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TIME-VARYING NONLINEAR DYNAMICAL SYSTEM ANALOG COMPUTING CO-SIMULATION

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Abstract. *Usually, analog computation is applied to solve linear time-invariant dynamical systems, due to the easy design of analog circuits to perform integrations, sums and gains. However, nonlinear dynamical systems demand much more complicated mathematical operations. This work presents a technique of co-simulation for a time-varying nonlinear dynamical system based on integrating analog computation with data acquisition system and control software in order to modify the dynamical system gains. This technique was applied to simulate a bench with three connected tanks, where the main objective was to control the level central tank. There are two pumps to supply water to the left and right tanks and this last has a return to reservoir. The pump flows are the control variables. The right tank and the flow from it to the reservoir are calculated by an analog computation simulator. The controller was developed using the LQG/LTR technique and the algorithm is responsible to communicate with the plant and the simulator, and to run the control loop.*

Keywords: *nonlinear dynamic systems, analog computation, simulation, co-simulation*

1. INTRODUCTION

Analog computation has been widely used in order to solve linear time-invariant dynamical systems, since analog circuits can be easily designed to perform integrations, sums and gains. However, for nonlinear dynamical systems much more complicated mathematical operations are needed. This work presents a technique of co-simulation for a time-varying nonlinear dynamical system based on associating analog computation with data acquisition system and control software in order to modify the dynamical system gains.

All the application tools are integrated by OPC[1,2] communication, it includes the plant simulator, the analog computer and the control algorithm. The plant is designed in LabVIEW™ application with the same dynamical nonlinear equations proposed by the experimental setup manufacturer.

The analog computer is based on the electronic behavior of operational amplifiers configurations, performing operations such as integrations, sums, gains, signal inversions, being able to compute the linear part of the plant (Malvino, 1997; Ogata, 1982). .

In order to be implemented by an analog computer the time-varying nonlinear systems must be continuous. The time constants are defined by the capacitors and resistors associations in the integration configuration, in this work they are selected in order to obtain a time constant equal to one second. However the association can be adjusted, by replacing the components, in order to scale the time constant, speeding up or slowing down the computations. In addition, time-varying behaviors are included by analog multipliers, with externally computed variant gain, in the feedback loop of the operational amplifiers

1 OLE for Process Control (OPC).
2 Object Linking and Embedding (OLE).

Nonlinear behaviors, such as saturation and dead-zone, can be implemented directly in the electronic circuit. For more complex nonlinear functions, such as trigonometric and logarithmic functions, the implementation circuitry is also much more complex. The advantages of using analog computation instead of digital is the absent of Analog to Digital (AD) and Digital to analog (DA) conversions effects. In addition, sometimes, the digital simulations and calculations take too much time to get completed, and the numerical simulations become expensive. In analog computing it is possible to run the real time scale or faster, including time varying effects, without any concern about algorithm numerical stability. In reality, the analog circuit performing a mathematical model is itself an experimental setup.

This electronic circuit was applied to simulate a hydraulic experimental setup to check its similarity with the real system. The experimental setup (AMIRA, 2002) consists of a set of three interconnected tanks, a reservoir and two pumps to supply water for the tank on the left, and to the tank positioned to the right. The objective is to control the water level of the central tank. The three tanks are connected serially, and the one on the right is connected to the reservoir by a flow controlled valve. The inlet flow from the pumps are controlled and leaded to the left side and to the right side tank. The plant can be seen on Fig. 1. This experimental setup provides a nonlinear time-invariant model, however some adaptations can be introduced to reproduce time-varying behavior, such as the varying flow in a valve, adding extra water or producing air bubbles into the tanks.

The flows between the left side tank, the central tank and the right side tank, are changed manually as a way to introduce time varying dynamics to the plant. All the tanks are open, so there is not pressure actuating to the fluid. The variable to be controlled is the water level on the central tank, and the variable of actuation can be the flow to the right or to the left tanks, or both. As an example, it can be actuated directly with the same flow for the two valves.

The three tank system can represent an automatic device to supply water into the pressure vessel of a boiler, with a control technique based in three elements, whose objective is to keep the level within a secure level, since low water level cause damages to the boiler by overheating the metal barrier, whereas a high level can create an event of dragging water to the steam, damaging the turbine blades or reducing the machine efficiency.

The plant control system is implemented in mathematical application Scilab™, where the LQG/LTR controller performs the gain matrix calculation and run a loop for communication and feedback.

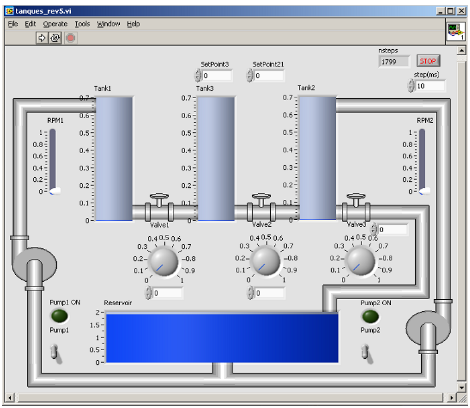

Figure 1. The three tank system plant LabVIEW™ Interface.

2. MATHEMATICAL MODEL OF THE THREE TANK SYSTEM

The analog computer is mounted based on three configurations of operational amplifiers. One is the integrator that receives the inlet/outlet flow of the tank and its output is the tank level. The sum configuration receives the flows from the central tank, from the pump and to the reservoir. The last one is a signal inverter. There is two multipliers, one working on the valve opening to the reservoir, and the other performing a of root square. The circuit representation is shown in Fig. 2, and the simulator design, in Fig. 3.

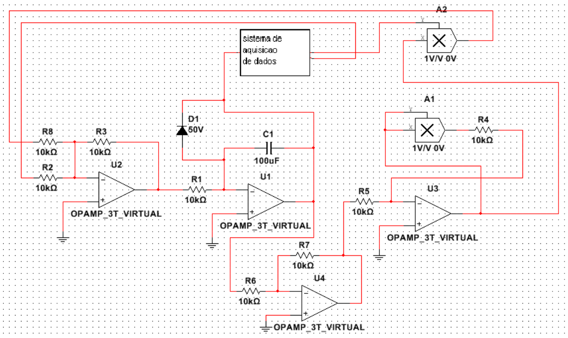

Figure 2. Analog computer

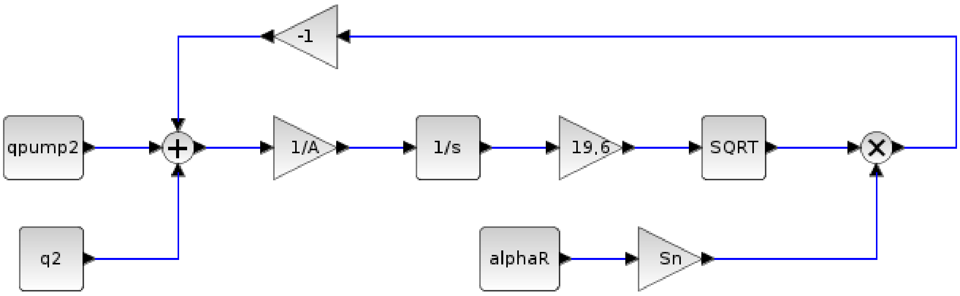

Figure 3. Analog computer simulator

The plant simulation runs in LabVIEW™ application. Basically there are two equations. The first is the flow calculation in function of the level difference between the tanks, and the second is the integrator for the water level calculation in each tank.

$$q_i = \alpha_i . S_n . sign(\Delta h_i) . \sqrt{2 . g . |\Delta h_i|} \tag{1}$$

$$A . \frac{dh_i}{dt} = \sum q_i \tag{2}$$

Where α_i is the opening percentage of the valve i, S_n is the total area of the valve, Δh_i is the water level difference between tanks, g is gravity acceleration, A is the tank area, $\sum q_i$ is the sum of the flow, q_i with i = 1,2 or 3, for valves between tanks 1 and 3, between tanks 3 and 2, and the valve to return to reservoir, and h_i with i = 1, 2 or 3, for the left, right and central tank levels. For the linearization, the following approximation is considered for the operation's points:

$$\sqrt{\Delta h} \approx \frac{10 \cdot \Delta h}{\sqrt{19.6}} \tag{3}$$

The linearization is made to project the LQG/LTR controller, and the following equations are found:

$$A . \frac{dh_1}{dt} = \alpha_1 \cdot Sn \cdot \sqrt{19.6} \cdot \frac{10 \cdot (h_3 - h_1)}{\sqrt{19.6}} + u_1 \cdot 10^{-4} \tag{4}$$

$$A . \frac{dh_2}{dt} = \alpha_2 \cdot Sn \cdot \sqrt{19.6} \cdot \frac{10 \cdot (h_3 - h_2)}{\sqrt{19.6}} + \alpha_3 \cdot Sn \cdot \sqrt{19.6} \cdot \frac{10 \cdot h_2}{\sqrt{19.6}} + u_2 \cdot 10^{-4} \tag{5}$$

$$A . \frac{dh_3}{dt} = \alpha_1 \cdot Sn \cdot \sqrt{19.6} \cdot \frac{10 \cdot (h_1 - h_3)}{\sqrt{19.6}} + \alpha_2 \cdot Sn \cdot \sqrt{19.6} \cdot \frac{10 \cdot (h_2 - h_3)}{\sqrt{19.6}} \tag{6}$$

After some mathematical reasoning, the three tank is system is described by the following matrix equations:

$$\dot{h}=\begin{bmatrix}-0,0325 & 0 & 0,0325\\ 0 & -0,0649 & 0,0325\\ 0,0325 & 0,0325 & -0,0649\end{bmatrix}.h+\begin{bmatrix}0,0065 & 0\\ 0 & 0,0065\\ 0 & 0\end{bmatrix}.u \quad , \tag{7}$$

$$y=\begin{bmatrix}0 & 0 & 1\\ 1 & -1 & 0\end{bmatrix}.x \quad . \tag{8}$$

3. THE LQG/LTR CONTROLLER DESIGN

The controller is designed based on the procedure shown in (Cruz, 1996) and (Sinha, 2007). First step is to add integrators to the system and find the L matrix to match the singular values to all frequencies.

$$L_L=-\left(C\cdot A^{-1}\cdot B\right)^{-1} \tag{9}$$

$$L_H=-\left(A^{-1}\cdot B\cdot L_L\right) \tag{10}$$

$$L=\begin{bmatrix}L_L\\ L_H\end{bmatrix} \tag{11}$$

The next step is to calculate the LQG/LTR compensator K(s) with this equation:

$$K(s)=-G(sI-A+BG+HC)^{-1}H \tag{12}$$

given that the state regulator gain matrix G is given by:

$$G=\rho^{-1}B^TP \quad , \tag{13}$$

where P is the solution of the following algebraic Riccati equation:

$$PA+A^TP-\rho^{-1}PBB^TP+C^TC=0\,. \tag{14}$$

The state observer gain matrix H is given by:

$$H=\Sigma C^T\Theta^{-1} \tag{15}$$

where $\sum$ is the solution of the filter algebraic Riccati equation:

$$A\Sigma+\Sigma A^T+LL^T-\Sigma C^T\Theta^{-1}C\Sigma=0 \quad . \tag{16}$$

4. RESULTS

The system was simulated with the parameters: 50% and 70% opened for the valves between tanks 1 to 3, and 3 to 2, respectively (Fig. 4). After that, these percentages have been changed, that is, 70% and 50%, respectively (Fig. 5). This action makes the system to have time variant behavior.

The set point was 0.4 m to the central tank, and 0.2 m to the difference between the first and the last tank. The change dynamics occurred after 700s after the data acquisition starts. As it can be seen the controller is able to regulate the system in both situations.

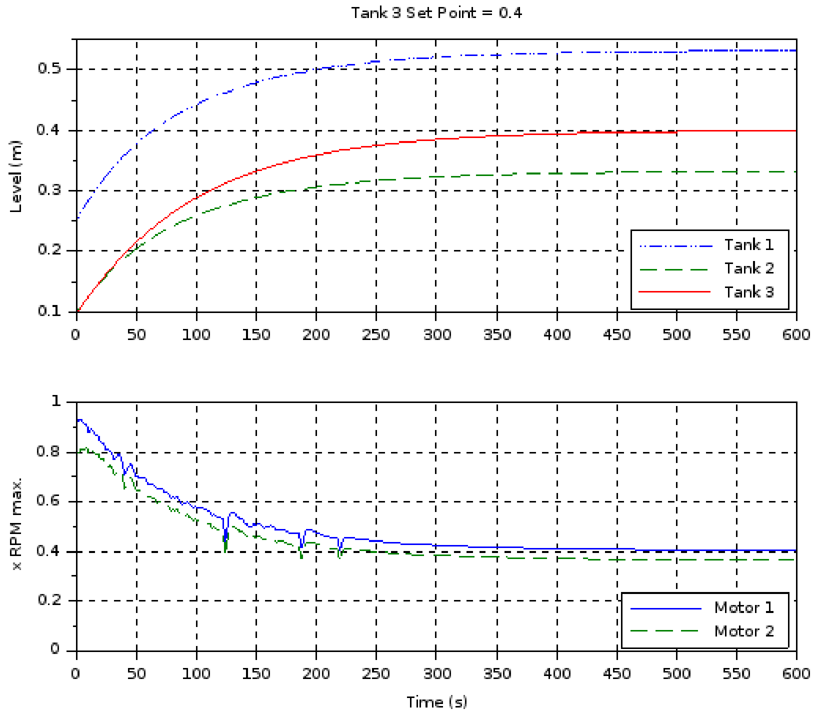

Figure 4. Simulation before the change in dynamics

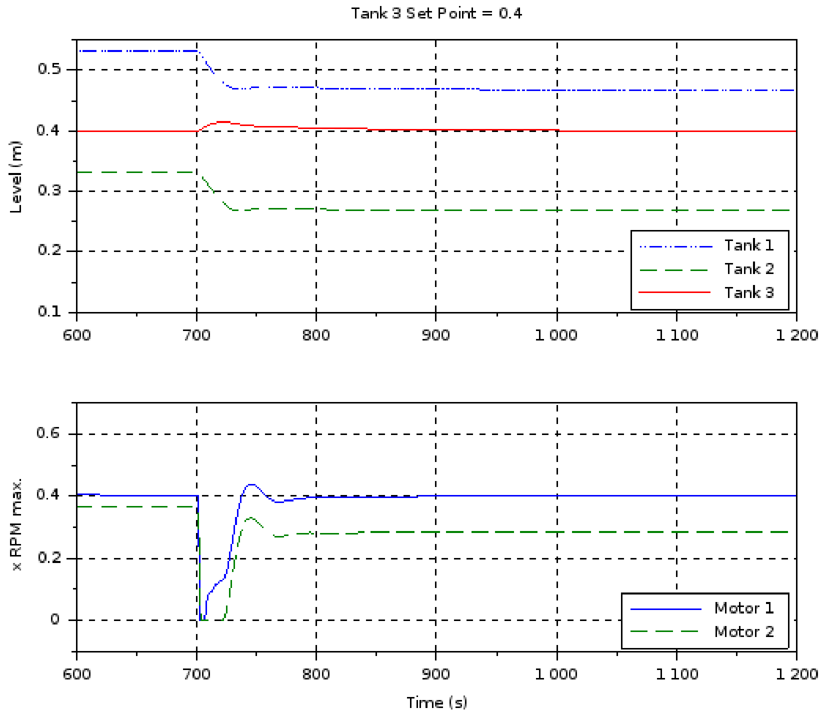

Figure 5. Simulation after the change in dynamics

An additional situation was performed, changing the set point relative to the difference between the left side and the right side tanks from 0.2 m to 0.1 m. This situation shows the behavior of the pumps attempting to get to the new set point (Fig. 6). This change occurred 700 s after the data acquisition starts. As it can be noticed the controller was effective in regulating tanks levels.

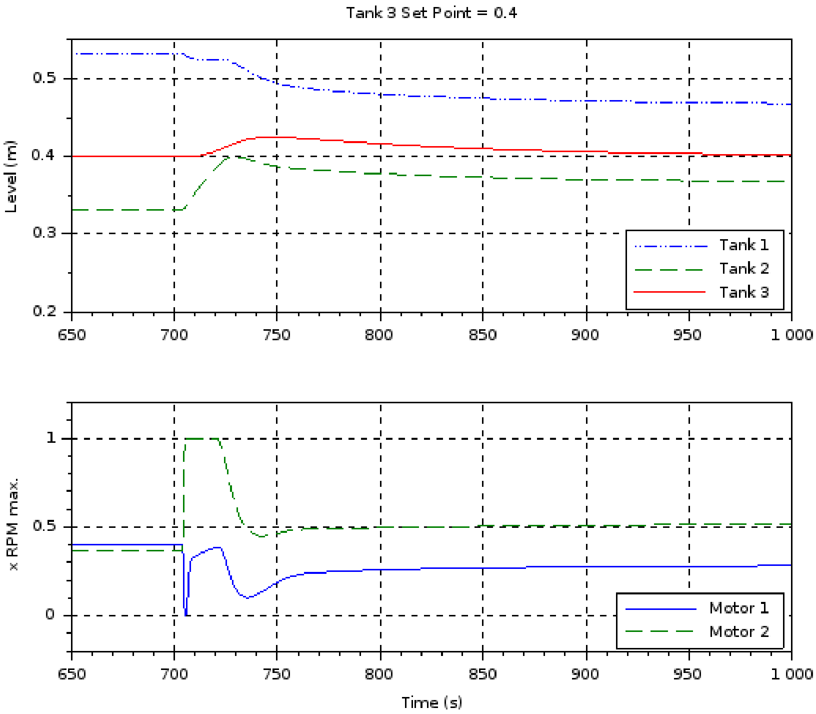

Figure 6. Simulation after the change in set point

5. CONCLUSION

The LQG/LTR controller had success to regulate the initial configuration and after the time varying changes in the system and new set points. With the real electronic circuit, the expected result is a better approximation of the analog computation to the real system with regard to the digital simulation. Another result is to verify the interference of the gain and parameter updates in the conventional electronic circuit of the analog computer. The stability and convergence of the circuit will be evaluated. In last stage, some controllers will be applied to the simulated system and the response will be compared with that one from the real experimental setup.

This work presented a technique to simulate dynamical systems combining analog computation, time-varying model gains, and parameters updates during the simulation from a digital algorithm. The model from the real bench and its linearization, is also shown, as a way to design the LQG/LTR controller. Currently, an electronic circuit simulator was used to make this function and communicate with the plant. All the data and control informations are interchanged between the modules by industrial protocol OPC.

Additionally, an electronic circuit, such as an analog computer, can be used in order to simulate the plant dynamics, including nonlinear behaviors, without adding the discretization effects, and consequently numerical computing stability problems.

6. REFERENCES

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7. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.