15)$$

$$K_{p2} = \frac{21}{7159}\ ;\ K_{I2} = \frac{-1.5626}{7159}\ ;\ K_{d2} = 0 \quad (16)$$

4. NON-LINEAR CONTROLLER BASED ON THE BACKSTEPPING TECHNIQUE

The model given by Eq. (9) will be rewritten as follows:

$$\dot{x}_1 = a{x_2}^2 + \frac{1}{mr}\,u_1 \quad (17)$$

$$\dot{x}_2 = -\frac{ma}{ma^2+I}x_1x_2 + \frac{L}{2r(ma^2+I)}\,u_2 \quad (18)$$

For the first step, considering only the linear velocity (x_1) and a reference velocity (x_{1ref}), the system error is defined as:

$$z_1 = x_{1ref} - x_1 \quad (19)$$

whose derivate is:

$$\dot{z}_1 = \dot{x}_{1ref} - (a{x_2}^2 + \frac{u_1}{mr}) \quad (20)$$

Proposing a Lyapunov candidate function (Vc) must be positive definite and its its definite negative time derivative, being candidate:

$$V_c(z_1) = \frac{1}{2}{z_1}^2 \quad (21)$$

and whose time derivative is:

$$\dot{V}_c(z_1) = z_1\dot{z}_1 = z_1[\dot{x}_{1ref} - a{x_2}^2 - \frac{u_1}{mr}] \quad (22)$$

The stability of z1 is obtained by introducing a control input where:

$$u_1 = mr[-a{x_2}^2 + \dot{x}_{1ref} + K_4z_1] \quad (23)$$

then Eq. (22) becomes:

$$\dot{V}_c(z_1) = -K_4{z_1}^2 < 0\ \ (K_4 > 0)$$

The next step, considering the angular velocity (x_2), the system error and its derivative is given by equations (24) and (25), respectively:

$$z_2 = x_{2ref} - x_2 \tag{24}$$

$$\dot{z}_2 = \dot{x}_{2ref} + \frac{ma}{ma^2 + I} x_1 x_2 - \frac{L}{2r[ma^2 + I]} u_2 \tag{25}$$

For the final controller design, a new Lyapunov function candidate is choosen as follows:

$$V_c(z_1, z_2) = \frac{1}{2} {z_1}^2 + \frac{1}{2} {z_2}^2 \tag{26}$$

whose time derivative is:

$$\dot{V}_c(z_1) = z_1 \dot{z}_1 + z_2 \dot{z}_2 = z_1 [\dot{x}_{1ref} - a{x_2}^2 - \frac{1}{mr} u_1] + \ z_2 [\dot{x}_{2ref} + \frac{ma}{ma^2 + I} x_1 x_2 - \frac{L}{2r(ma^2 + I)}\ u_2] \tag{27}$$

If we choose:

$$u_2 = \frac{2r(ma^2 + I)}{L} \left(\dot{x}_{2ref} + \frac{ma}{ma^2 + I} x_1 x_2 + K_5 z_2 \right) \tag{28}$$

we obtain:

$$\dot{V}_c(z_1) = -K_5 {z_2}^2 < 0 \tag{29}$$

where K_5 is positive constant that guarantees the asymptotic convergence of $z_2 \to 0$.

$$\tau_D = \frac{1}{2}[u_1 + u_2] = \ \frac{1}{2}\left[mr\left(\dot{x}_{1ref} - a{x_2}^2 + K_4 z_1\right) + \frac{2r(ma^2 + I)}{L}\left(\dot{x}_{2ref} + \frac{ma}{ma^2 + I} x_1 x_2 + K_5 z_2\right)\right] \tag{30}$$

$$\tau_E = \frac{1}{2}[u_1 - u_2] = \ \frac{1}{2}\left[mr\left(\dot{x}_{1ref} - a{x_2}^2 + K_4 z_1\right) - \frac{2r(ma^2 + I)}{L}\left(\dot{x}_{2ref} + \frac{ma}{ma^2 + I} x_1 x_2 + K_5 z_2\right)\right] \tag{31}$$

The fig 3 represents the kinematic and dynamic models of WMR.

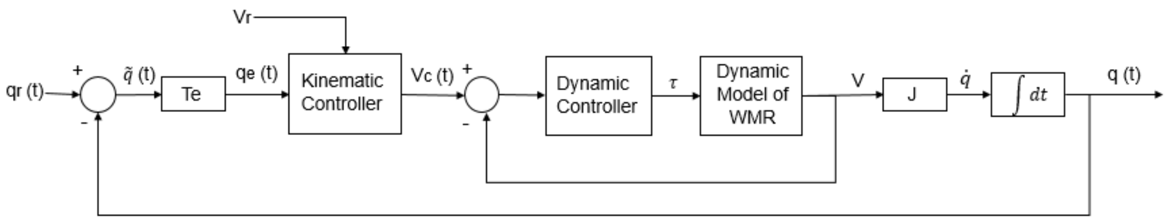

Figure 3. Complete structure of the wheeled mobile robot.

5. PARAMETERS

The parameters of the kinematic and dynamic controllers are determined using the Simulink® software. For this, the complete structure of the WMR (Fig. 3) is implemented, the initial and reference conditions are defined as:

- Linear and angular reference velocities are $v_r = 1.4\ m/s$ e $\omega_r = 0\ rad/s$.
- Reference trajectory is a straight line with a slope of 45 °.
- Inicial posture is $q = [0\ 0\ 0]$;

The way of determining the parameters was exhaustive, obtaining: K1 = 45, K2 = 20, K3 = 50, K4 = 200, K5 = 60.

6. RESULTS AND DISCUSSION

6.1. Dynamic controller based on Backstepping technique.

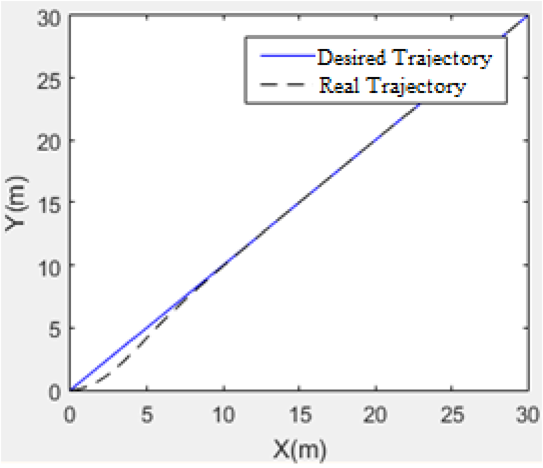

Figure 4. Rectilinear trajectory.

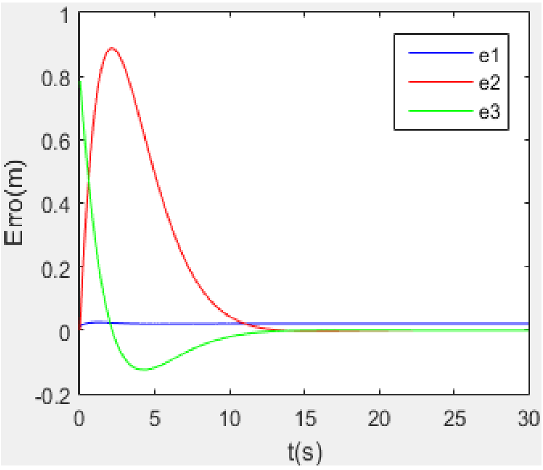

Figure 5. Tracking Error (e_1 – Error in position x, e_2 – Error in position y -, e_3 – Error in orientation).

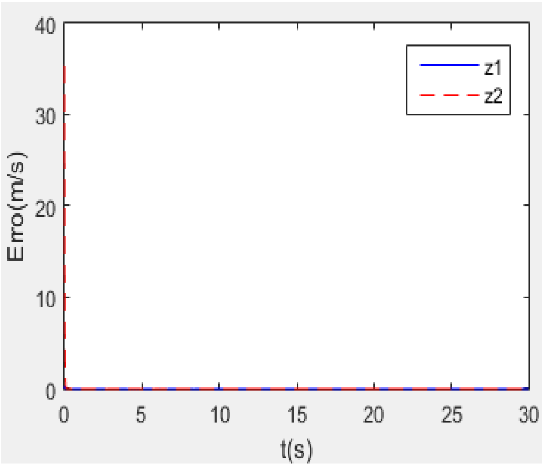

Figure 6. Convergence of robot velocity to controller velocity (z_1 – Error in linear velocity and z_2 –Error in angular velocity).

Now, changing the initial posture of the robot to q = [15 10 30], we have:

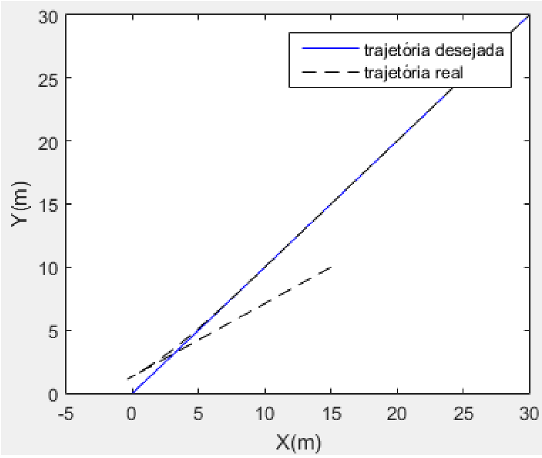

Figure 7. Linear trajectory for initial posture changed to [15 10 30 °].

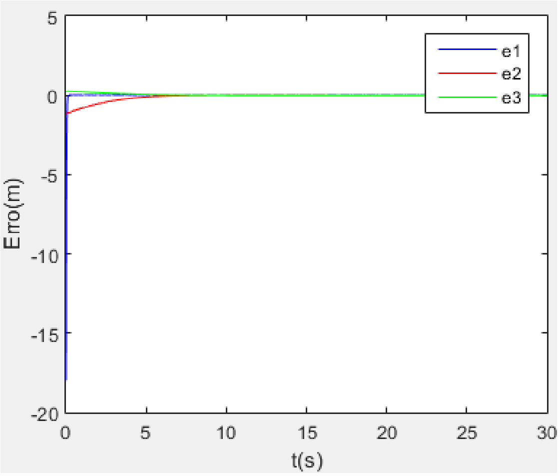

Figure 8. Tracking Error for initial posture changed to [15 10 30 °] (e_1 – Error in position x, e_2 – Error in position y -, e_3 – Error in orientation).

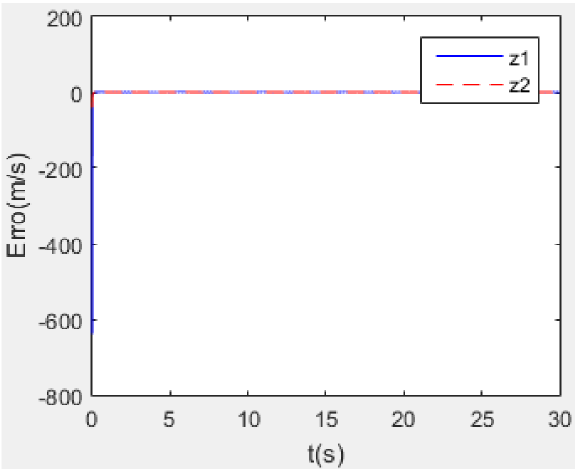

Figure 9. Convergence of robot velocity to controller velocity for initial posture changed to [15 10 30 °] (z_1 – Error in linear velocity and z_2 –Error in angular velocity).

6.2. Controller based on PID

Designed PID was applied under the same conditions presented in section 5.

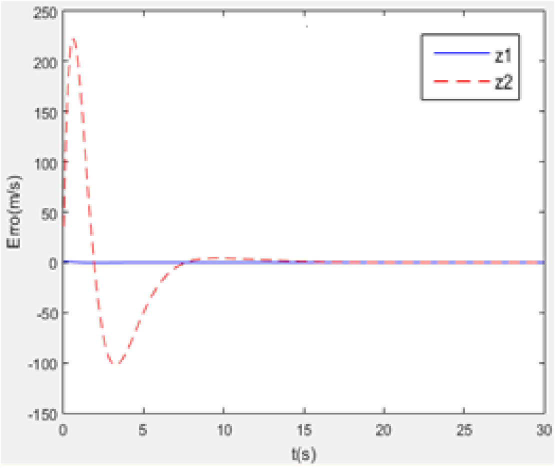

Figure 10. Error of the linear (z_1) and angular (z_2) velocities without considering the coupling.

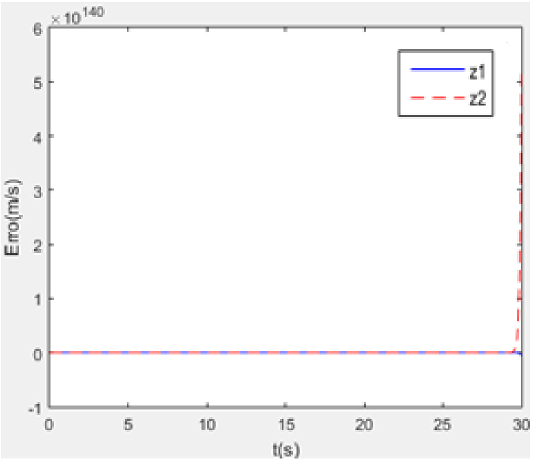

Figure 11. Errors of the linear (z_1) and angular (z_2) velocities considering the coupling.

Note that the designed controllers (Gc1 and Gc2) meet the meshes 1 and 2 when the coupling is not considered, ie G12 (s) and G21 (s) = 0, as can be seen in figure 4 a). However, applying these same controllers in the coupled system, the error presents an inappropriate behavior at the end of the trajectory, assuming very high values, as seen in figure 4 b).

The process of adjusting controller gains is performed until a satisfactory response is achieved. For gains $K_{P1} = 0.7596$; $K_{I1} = 1.165$; $K_{P2} = -2.613x10^{-5}$; $K_{I2} = -1.603x10^{-3}$, we have the results shown in figures 8 to 10.

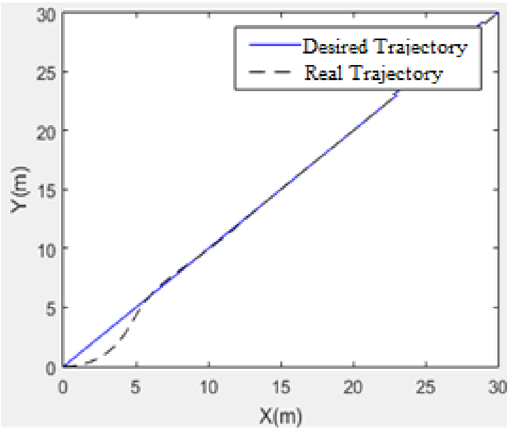

Figure 12. Rectilinear trajectory using PID.

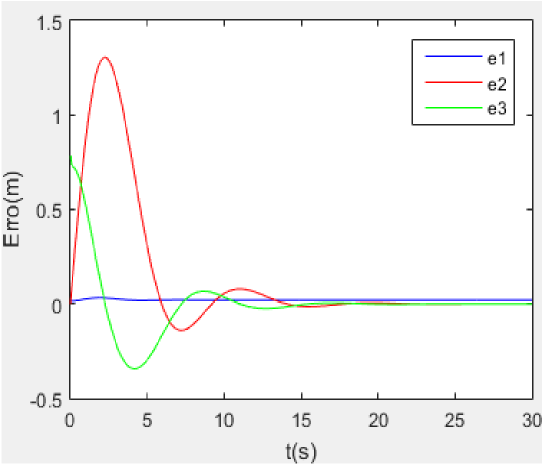

Figure 13. Tracking Error using PID (e_1 – Error in position x, e_2 – Error in position y -, e_3 – Error in orientation)

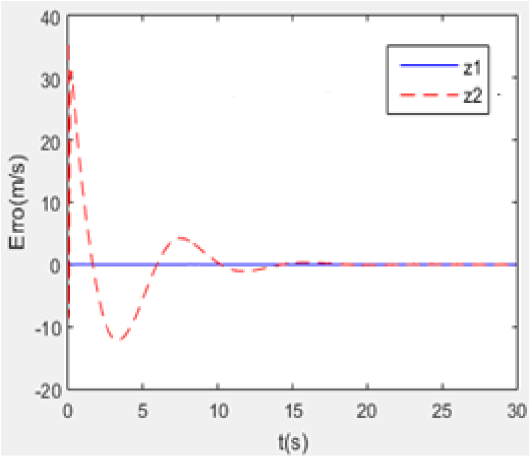

Figure 14. Convergence of robot velocity to controller velocity using PID.

7. CONCLUSIONS

The dynamic controller designed by both PID and Backstepping presented satisfactory results in the convergence of robot velocity to controller velocity, consequently followed the proposed trajectory. In spite of reaching the objective, the Backstepping technique generated a settling time close to zero, unlike the PID control that generated a settling time of 10 s. At the point of view of the maximum overshoot of the control input (torque), the Backstepping presented the value of 0.8 N.m and the PID the value of 0.3 N.m

The difficulties encountered were the tuning of the PID and the determination of the constants of the non-linear controller. In the case of PID tuning, to apply a monovariable control technique, which is usually simpler, it is possible to use uncouplers in the system or to design PID controllers to meet their respective loops, loop 1 and loop 2, and then check the influence on the other output.

Several simulations were performed in order to achieve stabilization of the system, and the values of the constants were found empirically. For the constants of the non-linear controller, one can use search algorithm, like the genetic algorithm, thus obtaining the exact values.

The study focused on the modeling and control of this system and it contributes to the modeling analysis of dynamics for robots with center of mass and center of rotation separated at a distance 'a', case little studied in mobile robots. In order to compare performance, another non-linear technique can be used and infer which is the most adequate and efficient to its needs.

8. REFERENCES

CHIH-YANG, C.; TZUU-HSENG, S. L; YING-CHIEH, Y.; CHA-CHENG, C. **Design and Implementation of an adaptive sliding-mode dynamic controller for wheeled mobile robots.** *Elsevier*, set/2012. <www.elsevier.com/locate/neucom>.

FIGUEREDO, L. C.; JOTA, F. G. **Introdução ao controle de sistemas não-holonômicos.** *Sba Controle & Automação*, Campinas, v.10, n.3, setembro/2014. Available in: < http://www.scielo.br/scielo.php?script=sci_arttext&pid=S0103-17592004000300002>. Access on: September 10, 2016.

MOHARERI, O.; DHAOUADI, R.; RAD, A. B. **Indirect adaptive tracking control of a nonholonomic mobile robot via neural networks.** *Elsevier*, mar/2012. Available in: <www.elsevier.com/locate/neucom>.

VIEIRA, F. C. **Controle Dinâmico de Robôs Móveis com Acionamento Diferencial.** Natal, 2005. Dissertação (Mestrado em Engenharia Elétrica) – Universidade Federal do Rio Grande do Norte, 2005.

ZIDANI, G.; DRID, S.; CHRIFI-ALAOUI, L.; BENMAKHLOUF, A. e CHAOUCH, S. **Backstepping Controller for a Wheeled Mobile Robot**. 4th International Conference on Systems and Control, 28-30 de abril, 2015 Sousse, Tunisia.

9. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.