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EVALUATION OF A MULTILEVEL ALGORITHM TO STUDY THE HYDRODYNAMIC BEHAVIOR OF JOURNAL BEARINGS

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Abstract. *This work aims to evaluate the Full Multigrid (FMG) algorithm performance when solving a journal bearing lubrication problem under static load condition. Journal bearing's hydrodynamic behaviour is described through the classical Reynolds equation, which is evaluated with the Finite Volume Method (FVM). The discrete equations are then solved with FMG. Approximated error norm's convergence behaviour is observed while varying the number of levels, number of pre and post-relaxations and type of correction cycle, which can be type V or type W. Results show that a W-cycle doesn't diminishes significantly the error. Also, a greater number of pre and post-relaxations provides a faster convergence of the approximated error norm. Furthermore, computational time spent for the FMG algorithm to solve a problem seems to be largely defined for coarsest grid's number of volumes, which is directly related to the number of levels used in FMG.*

Keywords: *journal bearing, hydrodynamic lubrication, multilevel method, Full Multigrid algorithm*

1. INTRODUCTION

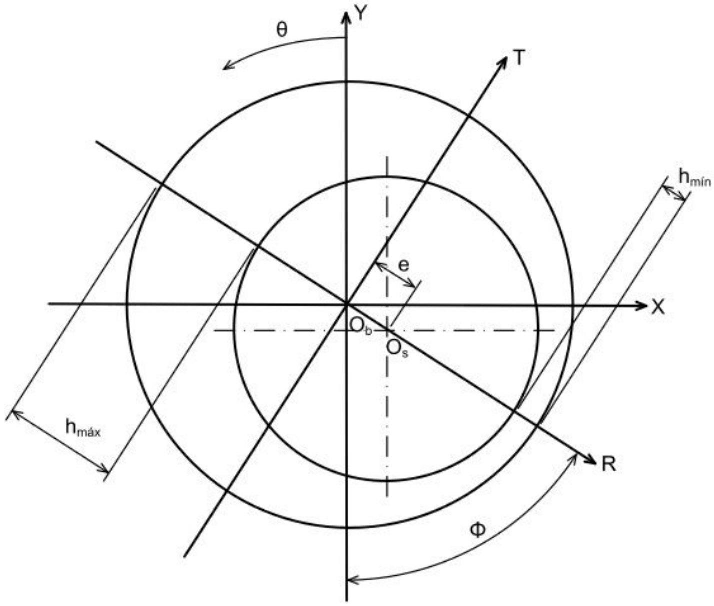
Since analytical solutions for differential equations can be obtained only under the assumption of specific hypothesis, numerical methods are usually applied in order to find numerical solutions to those equations for the cases closer to engineering's reality (Frêne et al., 1990). Therefore, physics of the studied phenomena is enabled to be described in a more realistic way and numerical discretization methods as the FVM, for instance, show themselves as powerful mathematical tools. As a result, the problem resumes into solving a system of linear equations.

Specifically for a hydrodynamic lubrication problem, numerical discretization method's computational mesh is usually not too refined and the resultant system of linear equations can be solved with, for instance, Gauss-Seidel method. However, in some applications such as in journal bearing's surface texturing, computational grid is highly refined and iterative methods are not capable to solve it without a large and sometimes even impracticable computational cost (Gropper et al., 2016). In this case, is necessary to apply a more efficient solver for the problem's discrete formulation. Multilevel method is an adequate mathematical tool in such circumstances, reducing considerably the computational cost and even improving discretization errors, but at a price of a laboring coding task.

The first part of this paper presents a description of the lubrication phenomena occurring on a journal bearing, through the Reynolds equation. Next are discussed the discrete formulation of the problem obtained through the Finite Volume Method (FVM) and the multilevel technique used to solve it. Then the main part of the study is exposed which consists of the convergence analysis of a FMG algorithm through the approximated error norm adoption as criteria.

2. METHODOLOGY

Figure 1 shows a schematic representation of a journal bearing, in which e is the shaft's eccentricity and ϕ is the attitude's angle. Thus, the pair (e,ϕ) indicates shaft's position relative to the bearing in polar coordinates.



(a)

(b)

Figure 1. Scheme of a journal bearing

2.1 Classical Reynolds equation and its discretization

Pressure distribution in journal bearing's lubricant film can be evaluated from classical Reynolds equation, under static load, as follows:

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(h^3\frac{\partial p}{\partial z}\right)=6\mu\omega R\frac{\partial h}{\partial x} \quad (1)$$

where p is the pressure on the lubricant film, μ is the lubricant's dynamic viscosity, ω is the shaft's rotational speed and R is the shaft's radius. The lubricant film thickness h for a smooth journal bearing can be defined as:

$$h(\theta)=C_r+e_x\sin(\theta)-e_y\cos(\theta) \quad (2)$$

being C_r the journal bearing's radial clearance, e_x and e_y the eccentricities in X and Y directions, respectively, and θ the angular coordinate established on inertial coordinate system XY from Fig. 1(a). In this study, Reynolds cavitation condition was adopted for Eq. (1).

Then, the FVM is applied to solve numerically Eq. (1). The partial derivatives that describe the pressure gradient in the boundaries of the volumes are approximated by the central differences scheme. Thus, the integration of the Reynolds equation leads the linear equation presented as:

$$C_P p_P = C_N p_N + C_S p_S + C_E p_E + C_W p_W + B \quad (3)$$

with

$$C_{E,W} = \frac{\Delta z}{\Delta x} h_{e,w}^3 \tag{4}$$

$$C_{N,S} = \frac{\Delta x}{\Delta z} h_{n,s}^3 \tag{5}$$

$$C_P = C_N + C_S + C_E + C_W \tag{6}$$

$$B = 6\mu\omega R \Delta z (h_w - h_e) \tag{7}$$

where C are pressure's coefficients, B is the source term, Δx and Δz are, respectively, volume's size in x and z directions as shown in Fig. 1(b). Sub index in Eqs. (3) - (7) means the cardinal orientations used in the computational mesh.

2.2 Multilevel method applied to solve the resultant linear system of equations

Such system of linear equations can be solved through multilevel scheme. First step is rewrite Eq. (3) as follows:

$$L^h u^h = f^h \tag{8}$$

being L^h an operator comprehending pressure coefficients C and source term B, u^h a vector with pressure nodal values and f^h a vector initially null. The FVM's mesh is indicated by h. An immediately FVM coarser grid H is related to that one by $H = 2h$ in each direction, i.e. it has a number of volumes four times smaller in two-dimensional case. Considering the Reynolds cavitation condition imposed to Eq. (1), L^h becomes non-linear and Eq. (8) can be rewritten as:

$$L^h \left\langle v^h + \tilde{u}^h \right\rangle = L^h \left\langle \tilde{u}^h \right\rangle + r^h \tag{9}$$

where $\tilde{u}^h$ is an approximation to u^h, $v^h = u^h - \tilde{u}^h$ is an error in this approximation and $r^h = f^h - L^h \tilde{u}^h$ is the residual. Notation $\langle \cdot \rangle$ is purposed to indicate L^h as non-linear. Approximated solution $\tilde{u}^h$ is obtained performing v_1 pre-relaxations with Gauss-Seidel method over the finer grid h. Then, Eq. (9) is solved on coarser grid as follows:

$$L^H \left\langle \hat{u}^H \right\rangle = \hat{f}^H \tag{10}$$

with,

$$\hat{u}^H = I_h^H (v^h + \tilde{u}^h) \tag{11}$$

$$\hat{f}^H = L^H \left\langle I_h^H \tilde{u}^h \right\rangle + I_h^H r^h \tag{12}$$

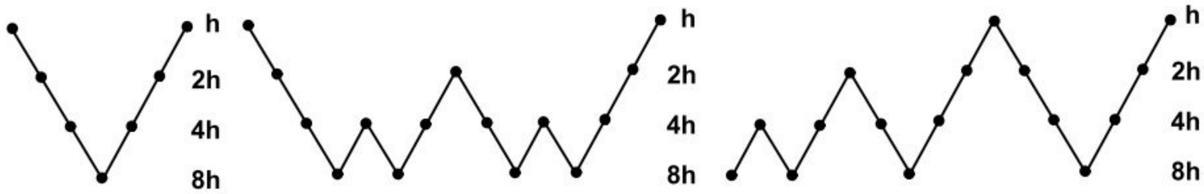
where I_h^H is a full weighting restriction operator. Notice that L^H is the L^h operator written on mesh H nodes. Once coarser grid problem given by Eq. (10) is solved through Gauss-Seidel method, $\tilde{u}^h$ can be corrected as follows:

$$\overline{u}^h = \tilde{u}^h + I_H^h (\underline{\tilde{u}}^H - I_h^H \tilde{u}^h) \tag{13}$$

where I_H^h is a linear interpolation operator and $\underline{\tilde{u}}^H$ is the encountered approximation to $\hat{u}^H$. At last, v_2 post-relaxations are performed on the corrected solution $\overline{u}^h$. Equations (9) - (13) illustrate correction scheme *FAS* (*Full Approximation Scheme)* used to adequately solve the presented lubrication problem with Reynolds cavitation condition.

The multilevel resolution process was explained only for two related grids or levels, but it is extended for as many levels as is possible to choose only changing the number of restrictions until achieve and solve coarsest grid and also the necessary number of interpolations until correct finest grid's searched solution.

The FMG algorithm was implemented allowing the use of as many levels as necessary to solve efficiently the lubrication problem and according to the proposed algorithm by Venner and Lubrecht (2000). Thus, it works with a multi-level correction cycle V or W, depending on the accuracy needed for the solutions on intermediate levels. Figure 2 shows flow diagrams for both mentioned correction cycles and the FMG with V-cycle. The *FAS* is the correction scheme used in the code.



(*a*) (*b*) (*c*)

Figure 2. Flow diagrams for (2) V-cycle; (b) W-cycle; (c) FMG V-cycle. Source: adapted from Briggs (2000)

Notice that in Fig. 2 the black dots are the levels and ascendant direction means interpolation while the descendent one, restriction. As can be seen FMG starts by solving the problem on coarsest grid, with Gauss-Seidel relaxations, and the correspondent solution then serves as an initial approximation, through interpolation operator, for solving the immediately next finer grid, which will be solved through V or W cycles according to the multilevel technique, and so on until finest grid is reached. Since the discrete formulation is solved on all levels, an approximated error norm can be evaluated on all of them, except on finest grid, which is defined as:

$$aen^H = \frac{1}{\left(N_x/2 - 1\right)\left(N_z/2 - 1\right)} \sum_{i=1}^{N_x/2-1} \sum_{j=1}^{N_z/2-1} \left|\bar{u}_{i,j}^H - \bar{u}_{2i,2j}^h\right| \tag{14}$$

where N_x and N_z are the number of volumes in x and z directions, respectively, of the finer grid, $\bar{u}^H$ and $\bar{u}^h$ are converged solutions of two successive coarser and finer grids, respectively.

3. RESULTS AND DISCUSSIONS

The journal bearing used in the computational simulations has a radial clearance of $C_r = 90\mu m$, a length-diameter ratio of $L/D = 0.67$, shaft's eccentricity ratio of $\varepsilon = 0.5$ and the dynamic viscosity of its lubricant oil is $\mu = 0.051 Pa.s$. FVM finest grid has $N_x = N_z = 1024$ volumes and Gauss-Seidel method's convergence criteria is an accumulated absolute error of 10^{-1} on pressure field.

First test aims to verify the aen^H convergence for each level when FMG algorithm has a total number of 6 levels with 2 pre-relaxations and only 1 post-relaxation, considering still the both V and W correction cycles. Furthermore, $\gamma = 2$ for W-cycle.

The number of cycles per level was controlled and established the same for all of them and results are shown on Fig. 3, which indicates no significant difference between V or W correction cycles regarding final value of aen^H. Thus, is sufficient to adopt only a V correction cycle. Also, aen^H decays with a ratio of 4 from lower to higher level, confirming achievement of discretization error in mesh levels which is of second order. Graph in Fig. 3(f) was obtained by extrapolation following that discretization error decay.

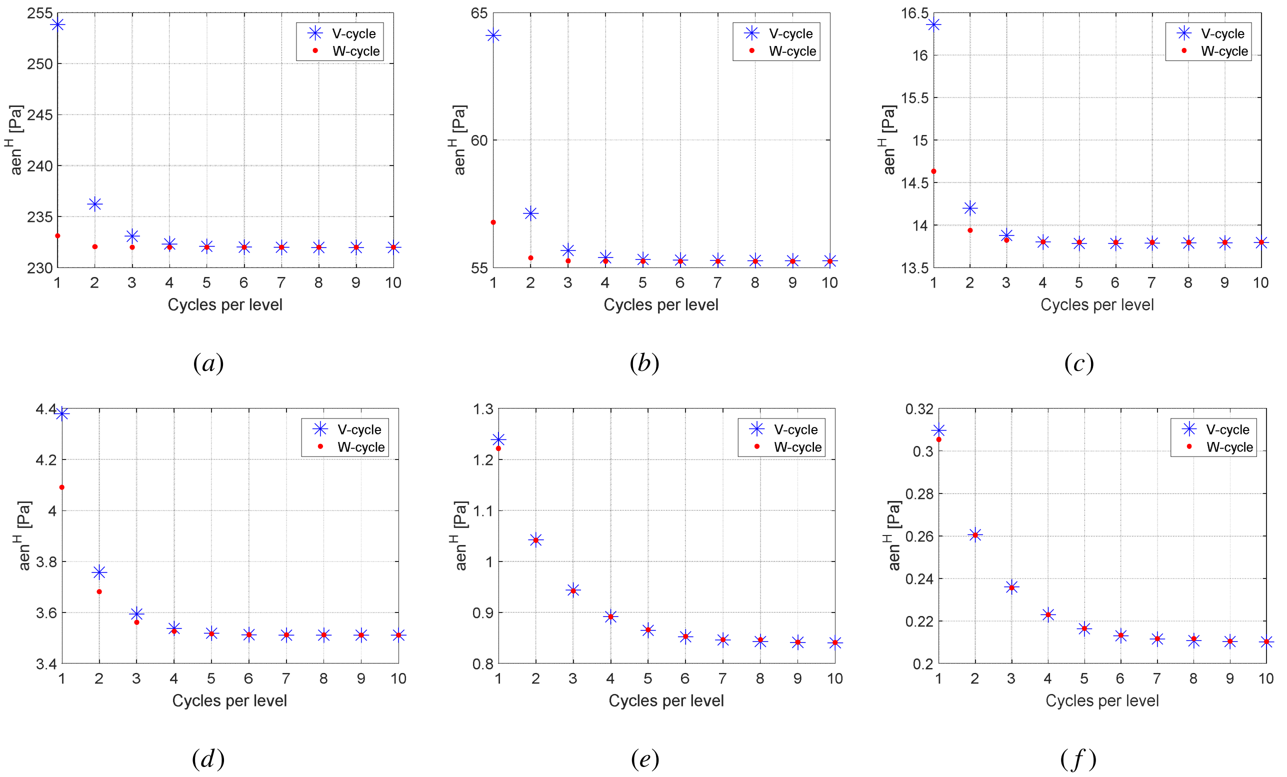


Figure 3. aen^H convergence as the number of cycles per level grows for FMG. Level: (a) 1^{st} ; (b) 2^{nd} ; (c) 3^{rd} ; (a) 4^{th} ; (b) 5^{th} ; (c) 6^{th} .

A second test was then performed consisting of the same lubrication problem being solved for 4, 5 and 6 total number of levels and different values of (v_1, v_2) , whose results can be observed in Fig. 4.

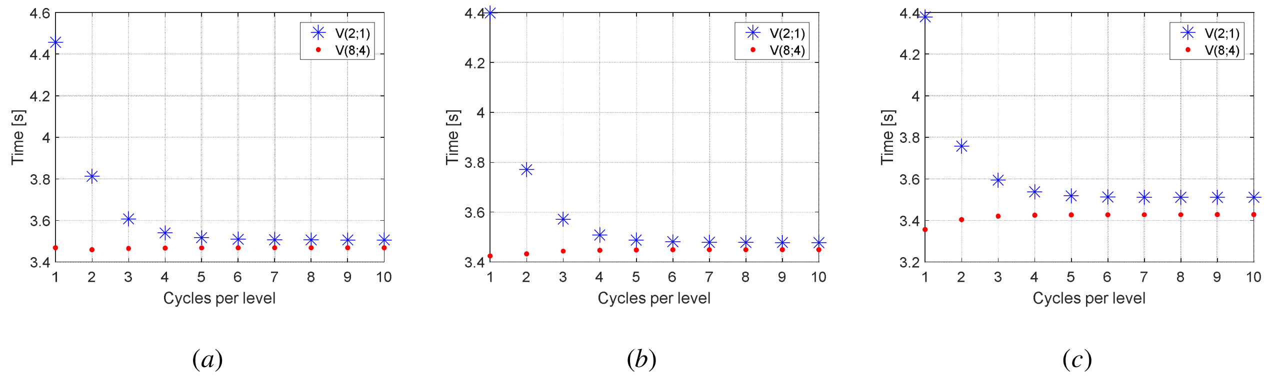


Figure 4. aen^H convergence for the grid with $256x256$ volumes. Total number of levels: (a) 4 ; (b) 5 ; (c) 6 .

As can be seen the number of levels doesn't changes significantly converged value of aen^H for same grid. However, it becomes clear that greater numbers of v_1 and v_2 provides faster convergence regardless of the chosen number of levels. Figure 5 shows the spent computational times for each case studied.

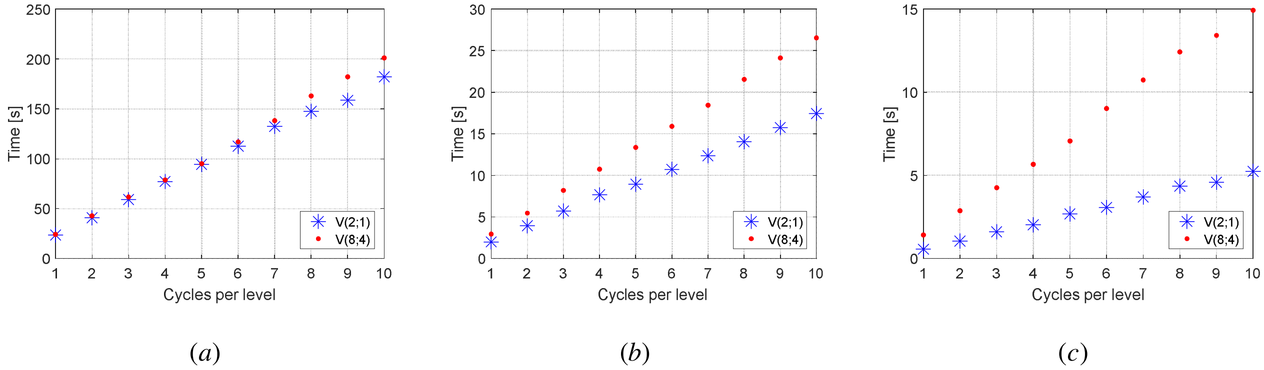


Figure 5. Computational time to solve the discrete formulation. Total number of levels: (a) 4 ; (b) 5 ; (c) 6 .

According to Fig. 5, results show that the choice for a greater number of levels to solve the problem is best regarding computational time, once coarsest grid to be solved through Gauss-Seidel method becomes the smallest one - in terms of total volumes' number - when compared to the choices for 4 or 5 total number of levels. Thus, the spent computational time is strongly influenced by the total number of levels. Moreover, although differences on computational time are verified according to the choice for (v_1, v_2) the same are small when compared to the influence that total number of levels can have over total amount of time spent on resolution. Clearly, simulations must be settled to work with a number of 6 and $(v_1, v_2) = (8, 4)$, i.e. greater numbers of levels and pre and post-relaxations.

4. CONCLUSIONS

In this study becomes clear that hydrodynamic lubrication problem in journal bearings can be solved accurately with a V correction cycle when using a FMG algorithm. Moreover, spent computational time can be reduced by increasing the number of levels in FMG, since coarsest level is solved by Gauss-Seidel method and practically decides the necessary amount of time to solve the problem. Also, greater numbers of pre and post-relaxations accelerates aen^H convergence, while adding a relatively smaller amount of computational time.

Finally, it is important to highlight that the FMG algorithm represents a powerful tool for the resolution of the lubrication problem in high discretization mesh conditions, due to mainly its high performance related to the computational time.

5. ACKNOWLEDGEMENTS

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7. RESPONSIBILITY NOTICE

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