24th ABCM International Congress of Mechanical Engineering
December 3-8, 2017, Curitiba, PR, Brazil

COBEM-2017-0289
OPTIMAL CONTROL APPLIED IN SLEWING CONTROL FOR FLEXIBLE STRUCTURE

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Abstract. *The study of slewing control of an active flexible structure has attracted the interest of researchers in the areas of engineering, aeronautics and aerospace. In this work was conducted mathematical modelling, numerical simulations and comparison between free dynamics, with linear control application and nonlinear control in the flexible structure system with active actuator. The linear control is designed using the control method of Linear Quadratic Regulator (LQR). The nonlinear control is designed using the control method of State Dependent Riccati Equations (SDRE). The control is proposed for two different applications; the first one is for the electric voltage of the motor and the second one is for vibration control and position of the structure (beam) applying an electric voltage in piezoelectric material. Numerical simulations demonstrated the effectiveness of the proposed control strategy, demonstrating that the linear and nonlinear controllers have the same effect on the active flexible structure system.*

Keywords: *LQR Control, SDRE Control, Active Flexible Structure System, Slewing Control, Piezoelectric Ceramic Materials.*

1. INTRODUCTION

Flexible structures are widely used in applications such as robotics, aeronautics and aerospace. The problems involving deformation and/or vibration in structural systems can be solved through the use of conventional control techniques. The control actuators can be DC motors and also intelligent materials that react to the passage of electric current inside, for example, the shape memory effect (SME) of metallic alloys and piezoelectric effect (PZT) of some ceramic materials. The dynamic of robotic manipulators is considered as nonlinear systems, and the controllers are usually nonlinear as well, which increases the complexity of the system and increases the final price of the project. Such study is very widespread in the control area with different researches on robotic manipulators, as presented by Garcia and Inman (1991) considering the slewing control of an active flexible structure by examining the governing equations of motion of an integrated actuator-structure system composed by a thin aluminum beam torque driven by an armature controlled electric motor and actuated by a piecewise distributed piezoceramic actuator. Juang *et al.* (1989) discussed several important issues related to slewing experiments with flexible structures including nonlinearities and calibration of actuators and sensors.

Nowadays, non-conventional actuators have become a highly attractive alternative to conventional actuators and the more promising are the ones based on the shape memory effect (SME) of metallic alloys and piezoelectric effect (PZT) of some ceramic materials (Tusset, et al., 2013).

Therefore, this work presents the study, modeling and simulations of the dynamics of robotic manipulators with an actuator motor. The proposal of control is through coating with piezoelectric ceramic in the structure considered as actuator of control in the dynamics of the beam, and the control of the electric voltage applied to the DC motor. The system control is designed in two different ways, through a linear state matrix and a nonlinear state matrix, through two control techniques that are Linear Quadratic Regulator (LQR) and State Dependent Riccati Equations (SDRE).

2. MATHEMATICAL MODELING

The studied model in the work consists of a DC motor (actuator) coupled to a beam of elastic material (structure) that rotates in relation to the actuator motor. The schematic drawing of the model is shown in Figure 1.

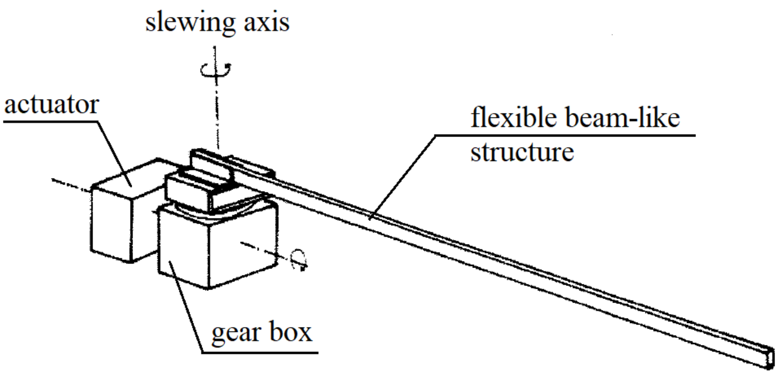

Figure 1. Schematic of a slewing flexible structure and actuator system (Fenili, 2000)

The governing equations of motion for the nonlinear and non-ideal system derived from the extended Hamilton principle is given by Eqs. (1) (Fenili and Balthazar, 2008).

$$\begin{aligned} \dot{\nu} &= -a_1\upsilon - a_2\dot{\theta} \\ \ddot{\theta} &= -b_1\dot{\theta} + b_2\upsilon + b_3 z \\ \ddot{z} &= -c_1\dot{z} - c_2 z - c_3\ddot{\theta} + \dot{\theta}^2 z - c_4\dot{\theta} \end{aligned} \tag{1}$$

Where the parameters of Eqs. (1) are denoted by:

$$a_1 = \frac{R}{L_m};\ a_2 = \frac{K_b\beta}{L_m};\ b_1 = \frac{c\beta^2}{I_e + \beta^2 I_m};\ b_2 = \frac{\beta K_t}{I_e + \beta^2 I_m};\ b_3 = \frac{EI\phi''(0)}{I_e + \beta^2 I_m};\ c_1 = \mu;\ c_2 = w^2 \text{ e } c_3 = \alpha$$

where z represents the amplitude of vibration of the beam, θ represent angular displacement, υ represent electric current in the motor armature, and the other parameters are defined in Tab. 2.

Equation (1) can be represented is state-space notation in Eq. (2), where: $x_1 = \upsilon$, $x_2 = \theta$, $x_3 = \dot{\theta}$, $x_4 =$ z and $x_5 = \dot{z}$.

$$\begin{aligned} \dot{x}_1 &= -a_1 x_1 - a_2 x_3 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= -b_1 x_3 + b_2 x_1 + b_3 x_4 \\ \dot{x}_4 &= x_5 \\ \dot{x}_5 &= -c_1 x_5 - c_2 x_4 + c_3 b_1 x_3 - c_3 b_2 x_1 - c_3 b_3 x_4 + x_3^2 x_4 \end{aligned} \tag{2}$$

2.1 Numerical simulations

The numerical simulations of Eq. (2) were carried out through the 4th order Runge-Kutta method considering the parameters of the system of Tab. (2).

Table 1. Parameters of the system

Parameter	Value	Means
L_m [H]	3.1×10^{-3}	Motor inductance
R [Ω]	1.914952	Armature resistance of the motor
K_b [v]	5.281×10^{-2}	Constant counter electromotive force of the motor
I_e [kg.m²]	6.540×10^{7}	Inertia associate with the motor shaft
I_m [kg.m²]	6.540×10^{-5}	Inertia of internal parts of the motor
c	4.629×10^{-3}	Inertial friction of motor
K_t [N.m]	5.281×10^{-2}	Motor torque constant
E [N/m²]	0.7×10^{11}	Young's modulus of the beam
I [kg.m²]	1.562×10^{-13}	Inertia of the cross section of the beam
L [m]	1.2	Length of the beam
μ [N.s/m]	0.1	Structural damping of the beam
w [rad/s]	11.3097	Natural frequency of the beam
α [rad/s]	0.821	Angula acceleration of the motor shaft
ϕ [rad]	4.8984	Vibrating beam forms
β	0.05	Ratio between the beam and motor
x_{10} [v]	0.2	Initial condition of x_1
x_{20} [rad]	0.002	Initial condition of x_2
x_{30} [rad/s]	0.1	Initial condition of x_3
x_{40} [m]	0.001	Initial condition of x_4
x_{50} [m/s]	0	Initial condition of x_5

where the dynamics of the system is presented in Figs. 2 (Tusset, et al.,2013).

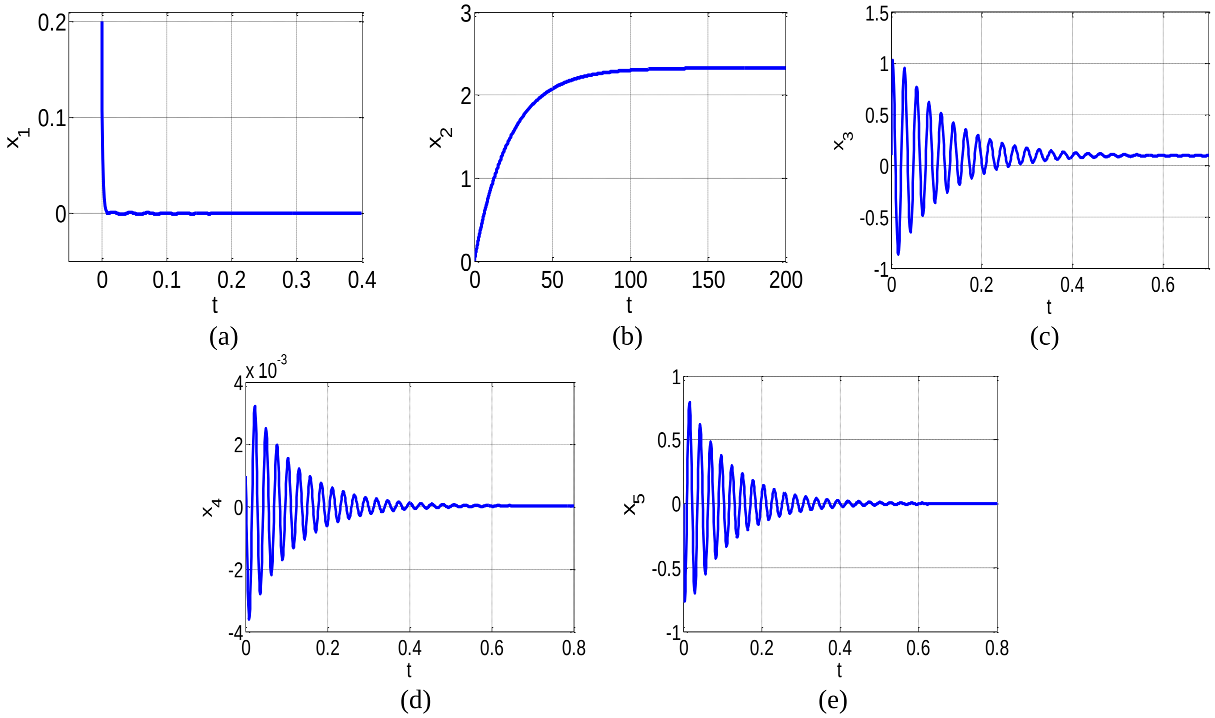

Figure 2. (a) Electric voltage in the motor. (b) Angular position displacement in the motor shaft. (c) Angular speed in the motor shaft. (d) Deflection of the beam. (e) Displacement speed of the beam (vibration).

3. PROPOSED CONTROL

The control proposal for this work was formulated from the concept of closed-loop control with state feedback. The project aims to apply the control using two different techniques to Eq. (2) system, one in a linear and other in nonlinear way, for comparison. The control is proposed for two different applications, the first one for the electric voltage of the motor and the second for control of vibration and position of the structure (beam) by applying an electric voltage in the piezoelectric material. The used control techniques are Linear Quadratic Regulator (LQR) and State Dependent Riccati Equations (SDRE).

The dynamic system defined by Eq. (2) with a control signal u can be parameterized in first order equations and written in the state dependent coefficient (SDC) form (Molter, *et al.*, 2010):

$$\dot{x} = A(x)x + Bu \tag{3}$$

A state feedback rather than output feedback is adopted to enhance the control performance. The non-quadratic cost function for the regulator problem is given by (Tusset, *et al.*, 2013):

$$J = \frac{1}{2}\int_{t_0}^{\infty} [x^T Q(x)x + u^T R(x)u]dt \tag{4}$$

where $Q(x)$ is a semi-positive-definite matrix and $R(x)$ a positive definite. There are weighting matrices on the outputs and control inputs, respectively. For a pointwise linear fashion, the matrices are assumed as constant control coefficients. Assuming full state feedback, the control law is given by (Nozaki, *et al.*, 2013):

$$u = -R^{-1}(x)B^T(x)P(x)x \tag{5}$$

The state dependent Riccati equation to obtain $P(x)$ is given by (Tusset, et al.,2013):

$$A^T(x)P(x) + P(x)A(x) - P(x)B(x)R^{-1}(x)B^T(x)P(x) + Q(x) = 0 \tag{6}$$

Equation (3) is controllable if the rank of the matrix M is n, as follows (Tusset, et al.,2013):

$$M = [B \;\; A_{nxn}B \;\; \dots \;\; A_{nxn}^{n}B] \tag{7}$$

The SDRE technique to obtain a suboptimal solution for dynamic control problem has the following procedure (Tusset, *et al.*, 2013):

Step 1. Define the state-space model with the state dependent coefficient form as in Eq.(3).

Step 2. Define $x(0) = x_0$, so that the rank of M is n and choose the coefficients of weight matrices Q and R.

Step 3. Solve the Riccati Eq. (6) for the state $x(t)$.

Step 4. Calculate the input signal from Eq. (5).

Step 5. Integer the Eq. (3) and update the state of the system $x(t)$ with this results.

Step 6. Calculate the rank of Eq. (7), if $rank = n$ go to step 4. If $rank < n$ using the last matrix A controllable obtained, and go step 4.

3.1 Position control of the motor shaft and structure vibration control with piezoelectric material

Considering the cases where the electric voltage of the motor can interfere in the angular position of the motor axis and consequently in the final position and speed of the beam, and in cases where a greater rigor is required in relation to the positioning and vibration of the structure, two control signals were developed. A study was carried out regarding intelligent materials and it was chosen to use piezoelectric materials to coat the structure.

The mathematical modeling for the piezoelectric was obtained through Shawry, *et al.* (2007) and Triplett and Quinn (2009), where the term given by $(d(x)/c)q$ represents the piezoelectric coupling for the mechanical component, where q represents the electric charge developed through the coupled circuit and $d(x)$ is a voltage-dependent coupling coefficient (mechanical stress) applied. Therefore, the electric voltage $\mathrm{V_p}$ passing through the piezoelectric material can be represented by:

$$\mathrm{V_p} = -\mathrm{d(x)x/C} + \mathrm{q/C} \tag{8}$$

Where C is the piezoelectric capacitance and x is the vertical displacement of the system. Hence, considering that:

$$\mathrm{V_p} = -\mathrm{R_p}\dot{\mathrm{q}} \tag{9}$$

Where R_p is the electrical resistance and $\dot{q}$ is the electric current, and substituting Eq. (9) into Eq. (8), it has:

$$R_p\dot{q} - d(x)x/C + q/C = 0 \tag{10}$$

In this work, it was used the dimensionless piezoelectric coupling coefficient, being approximated by (Triplett and Quinn, 2009), as:

$$d(x) = \theta_p(1 + \Theta|x|) \tag{11}$$

Where Θ is the nonlinear part of the piezoelectric coupling and θ_p represents the linear segment thereof. Thus Eq. (10) becomes:

$$\rho v' = \theta_p(1 + \Theta|x|)x - v \tag{12}$$

Where $\rho = RC\omega_0$ and ω_0 natural frequency of the system. From Eq. (3), coupling Eq. (12) to the system, it has Eq. (13) where it is found the two control designs, both for the motor and for the beam.

$$\begin{aligned}
\dot{x}_1 &= -a_1x_1 - a_2x_3 + d_1u_1 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= -b_1x_3 + b_2x_1 + b_3x_4 \\
\dot{x}_4 &= x_5 \\
\dot{x}_5 &= -c_1x_5 - c_2x_4 + c_3b_1x_3 - c_3b_2x_1 - c_3b_3x_4 + x_3^2x_4 + \left(d_2 + d_3|x_5|\right)u_2
\end{aligned} \tag{13}$$

The control signal u_1 represents the voltage applied to the motor terminals, the control signal u_2 represents the voltage applied to the piezoelectric material and $d_1 = \frac{1}{L_m}$, $d_2 = \frac{\theta_p}{\rho}$, $d_3 = \frac{\theta_p\Theta}{\rho}$ and $d_4 = (d_2 + d_3|x_5|)u_2$, where values are assumed as $d_2 = 2$ and $d_3 = 1.4$.

3.2 State Dependent Riccati Equations (SDRE)

As the first control technique, the State Dependent Riccati Equations (SDRE) was used, with the nonlinear matrix A as can be observed in Equation (14).

$$A(x) = \begin{bmatrix} -a_1 - x_2 & x_1 & -a_2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ b_2 & 0 & -b_1 & b_3 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -c_3b_2 & & c_3b_1 & -c_2 - c_3b_3 + x_3^2 & -c_1 \end{bmatrix}, \; B = \begin{bmatrix} \frac{1}{L_m} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & d_4 \end{bmatrix} \tag{14}$$

Defining:

$$x^* = \begin{bmatrix} 0 & 0 \\ 2 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \; Q = 10^4 \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 10^3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \text{ e } R = \begin{bmatrix} 10^{-4} & 0 \\ 0 & 10^{-4} \end{bmatrix} \tag{15}$$

Where x^* represents the desired states, Q and R are matrices of Eq. (4) used in calculating the Riccati matrix Eq. (6). The behavior of the system of Eq. (13) with the controls u_1 and u_2 concerning Eq. (14) and Eq. (15) can be seen in Figure 3.

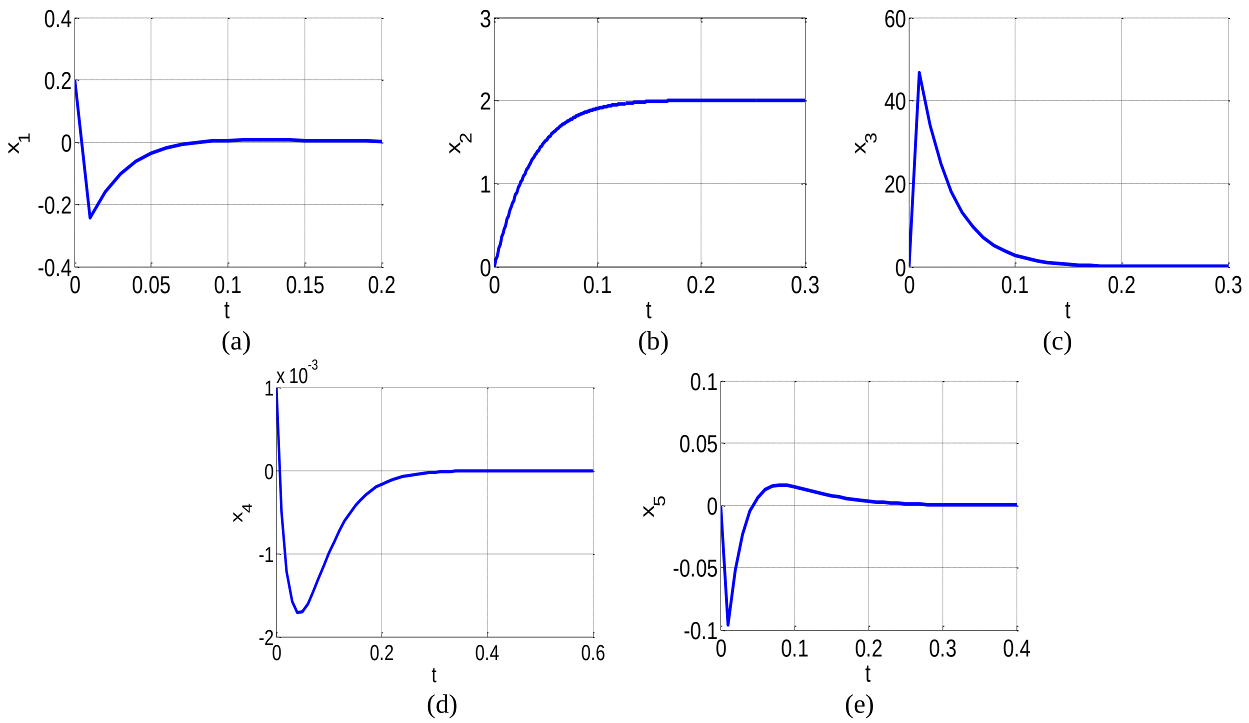

Figure 3. (a) Electric voltage in the motor. (b) Angular position displacement in the motor shaft. (c) Angular speed in the motor shaft. (d) Deflection of the beam. (e) Displacement speed of the beam (vibration).

3.3 Linear Quadratic Regulator (LQR)

As a second control technique, the Linear Quadratic Regulator (LQR) was applied, where the Jacobian matrix was calculated to linearize the matrix A of Eq. (14) as can be observed in Equation (16).

$$A=\begin{bmatrix} -a_1 & 0 & -a_2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ b_2 & 0 & -b_1 & b_3 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -c_3b_2 & 0 & c_3b_1 & -c_2-c_3b_3 & -c_1 \end{bmatrix},\ B=\begin{bmatrix} \frac{1}{L_m} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & d_4 \end{bmatrix} \tag{16}$$

Defining:

$$x^*=\begin{bmatrix} 0 & 0 \\ 2 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},\ Q=10^4\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 10^3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \text{ e } R=\begin{bmatrix} 10^{-4} & 0 \\ 0 & 10^{-4} \end{bmatrix} \tag{17}$$

Where x^* represents the desired states, Q and R are matrices of Eq. (4) used in calculating the Riccati matrix Eq. (6). The behavior of the system of Eq. (13) with the controls u_1 and u_2 concerning Eq. (16) and Eq. (17) can be seen in Figure 4.

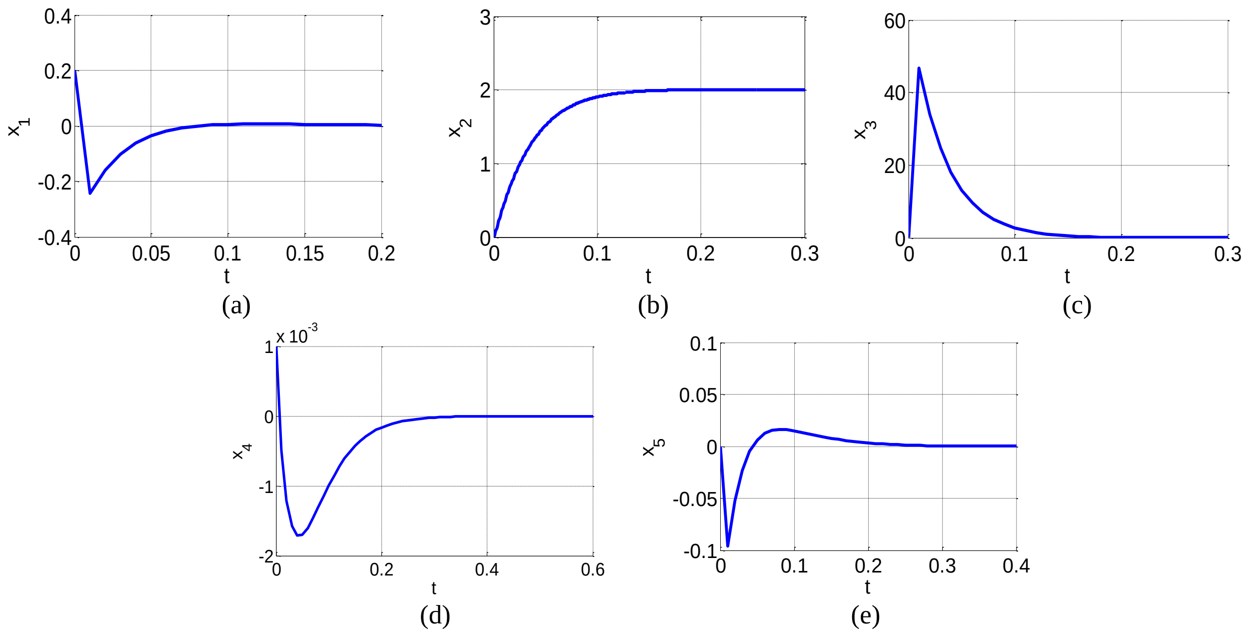

Figure 4. (a) Electric voltage in the motor. (b) Angular position displacement in the motor shaft. (c) Angular speed in the motor shaft. (d) Deflection of the beam. (e) Displacement speed of the beam (vibration).

4. RESULTS AND DISCUSSION

According to the results presented in the numerical simulations, it can be observed that after the application of the controls in the motor and the structure, there is an evident reduction of the considered aspects. In order to compare the proposed control techniques, it was considered the superimposed control charts as shown in Fig. 5. Figures have subtitles for the systems with influence of the control (With C.) and without influence of the control (Without C.).

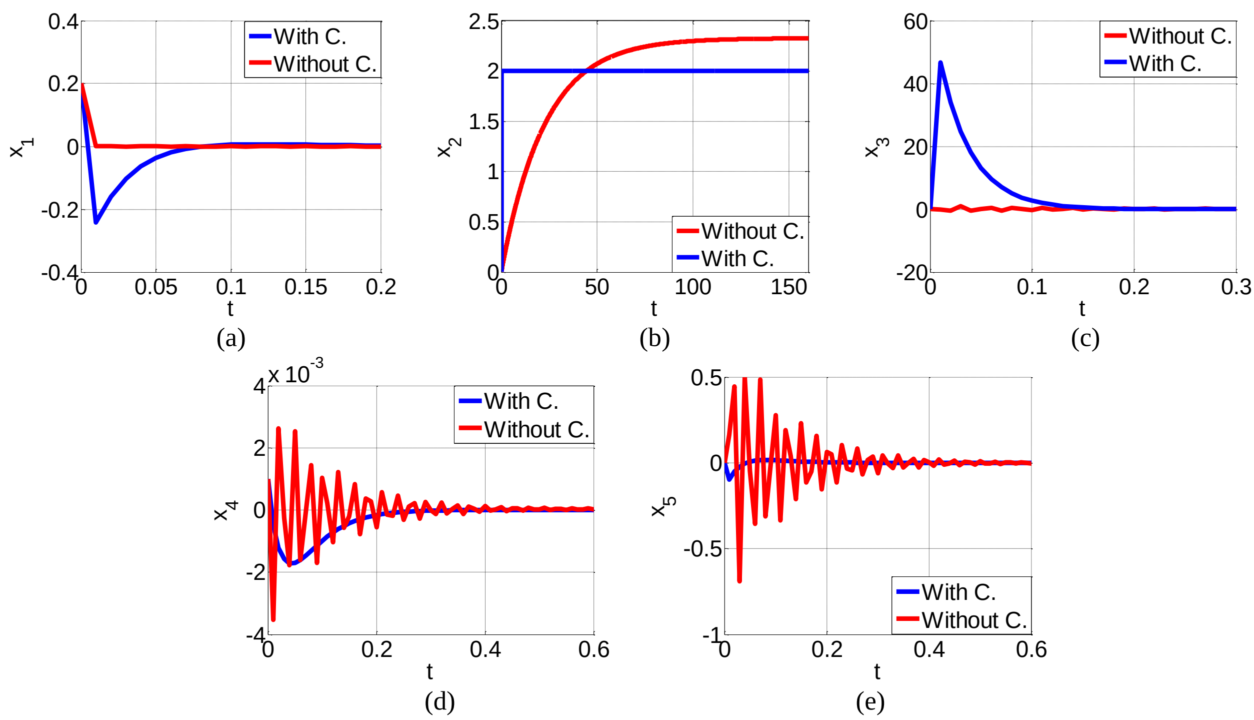

Figure 5. (a) Electric voltage in the motor. (b) Angular position displacement in the motor shaft. (c) Angular speed in the motor shaft. (d) Deflection of the beam. (e) Displacement speed of the beam (vibration).

The two techniques of control overlap, LQR and SDRE, have the same behaviour for the studied system, with the parameters stipulated in the work. There was a reduction of the considered aspects comparing the system without and with control, where the two proposed control are efficient for the system. The dynamic response of the system improved significantly with the control designed in the work, as shown in Fig. 5d and 5e, where the control is efficient to reduce the vibration and displacement of the structure and for the motor the control proved to be quite effective, as observed in Fig. 5a, 5b and 5c.

5. CONCLUSIONS

From mathematical modelling and numerical simulations presented in this work, through the nonlinear dynamics of robotic manipulators, it is possible to design a linear control using the technique Linear Quadratic Regulator (LQR) and a nonlinear control using the State Dependent Riccati Equations (SDRE) technique to operate in the system with the same efficiency. As shown, linear control (LQR) has the same behaviour as nonlinear control (SDRE) and this implies possible cost reductions and energy reductions employed to control the system.

For future work, it can be designed a position of fixation of the piezoelectric in relation to the points of greater vibrations and displacements of the beam to decrease the amount of material to be used and carry out the experiments in laboratory to verify the efficiency of the controllers, in practice.

6. ACKNOWLEDGEMENTS

The authors acknowledge support by CNPq (GRANT:306525/2015-1) and (GRANT:447539/2014-0), CAPES and FAPESP (GRANT 2015/20363-6) both Brazilian research funding agencies.

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