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COBEM-2017-2283 USE OF GAMMA THETA MODEL TO DETERMINE THE AERODYNAMICS CHARACTERISTICS OF NACA 23012

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Abstract. *In the flow around aerofoils the laminar region may correspond up to 30% of the flow. The regular models of RANS(Reynolds Averaged Navier-Stokes) like k-ϵ, k-ω and SST are incapable of predict the laminar-turbulent transition accurately because they consider a fully turbulent flow. The SST Gamma Theta model was created, by Dr. Florian Menter and Dr. Robin Blair Langtry, to identify the laminar region and the transition process. The objective of this study is analyse the flow around NACA 23012 aerofoil using numerical simulation with SST Gamma Theta model and ANSYS CFX in order to determine the aerodynamics characteristics of the airfoil. Simulation's results were compared with experimental data of NACA Report 586 and showed good behaviour of SST Gamma Theta Model.*

Keywords: *Numerical Simulation, Transition Model, NACA 23012 Aerofoil*

1. INTRODUCTION

Nowadays there are several techniques for solving turbulent flows, such as: Direct Numerical Simulation (DNS), Large Eddy Simulation (LES) and Reynolds Averaged Navier-Stokes (RANS). In the field of engineering, in most cases, the interest is focused on the average of the value and in a short time to obtain them. Obtaining an average value with good accuracy and in a short time is the best description of RANS method. There are several studies about RANS modelling turbulent flows, but the majority assume that the flow is completely turbulent, which is not always true. It is important to know that for some cases the laminar region can not be overlooked. The use of models that consider the flow completely turbulent will entail considerable errors in the obtained results. It is the case of airfoils. An airfoil is defined as the cross section of a body that is placed in an airstream in order to produce a useful aerodynamic force in the most efficient manner possible (Abrahams and Cladwell, 2005). Airfoils can be use in cross sections of wings, propeller blades, wind turbine blades, compressor and other, so it is important to evaluate their aerodynamic characteristic correctly. In order to correctly evaluate these characteristics it is necessary to take into account the laminar region of the flow on the airfoil and consequently it is necessary to analyse the laminar-turbulent transition. Transition is the phenomenon which occurs in trough different mechanisms in different applications (Menter *et al.*, 2006). And according to Uranga (2010), the strongest factors affecting transition process are roughness of the wall or surface where the flow passes, adverse pressure gradient and freestream turbulence. With this the need to develop a model that consider the laminar-turbulent transition process arose. Along with other models, which consider the transition, the γ-θ model appears, which will be used in this work to determine the aerodynamic characteristics of the NACA 0012 airfoil. The obtained results will be compared with the experimental data of the characteristics.

2. THEORETICAL REFERENCE

2.1 Transition Process

The boundary layer laminar-turbulent flow is an extremely complex process, even in simple geometries and with a very calm external flow. Several factor have influence on the transition process of laminar boundary layers (White, 2003), (Schmidt and Patankar, 1991). They are: Turbulent intensity of the free current, pressure gradients, wall roughness, streamlines curvature, Mach number, wall suction or blowing and wall heating or cooling. The laminar-turbulent trasition process in boundary layers are classified according to the phenomena that mostly trigger the process (White, 2003). In the case of the airfoils the transition process is the Natural process. The initial researches on Natural transition were based on the theory of inverse stability which suggest that all flows were unstable only if there was an inflection point in the velocity profile. Subsequently raised the theory that the laminar boundary layer can be destabilized by the presence of viscous waves of instability, these waves were called Tollmien-Schlihting waves. Although the waves were proven physically by Prandtl and mathematically by Tollmien, the initial experiments on transition had very high levels of disturbance in free flow which made it difficult to prove it experimentally. It was only in 1984 that Schubauer and Skramstad documented the existence of Tollmmien-Schlichting waves in a better quality wind tunnel. When the free stream has a low turbulent intensity the boundary layer becomes linearly unstable and when it reaches a critical Reynolds value the Tollmien-Schlichting (T-S) waves begin to grow slowly. Because of this slow growth the transition occurs only after the T-S waves become non-linear and inverse inverse mechanisms results in three-dimensional perturbations, called transverse vorticity (Klebanoff *et al.*, 1962). Subsequently the three-dimensional vortices break and the turbulent flow begins. Fig.1 show all these steps.

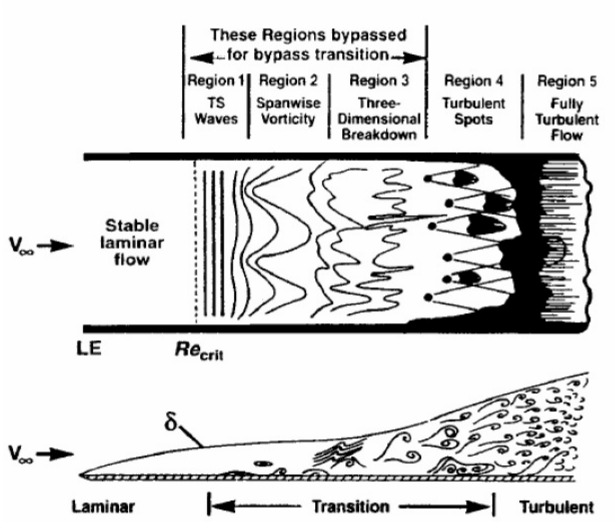

Figure 1. Natural transition process. Source:(Schlichting *et al.*, 1955)

2.2 γ-θ Model

One of the most common ways of predicting the boundary layer transition onset location in industrial CFD codes is to employ empirical correlations. The empirical correlations usually relate the freestream turbulence intensity (Tu) to the transition Reynolds number based on the momentum thickness Reynolds number (Re_θ).

The Gamma-Theta model is called semi-empirical because it is not aim to physically model the transition. The whole physical part of the process is left to the empirical correlations. The model can be fully calibrated, independent of the applied turbulence model, with internal transition start and transition length corrections. The correlations can also be extended to flows with low free-stream turbulence intensity or flows with cross flow instability. Therefore, this formulation provides a flexible environment for engineering transition predictions with the ability to capture multiple transition modes simply by adding correlations.

According to Oliveira Jr (2014) , these correlations generally provide values of Re_{xt}, $Re_{\theta t}$, Re_{xT} and $Re_{\theta T}$, where Re_x is the Reynolds number based on position x along the surface and Re_θ based on the momentum thickness given by Equations 1 and 2, respectively. The subscript "t" indicates the point where the transition actually begins, and the subscript "T" indicates the point where the boundary layer effectively completes the transition process.

$$Re_x = \frac{\rho U x}{\mu} \tag{1}$$

$$Re_\theta = \frac{\rho U \theta}{\mu} \tag{2}$$

Where: ρ is density of the fluid, μ dynamic viscosity, U velocity of reference and θ is the momentum thickness given by Equation 3.

$$\theta = \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy \approx \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy \tag{3}$$

Being δ the boundary layer thickness.

The main experimental correlation used in the development of the Gamma-Theta model was that of Abu-Ghannam and Shaw (1980). An extensive study on the transition of the boundary layer was made in this literature, arriving at the important conclusion that the history of the flow of the laminar boundary layer significantly changes the value of local Re_θ (Reynolds of transition based on the momentum thickness). In this way, correlations have been developed to obtain $Re_{\theta t}$ and $Re_{\theta T}$, given by Equations 4 and 5.

$$Re_{\theta t} = 163 + exp\left[F(\lambda_\theta) - \frac{F(\lambda_\theta)}{6.91}Tu\right] \quad (4)$$

$$F(\lambda_\theta) = \begin{cases} 6.91+12.75\lambda_\theta+63.64\lambda_\theta{}^2 \ para\ \lambda_\theta<0 \\ 6.91+2.48\lambda_\theta-12.27\lambda_\theta{}^2 \ para\ \lambda_\theta\geq 0 \end{cases} \quad (5)$$

Where λ_θ is the coefficient of pressure gradient in the free stream, given by Equation 6.

$$\lambda_\theta = \frac{\theta^2}{\nu}\frac{dU}{dx} \quad (6)$$

The vast majority of the correlations to predict the point at which the boundary layer transition begins provides this criterion in the form of a value of $Re_{\theta c}$. The calculation of this parameter is a non-local operation, involving the integral shown in Equation 3, which makes its application in modern CFD codes very difficult. To solve this problem, the Gamma-Theta model uses correlations of $Re_{\theta c}$ with locally calculated Reynolds.

One way to calculate the Reynolds locally is based on the local shear rate of the flow or vorticity, given by Equation 7.

$$Re_v = \frac{\rho y^2}{\mu}|\frac{du}{dy}| = \frac{\rho y^2}{\mu}\overline{S} \quad (7)$$

Where $\overline{S}$ is the local magnitude of shear rate tensor and y the normal distance to the nearest wall.

The Figure 2 shows a graph of Re on the y-axis and Re_x on the x-axis. The first curve (shown with dash-dot) is the value of max (Re_v) along the x-direction (shown in its dimensionless Re_x). The second curve (with dotted line) is the distribution of the value of Re_θ at each x position. The third and most important curve of this graph is the distribution curve of the maximum Re_v value divided by the constant 2.193, also in its distribution along the x-direction. Note that the last two curves are coincident.

So there is a direct relationship between $Re\nu$ and $Re\theta$. With this relation is possible to write the Equation 8 and calculate a nonlocal variable through a local variable, making the application of these correlations possible in CFD codes.

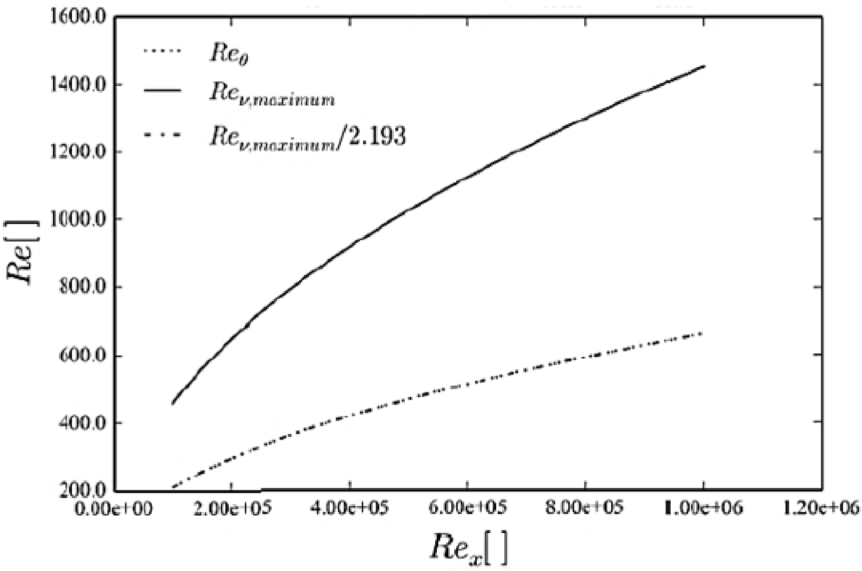

Figure 2. Relations between Reynolds number along the "x" direction of the plate. (Source: Oliveira Jr (2014))

$$Re_\theta = \frac{max(Re_v)}{2.193} \quad (8)$$

In this way the problem of calculating Re_θ has been solved, but there remain the non-local quantities T_u and λ_θ, as well as the implementation of the integration of the transition model with the turbulence models used in CFD packages. The solution adopted by the Gamma-Theta model was the use of two additional transport equations: one for intermittence (γ) and one for $Re_{\theta c}(\theta)$, from these two equations comes the name of the model.

–> Intermittency Equation

The intermittence factor, or simply intermittence, is defined in (Schlichting *et al.*, 1955) as the fraction of time in which the flow is turbulent. The intermittence at a given point depends on the conditions of the flow around it and the history of the portion of the fluid under evaluation, so a transport equation is used for the solution of this variable, given by Equation 9.

Intermittency transport equation used to trigger the transition onset. The intermittency function is coupled with the SST turbulence model. It is used to turn on the production term of the turbulent kinetic energy downstream of the transition point in the boundary layer. This approach can capturing the effect of large freestream turbulence levels and the subsequent increase in the laminar skin friction and heat transfer.

$$\frac{\partial(\rho\gamma)}{\partial t} + \frac{\partial(\rho\overline{u_j}\gamma)}{\partial x_j} = P_\gamma - E_\gamma + \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_T}{\sigma_\gamma}\right)\frac{\partial\gamma}{\partial x_j}\right] \tag{9}$$

The first and second terms are the local variation and the advective transport of γ and the last term of the equation is molecular and turbulent diffusion of γ.

P_γ is the transition source term given by Equation 10.

$$P_\gamma = F_{length} c_{\alpha 1} \rho S \sqrt{\gamma F_{onset}}(1 - c_{e1}\gamma) \tag{10}$$

Where S is the strain rate magnitude. This term is designed to be equal to zero (due to the Fonset function) in the laminar boundary layer upstream of transition and active everywhere the local vorticity Reynolds number exceeds the local transition onset criteria. The magnitude of this source term is controlled by the transition length function (Flength). The density and strain rate are present in order to have the correct units for the source term. The last term in Eq. 10 is used to limit the maximum value of the intermittency so that it cannot exceed a value of one.

The function F_{onset} and her terms are shown in Equations 11 to 15.

$$F_{onset} = max(F_{onset2} - F_{onset3}; 0) \tag{11}$$

$$F_{onset1} = \frac{Re_v}{2.139 Re_{\theta c}} \tag{12}$$

$$F_{onset2} = min[max(F_{onset1}; F^4_{onset1}); 2] \tag{13}$$

$$F_{onset3} = max\left[1 - \left(\frac{R_T}{2.5}\right)^3; 0\right] \tag{14}$$

$$R_T = \frac{\rho k}{\mu w} \tag{15}$$

As can be seen from Equation 12, the correlation between Re_v and Re_θ is used in the equation of the function F_{onset}. Another dimensionless function of great importance present in the term of production is F_{length}, which controls the magnitude of the term of production. Physically, this function controls the length of the transition zone. In the $\gamma - \theta$ model this function was obtained through a curve fit using the comparison of simulations with experimental transition results on a flat plate.

E_γ is the destruction/relaminarization source term, given by Equation 16. This term acts like a sink term and ensures that the intermittency remains close to zero in the laminar boundary layer.

$$E_\gamma = c_{\alpha 2} \rho \Omega \gamma F_{turbulent}(c_{e2}\gamma - 1) \tag{16}$$

Where: $\overline{S}$ is the local magnitude of the shear rate tensor, Ω is the vorticity magnitude.

The dimensionless function $F_{turbulent}$, given by Equation 17, activates the term of destruction in laminar boundary layers and cancels this term outside the boundary layer ($\mu_T \gg \mu$).

$$F_{turbulent} = e^{-(\frac{Rt}{4})^2} \tag{17}$$

The constants used in the terms of production and destruction of the transport equation of γ are: $C_{a1} = 2$; $C_{a2} = 0.06$; $C_{e1} = 1$; $C_{e2} = 50$; $\sigma_\gamma = 1$

–> $\tilde{Re}_{\theta t}$ Equation

According to Oliveira Jr (2014), one of the main difficulties in implementing a transition model for CFD codes based on empirical correlations is the fact that information outside the boundary layer (Tu and λ_θ) influences the transition process.

This transfer from the free stream to the boundary layer is a non-local operation solved in the Gamma-Theta model through use of an additional transport equation. This equation is solved in terms of the transition onset momentum-thickness Reynolds number ($Re_{\theta t}$) that marks the effective start of the $\tilde{Re}_{\theta t}$ transition, given by Equation 18.

According to Langtry (2006), this additional transport equation is an essential part of the model as it ties the empirical correlation to the onset criteria in the intermittency equation and allows the modelś use over multiple geometries.

$$\frac{\partial(\rho \tilde{Re}_{\theta t})}{\partial t} + \frac{\partial(\rho \overline{u_j} \tilde{Re}_{\theta t})}{\partial x_j} = P_{\theta t} + \frac{\partial}{\partial x_j}\left[\sigma_{\theta t}(\mu + \mu_T)\frac{\partial \tilde{Re}_{\theta t}}{\partial x_j}\right] \tag{18}$$

This equation calculates the value of $\tilde{Re}_{\theta t}$ outside the boundary layer, based on the pressure gradient values, turbulent intensity of the free stream, and transports this information to the boundary layer through its diffusion term.

$P_{\theta t}$ is the production term, given by Equation 19.

$$P_{\theta t} = c_{\theta t}\frac{\rho}{\tau}(Re_{\theta t} - \tilde{Re}_{\theta t})(1 - F_{\theta t}) \tag{19}$$

The first part of Equation 19 is the ratio of the density ρ to a time scale τ. This time scale is defined by dimensional analysis and aims to make the production term of the same order as the advective and diffusive terms of the transport equation of $\tilde{Re}_{\theta t}$. The time scale τ is given by Equation 20.

$$\tau = \frac{500\mu}{\rho U^2} \tag{20}$$

The second part of the production term is the difference between the $Re_{\theta t}$ calculated from the correlations and the $\tilde{Re}_{\theta t}$ effectively transported

The third part of the term is the function $F_{\theta t}$, which disables the production term inside the boundary layer, so that only the diffused $\tilde{Re}_{\theta t}$ information of the free stream arrives. $F_{\theta t}$ is given by Equation 21, is zero in the free current and goes to 1 in the boundary layer.

$$F_{\theta t} = min\{max\left[F_{wake}e^{-\left(\frac{y}{\delta_{wake}}\right)^4}; 1 - \left(\frac{y - \frac{1}{c_{e2}}}{1 - \frac{1}{c_{e2}}}\right)^2\right]; 1\} \tag{21}$$

Where F_{wake} serves to search for regions of high turbulent intensity (wakes, for example) and keep the function inactive in these zones.

The constants of these equations are: $c_{\theta t} = 0.03; \sigma_{\theta t} = 2$

3. METHODOLOGY

As the purpose of the study is to determine the aerodynamic characteristics of NACA 23012 through the *SST* γ-θ transition model it is necessary to compare the results obtained through the numerical simulation with the results obtained trough the experimental methodology performed by Jacobs and Sherman (1937) in his experiment in NASA's variable density wind tunnel for a Reynolds number of $3.1 \cdot 10^6$. The computational mesh was done in Pointwise software and it is a *O-grid* type, being the minimum distances of the airfoil surfaces to the walls equivalent to 100 chords, according to the validation method describe by Rumsey (2014). Within the theoretical boundary layer there are more than 15 layers of finite volumes and the spacing close to the wall is 0.001mm thus ensuring a dimensionless distance to the wall (y^+) around 1, value that is required for the turbulence *SST* γ-θ model. Fig.3 show mesh details.

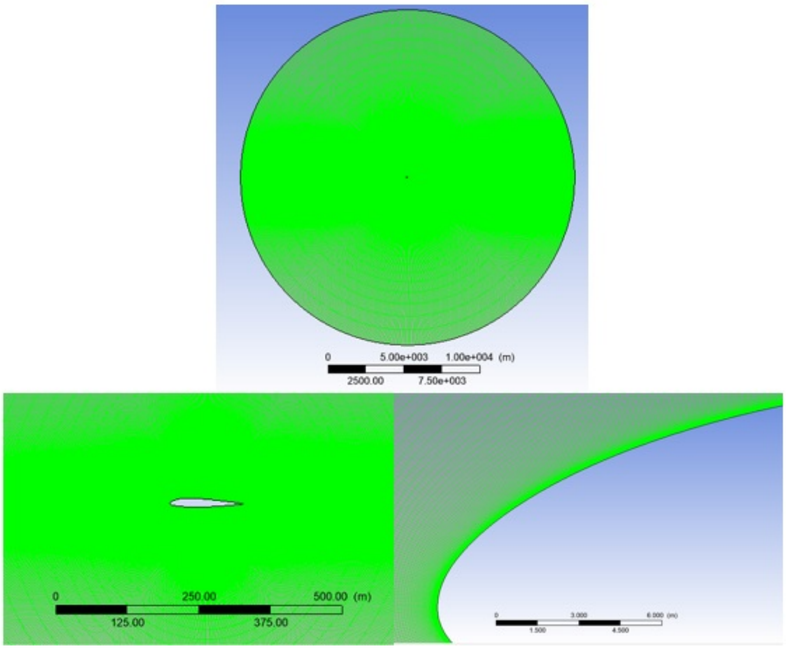

Figure 3. Mesh details.

The commercial package ANSYS CFX was used in the simulations. The working fluid was ideal gas air at 20 atm flowing in steady state. The surfaces of the airfoil were considered smooth and non slip. Symmetry conditions were applied to the lateral walls of the domain in order to simulate a two-dimensional flow. A velocity of 11.9 m/s was considered, equivalent to 0.03 Ma. Simulations for several angles of attack were performed in order to raise the $C_l \times \alpha$ and $C_d \times \alpha$ curves. The first simulations were then performed and with the results was possible to note that some adjustments of the boundary conditions would be necessary. The first adjustment was the value of the Eddy Viscosity Ratio in order to obtain results that are closer to reality. In the search for this value a iterative process was performed to reach the optimal value. More simulations were then performed and with the news results it was clear that the transition point differed from the experimental data since the values of C_l and C_d, although close to the experimental values, still presented relative high errors. Through researches it was revealed that the γ-θ model constants already configured in ANSYS CFX differed from the recommended values for a better performance of the model. These constantes were adjusted to the values recommend in the literature. Then came the knowlefge that the γ-θ model need to be calibrate because the model was modelled for a bypass transition, a different transition type from the studied case. It is necessary to carry out a study of the model correlations in order to obtain calibrated values for the constants, to made this calibrations some studies like (Malan *et al.*, 2009), (Tomac *et al.*, 2013), among others. To make sure that the changes in the constants and the model calibration were correct an initial test was performed with a flat plate, where the results obtained through the numerical simulation were compared with the results obtained experimentally by ERCOFTAC (European Research Community on Flow, Turbulence and Combustion). Considering the affinity with the studied case it was decided to use the T3AM case, since it is the only case that presents a natural transition. Details of the geometry and mesh are show in the following figures.

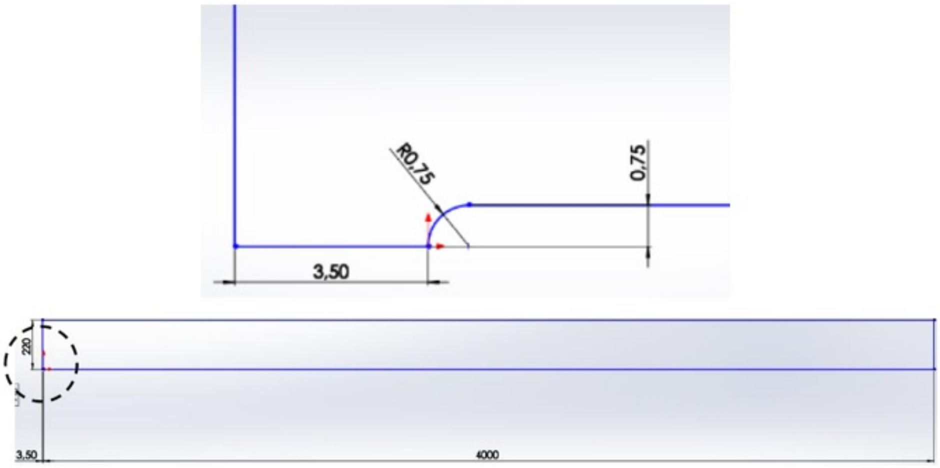

Figure 4. Geometry details.

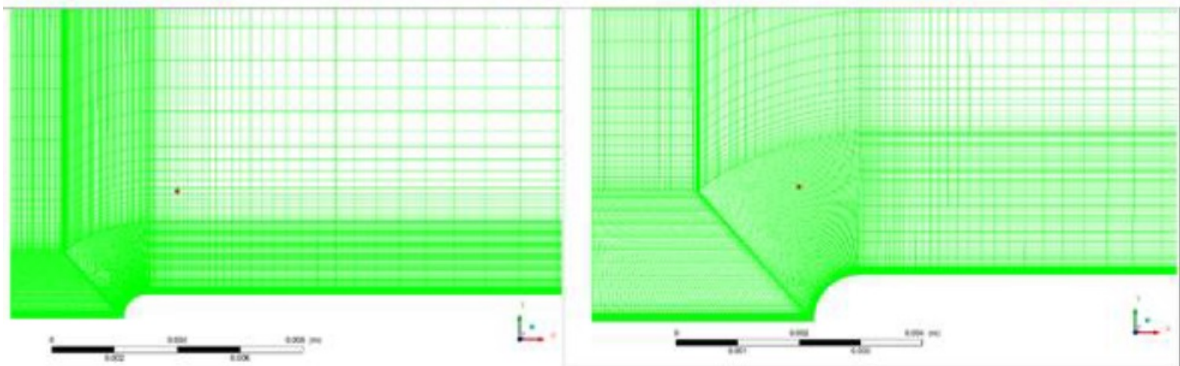

Figure 5. Flat Plate mesh details.

In order to ensure a good mesh quality for the flat plate case a maximum y^+ of 0.5 was used. As the analysis is also two-dimensional, only one element was used in the direction of the plate width. The results comparisons were made through the friction coefficient, given by:

$$C_f = \frac{\tau_\omega}{\frac{1}{2}\rho U_\infty^2} \tag{22}$$

where τ_ω is the wall shear stress, ρ is the specific mass and U_∞ is the initial flow velocity. The obtained results are presented in Fig.6

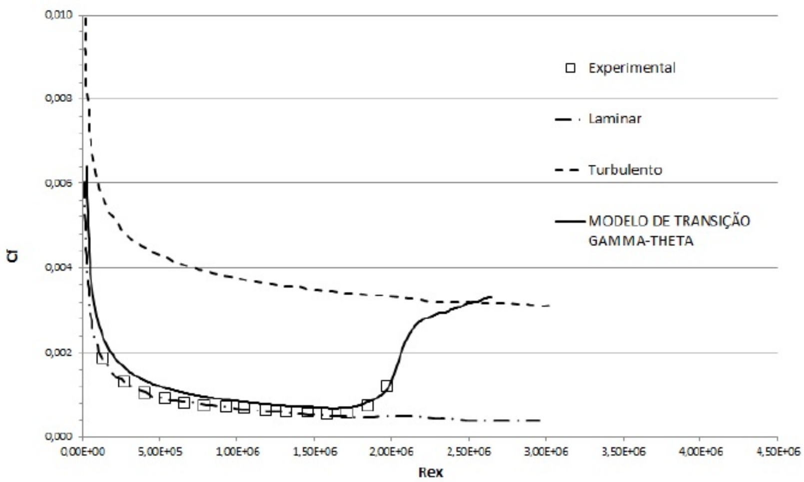

Figure 6. Flat plate results.

It was possible to notice that in the laminar region there was a small difference in relation to the experimental curve, but the corrected model was able to predict with precision the location of the laminar-turbulent transition. Whit this it was possible to continue with the airfoil numerical simulations.

4. RESULTS

As previously mentioned this study aims to determine the aerodynamic characteristics of the NACA 23012 airfoil. For this was chosen to determine two of the most known characteristics, the lift coefficient C_l and the drag coefficient C_d, they are given by:

$$C_l = \frac{F_l}{\frac{1}{2}\rho V^2 A} \tag{23}$$

$$C_d = \frac{F_d}{\frac{1}{2}\rho V^2 A} \tag{24}$$

where F_l is the lift force, F_d is the drag force, V is the flow velocity and A is the aerofoil area. The experimental data was taken from Jacobs and Sherman (1937). And the numerical results were analysed through the ANSYS POST. The results are presented below:

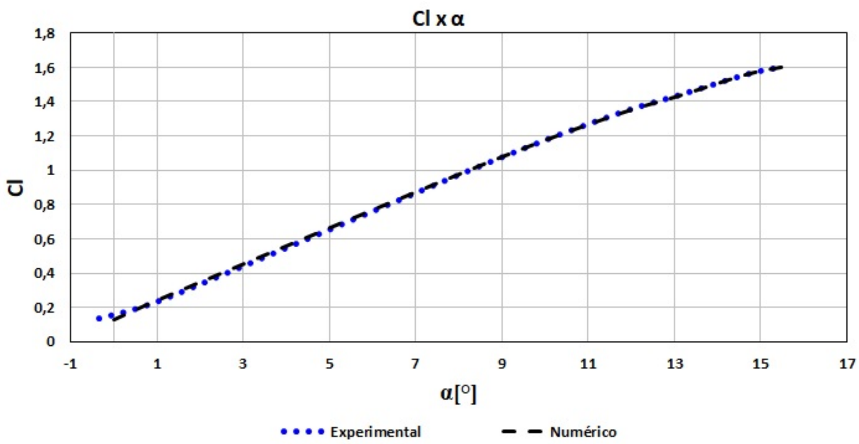

Figure 7. Lift coefficient comparison.

The highest relative error was 1.54%. This shows that the calibrated γ-θ model was extremely accurate in determining the lift coefficient. It is worth mentioning that to obtain the $C_l \times \alpha$ curve it was necessary to carry out simulations for different angles of attack. Regarding the drag the relative error reached a little less than 10% which was already expected since the comparison of C_d usually present the biggest errors.

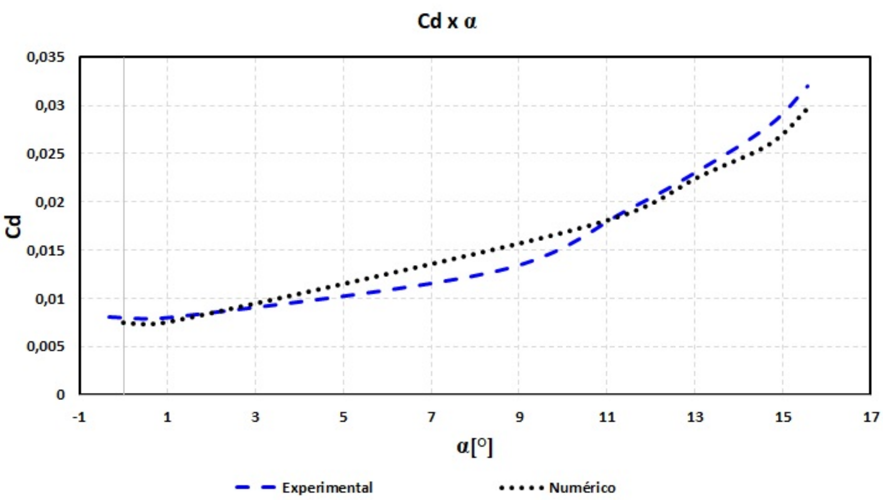

Figure 8. Drag coefficient comparison.

In order to compare the performance of the transition model was opted to perform new simulations using the SST turbulence model that considers the flow totally turbulent, the results are presented below.

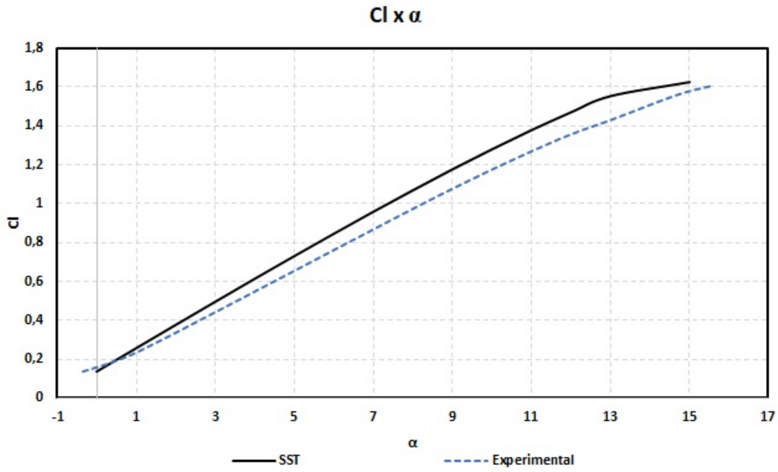

Figure 9. SST lift coefficient comparison.

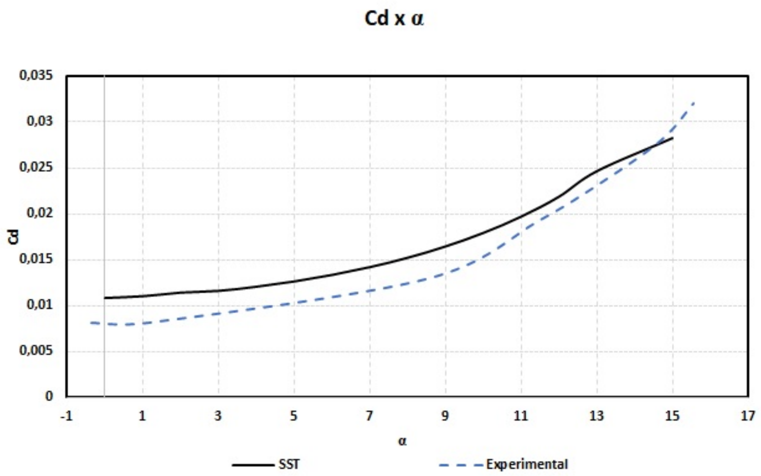

Figure 10. SST drag coefficient comparison.

It was observed that the results obtained using the gamma theta model were closer to the results obtained experimentally, which shows that disregarding the laminar part of the flow can cause discrepancies in the determination of airfoil's aerodynamics characteristics.

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