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SIMULTANEOUS ESTIMATION OF THERMAL PROPERTIES USING GENETIC ALGORITHM

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Abstract. This paper presents an experimental method for obtaining, simultaneously, thermal properties of solid materials using genetic algorithm (GA). The experimental method is based on the use of a surface probe that has one resistance heater, two thermocouples and one heat flux sensor. The technique relies on the minimization of multi-objective function with a quadratic error between measured and mathematical temperatures in same specific positions. Green's functions are used to determine analytical solution of temperature which is compared with the experimental measured, with that comparation we estimate thermal conductivity and thermal diffusivity.

Keywords: *Inverse Problems, Properties Estimation, Thermal Diffusivity, Thermal Conductivity.*

1. INTRODUCTION

Thermal properties measurements are extremely important for the prediction of thermal behaviour in various engineering applications. For example, the thermal behaviour of furnace walls, machining tools, engine body, generators or biological tissues. All of these examples, however, do not allow invasive testing or internal temperature measurements.

It can be observed that most of method found in literature use measurement data from interior locations or need two surfaces to estimate the properties. For example, classic methods, as flash method (Parker, 1961), Angstrom method (Zhu, 2016) and hot disk (Bohac, 2000). Others procedures use internal measurements to estimate thermal conductivity and diffusivity simultaneously, or external measurements to estimate only one thermal property, as (Mohamed, 2010), (Betta, 2009), (Neven, 2009), (Monde, 2010) and (Martinez, 2015).

Only one surface of probe are use and has one resistance heater, two thermocouples, and one heat flux sensor. Frontal surface of sample is partially heated while the others faces can be considered inactive or be exposing to an environment medium. Temperatures in two different points of surface are measured. An objective function defined as quadratic difference between experimental temperatures measurements and mathematical temperature solution are used to estimate thermal conductivity and thermal diffusivity The main difficulty in simultaneous parameter estimation is the problem of existence of a unique optimum solution, so, specific and appropriated search intervals are necessary.

Analytical solution of temperature was made based on Green's functions (GF), and specific boundary condition nomenclature was used (Beck, J.V. et al. 1999). Using GF is possible to find solutions of heat conduction problems of the most varied and complex types, such as three-dimensional, transient, heat-generating and non-uniform problems and which may still be subject to non-homogeneous boundary conditions varying with time and space.

In this study, we used the method of genetic algorithms. This method starts with a number of individuals (population) and each algorithm cycle (generation) individuals with higher numerical value tends statistically to replace those with smaller value, leading the function to a maximum value (Vanderplaats, 1984).

2. DIRECT PROBLEM

Direct problem is partial heating of a sample, initially at a uniform temperature, $T_o = T_\infty$ while all other surfaces are exposed to a convective environment $\left(h_1 = h_2 = h_3 = h_4 = h_5 = h_6\right)$, with specific nomenclature $X_{33}Y_{33}Z_{33}$ (Beck, J.V. et al.,1999). Figure 1, shows three-dimensional thermal model used.

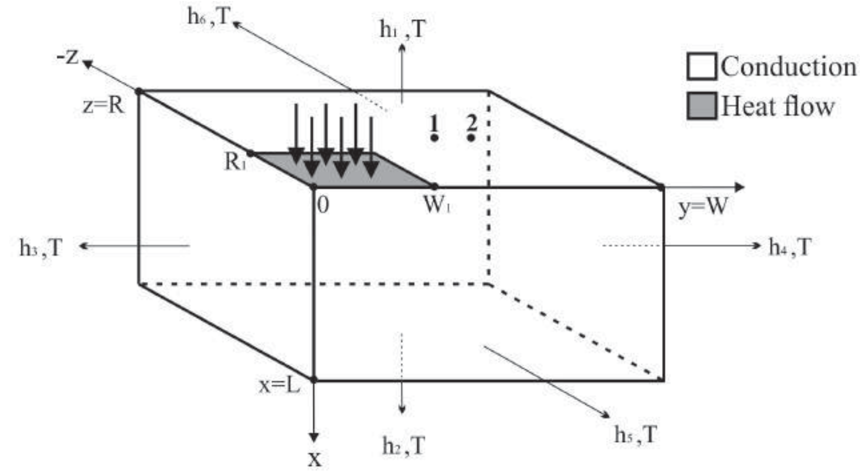


Figure 1. Direct problem $X_{33}Y_{33}Z_{33}$

Direct problem, presents in the Fig. 1, has diffusion equation describe as (Equation 1);

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{1}$$

With boundary condition at X, Y and Z coordinate, Equations 2, 3 and 4, respectively;

$$k\left.\frac{\partial T}{\partial x}\right|_{x=0} = -q(t) - h_1(T - T_\infty) \qquad -k\left.\frac{\partial T}{\partial x}\right|_{x=L} = h_2(T - T_\infty) \tag{2}$$

$$k\left.\frac{\partial T}{\partial y}\right|_{y=0} = h_3(T - T_\infty) \qquad k\left.\frac{\partial T}{\partial y}\right|_{y=W} = h_4(T - T_\infty) \tag{3}$$

$$k\left.\frac{\partial T}{\partial z}\right|_{z=0} = h_5(T - T_\infty) \qquad -k\left.\frac{\partial T}{\partial z}\right|_{z=R} = h_6(T - T_\infty) \tag{4}$$

With initial condition;

$$T(x,y,z,0) = F(x,y,z) - T_\infty \tag{5}$$

2.1 ANALYTICAL SOLUTION

The analytical solution of the temperature was made based on Green's functions (GF). Using GF is easy to find solutions of heat conduction problems of the most varied and complex types, such as three-dimensional, transient, heat generating and non-uniform problems and which may still be subject to non-homogeneous boundary conditions varying with time and space (Beck, J.V. et al. 1999).

The solutions of equations (1) - (5) can be obtained using Green's functions $\theta = T - T_\infty$, resulting in (6).

$$\theta(x,y,z,t)=\frac{8\theta_0}{LWR}\sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}e^{-\left(\frac{\alpha_m^2}{L^2}+\frac{\beta_n^2}{W^2}+\frac{\gamma_p^2}{R^2}\right)\alpha t}$$

$$\times\frac{\left[\alpha_m\cos\left(\frac{\alpha_m x}{L}\right)+B_1\sin\left(\frac{\alpha_m x}{L}\right)\right]}{(\alpha_m^2+B_1^2)\left[1+\frac{B_2}{(\alpha_m^2+B_1^2)}+B_1^2\right]}\frac{\left[\beta_n\cos\left(\frac{\beta_n y}{W}\right)+B_3\sin\left(\frac{\beta_n y}{W}\right)\right]}{(\beta_n^2+B_3^2)\left[1+\frac{B_4}{(\beta_n^2+B_4^2)}+B_3^2\right]}\frac{\left[\gamma_p\cos\left(\frac{\gamma_p z}{R}\right)+B_5\sin\left(\frac{\gamma_p z}{R}\right)\right]}{(\gamma_p^2+B_5^2)\left[1+\frac{B_6}{(\gamma_p^2+B_6^2)}+B_5^2\right]}$$

$$\times\frac{LWR}{\alpha_m\beta_n\gamma_p}\left[\alpha_m\sin(\alpha_m)-B_1(\cos\alpha_m-1)\right]\left[\beta_n\sin(\beta_n)-B_3(\cos\beta_n-1)\right]\left[\gamma_p\sin(\gamma_p)-B_5(\cos\gamma_p-1)\right]$$

$$\times\frac{8\alpha}{Lk}\sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\left[\alpha_m\cos\left(\frac{\alpha_m x}{L}\right)+B_1\sin\left(\frac{\alpha_m x}{L}\right)\right]}{(\alpha_m^2+B_1^2)\left[1+\frac{B_2}{(\alpha_m^2+B_1^2)}+B_1^2\right]}$$

$$\times\frac{\left[\beta_n\cos\left(\frac{\beta_n y}{W}\right)+B_3\sin\left(\frac{\beta_n y}{W}\right)\right]}{(\beta_n^2+B_3^2)\left[1+\frac{B_4}{(\beta_n^2+B_4^2)}+B_3^2\right]}\frac{\left[\gamma_p\cos\left(\frac{\gamma_p z}{R}\right)+B_5\sin\left(\frac{\gamma_p z}{R}\right)\right]}{(\gamma_p^2+B_5^2)\left[1+\frac{B_6}{(\gamma_p^2+B_6^2)}+B_5^2\right]}$$

$$\times\frac{\left(1-e^{-\left(\frac{\alpha_m^2}{L^2}+\frac{\beta_n^2}{W^2}+\frac{\gamma_p^2}{R^2}\right)\alpha t}\right)}{\left(\frac{\alpha_m^2}{L^2}+\frac{\beta_n^2}{W^2}+\frac{\gamma_p^2}{R^2}\right)}\frac{1}{\beta_n\gamma_p}\left[\beta_n\sin\left(\frac{\beta_n W_1}{W}\right)-B_3\left(\cos\left(\frac{\beta_n W_1}{W}\right)-1\right)\right]$$

$$\left[\gamma_p\cos\left(\frac{\gamma_p R_1}{R}\right)+B_5\sin\left(\frac{\gamma_p R_1}{R}\right)-1\right]\times\int_{\tau=0}^{t}q(\tau)e^{\left(\frac{\alpha_m^2}{L^2}+\frac{\beta_n^2}{W^2}+\frac{\gamma_p^2}{R^2}\right)\alpha t}d\tau \tag{6}$$

Where α_m, β_n and γ_p are the eigenvalues of the problem. The eigenvalues are obtained by solving the transcendental equation in each direction using equation (7). The indices $m=1,\ldots,\mathrm{M}$, $n=1,\ldots,N$, and $p=1,\ldots,P$ the number of eigenvalues required for the convergence of the series, given the truncation error, ε desired.

$$\alpha_m\tan(\alpha_m)=\frac{\mathrm{hL}}{k}\qquad\beta_n\tan(\beta_n)=\frac{\mathrm{hW}}{k}\qquad\gamma_p\tan(\gamma_p)=\frac{\mathrm{hR}}{k} \tag{7}$$

2.2 INVERSE PROBLEM: GENETIC ALGORITHM (GA)

Temperatures are measured in two different positions around heater at same surface. Objective function (Equation (7)) is a quadratic error between measured and mathematical (Equation 6) temperatures in both positions. Thermal diffusivity α and thermal conductivity k are the values that minimize the objective function proposed.

$$S(\alpha, k) = \sum_{j=1}^{s} \sum_{i=1}^{N} \left[Y_{i,j} - \theta_{i,j} \right]^2 \tag{7}$$

Where i and j are respectively the surface and time index, $Y_{i,j}$ is the experimental temperature, $\theta_{i,j}$ is the calculated temperature by Equation (6), s is the number of temperature sensors (thermocouples), N is the total number of time measurements. Genetic Algorithms optimization technique was chosen to minimize the objective function proposed (Vanderplaats, 1984).

The method of genetic algorithms is based on the idea of evolution by natural selection where the more "adapted" continues. The method starts with a number of individuals (population) and each algorithm cycle (generation) individuals with higher numerical value tends statistically to replace those with smaller value, leading the function to a maximum value.

Each individual in the population is a set of values of the variables that compose the objective function which when applied generates a value for the function, represented by binary code (genetic code), each bit being a "gen". Three operators of evolution are then applied.

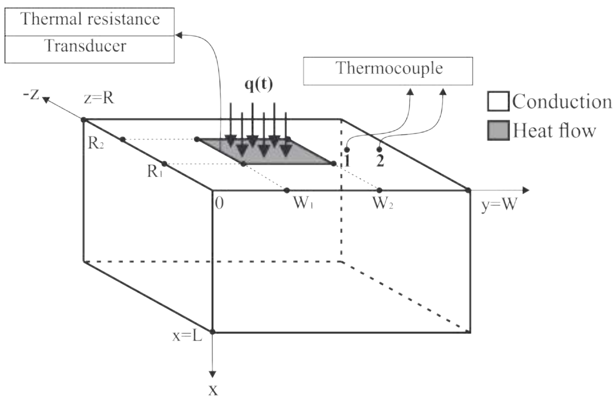
Reproduction. This operator is based on a lottery wheel where individuals which result in greater value of the objective function are more likely to form the next generation. After the assignment of probabilities, each individual in the current generation is replaced by another sorted individual, forming a new population with the same number of individuals as before.

Crossing. This operator mixes randomly portions of individuals in the population. Separating two individuals, parts of their chains are marked and swapped, creating two new individuals. This is done with all individuals in the population.

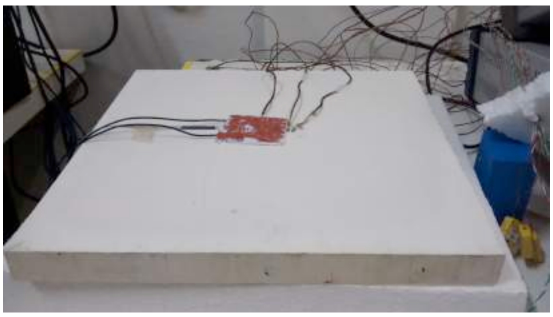
Mutation. This procedure is that which least interferes in the process and exists to generate or recuperate the individuals which have not been assessed. Inverting the 0 or 1 bit (gene) after a very large number of bits generated, for example, the bit is inverted after every 10,000 bits generated in each generation.

2.3 EXPERIMENTAL PROCEDURE

Experimental polyvinyl chloride (PVC) sample, initially in temperature equal to ambient temperature ($T_0 = T_\infty$), are partially heated for a heater covered with silicone rubber, with side dimensions 50-50mm and thickness 0,3mm. Thermopile transducer with dimensions 50x50mm and thickness 0,3mm measured heat flux while the temperatures were measured by type K surface thermocouples. The PVC sample (Figure 2.b) has a 25mm thickness, side dimensions 300-300mm and the center of the heat area is concentric to the access surface. Third independent runs for the PVC sample were realized. In this case, 300 points were taken with time intervals of 0,97s. The time duration of heating was 150s.



(a)



(b)

Figure 2. Experimental apparatus. a) Position of thermal resistance, transducer and thermocouple b) Instrumented PVC sample.

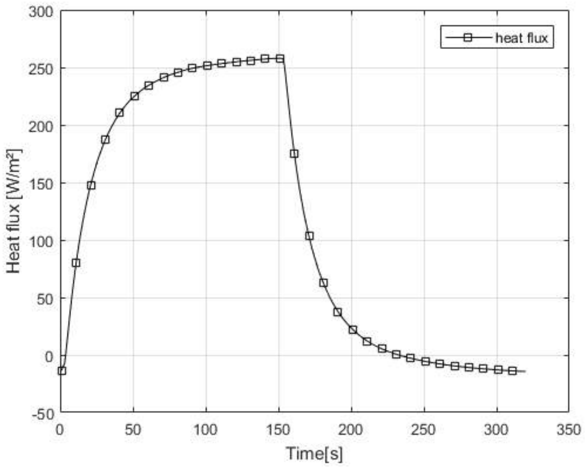
The position of the thermocouples and the transducer is show in Tab. 1.

Table 1. Experimental position of the thermocouples and transducer.

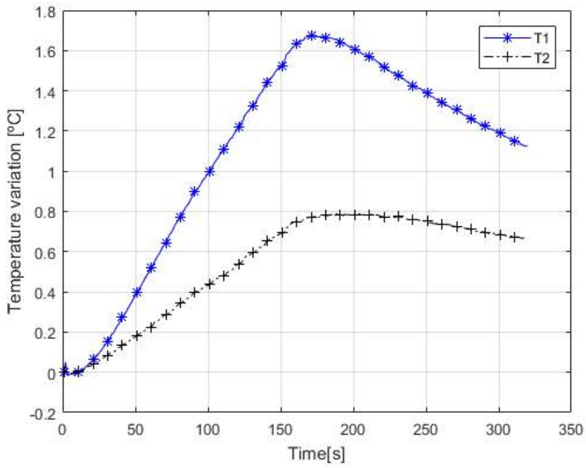
PVC	$x(mm)$	$y(mm)$	$z(mm)$
Thermocouple 1	177	25	150
Thermocouple 2	180	25	150
Transducer center	150	25	150

3. RESULTS AND DISCUSSION

Figure 3 shows heat flux variations and temperatures measured in positions 1 and 2 to 30 independents experiments realizes on PVC sample.



(a)



(b)

Figure 3. Experimental data. a) Heat flux in the PVC sample b) Temperatures in positions 1 and 2.

Temperature variations shows in Figure 3.b were used in equation 7 to estimate thermal conductivity and thermal diffusivity, simultaneously, using GA. As discussed, in multi-objective optimization, has a problem of unique solution existence and local minimums. To solve this problem, a specific search interval was defined and the results were compered with a minimization using extended search interval.

First optimization defined as search interval the polyvinyl chloride (PVC) thermal properties measured by (Maksimov, 2002), where diffusivity and conductivity range are, respectively, $1{,}032-1{,}185*10^{-7}[m^2/s]$ and $0{,}151-0{,}210[\mathrm{W/mK}]$. The estimated values were $1{,}48*10^{-7}[m^2/s]$ and $0{,}159[\mathrm{W/mK}]$, with 95% of confidence interval obtained by 30 independent experiments realized.

Second optimization used as search interval range to thermal diffusivity and conductivity, respectively, $1*10^{-8}-1*10^{-3}\left[m^2/s\right]$ and $0{,}1-500\left[W/\mathrm{mK}\right]$. The estimated values was $1{,}699*10^{-6}[m^2/s]$ and $4{,}01[\mathrm{W/mK}]$ to thermal diffusivity and conductivity, respectively. Results to first optimizations are in accordance with literature (Borges, 2006) $0{,}159[\mathrm{W/mK}]$ and $1{,}157*10^{-7}[m^2/s]$ thermal conductivity and thermal diffusivity respectively.

4. CONCLUSIONS

This work describes an experimental technique for simultaneously determining the thermal diffusivity, and the thermal conductivity, of solid materials using genetic algorithm (GA). Satisfactory results are obtained only when using an interval with an appropriate search interval as this avoids local minimums. When using very extensive search intervals the optimization problem converges to results very different from those defined in the literature.

5. ACKNOWLEDGEMENTS

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