24th COBEM - 2017

24th ABCM International Congress of Mechanical Engineering
December 3-8, 2017, Curitiba, PR, Brazil

COBEM-2017-1380
NON-IDEAL MAGNETOPIEZOELASTIC ENERGY HARVESTING WITH NONLINEAR PIEZOELECTRIC COUPLING

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Abstract. *We examine the behavior of a non-ideal magnetopiezoelastic energy harvesting with nonlinear piezoelectric coupling and driven by a non-ideal energy source. The principal focus in the research is on the effects of synchronization of two cantilever beams and the interaction with the energy source. The forced Duffing equation includes a relative mistuning in the stiffness of the harvesting oscillators. The maximum harvested power was obtained when the angular frequency of the DC motor was approximately with the typical resonance curve. We found a chaotic solution in vicinity of the resonance region of the system operation with the influence of the nonlinear piezoelectric coupling parameter. The maximum dimensionless harvested power over the piezo patches was achieved in a periodic regime.*

Keywords: *Energy harvesting, piezoelectric, chaos, nonlinear vibrations, nonlinear dynamics*

1. INTRODUCTION

This paper presents a numerical study of the nonlinear dynamics of a non-ideal magnetopiezoelastic energy harvesting with nonlinear piezoelectric coupling and driven by a non-ideal energy source (e.g. a DC motor), as shown in Fig. 1. The model considers the mistuning in the stiffness parameter. It should be considered in any realistic system as stated by Litak (2014). The nonlinear piezoelectric coupling is considered in the model like in the works of (Erturk and Inman, 2011; Triplett and Quinn, 2009) and (Iliuk *et al.*, 2013). A investigation of the behavior of the vibrational system considering the influence of the nonlinear piezoelectric coupling term was performed to check the contribution of this parameter in the adjust of the nonlinear piezoelectric response.

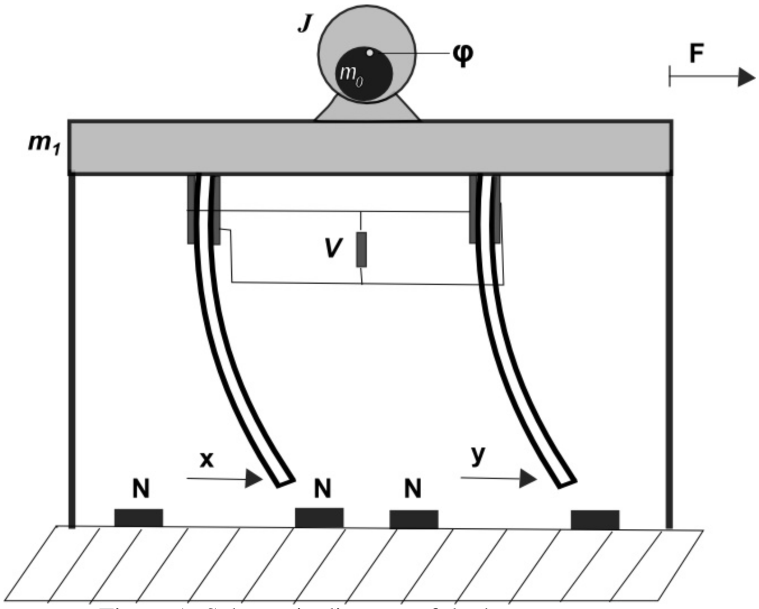


Figure 1: Schematic diagram of the harvester system

2. THEORETICAL PROCEDURE

The mathematical model may be written (Litak, 2014; Erturk and Inman, 2011; Triplett and Quinn, 2009) and (Iliuk *et al.*, 2013) as the following dimensionless equations:

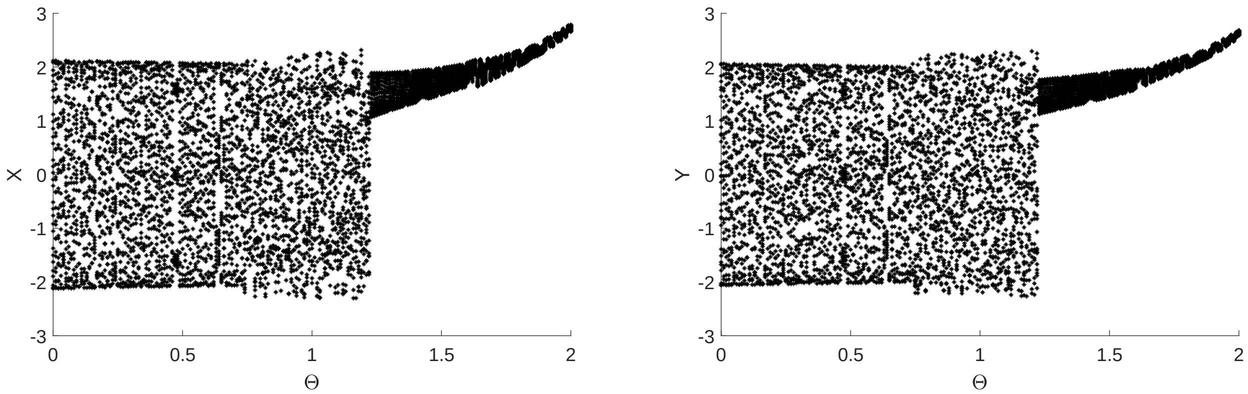
$$\begin{aligned} x'' + 2\alpha x' - \tfrac{1}{2}x(1-x^2) - \theta(1+\Theta|x|)v &= \delta_1\varphi''\sin\varphi + \delta_1\varphi'^2\cos\varphi \\ y'' + 2\alpha y' - \tfrac{1}{2}\epsilon y(1-y^2) - \theta(1+\Theta|y|)v &= \delta_1\varphi''\sin\varphi + \delta_1\varphi'^2\cos\varphi \\ \varphi'' &= \rho_1\cos\varphi\bullet'' - \rho_3\varphi' + \rho_2 \\ v' &= (\theta(1+\Theta|x|)x + \theta(1+\Theta|y|)y - v)/\rho \end{aligned} \tag{1}$$

where the $\bullet$ is replaced by x or y to coupling the energy source equation, x and y are the dimensionless transverse displacements of the beam tips, v is the dimensionless voltage across the load resistor, $\theta(1+\Theta|x|)$ and $\theta(1+\Theta|y|)$ are the dimensionless nonlinear piezoelectric coupling term in Eq. (1), ρ is the reciprocal of the dimensionless time constant of the electrical circuit, R is the load resistance. Finally, ϵ is the stiffness mistuning parameter which should be considered in any realistic system (Litak, 2014), and the excitation following is the linear function $-\rho_3\varphi' + \rho_2$. A series of simulations, using Runge-Kutta's 4th order integrator, in Matlab environment, were performed considering the initital conditions [x, x', y, y', ϕ, ϕ', v] = [0, 0, 0, 0, 0, 0, 0] and the numerical values of the parameters: $\alpha = 0.01$, $\delta_1 = 0.40$, $\epsilon = 1.1$, $\rho = 1.0$, $\rho_1 = 0.60$, $\rho_2 = 1.5$, $\rho_3 = 1.5$, $\theta = 0.2$.

3. RESULTS AND DISCUSSION

The results of the dynamical behavior of the Eq. (1) are presented in this section.

Considering the hypotheses of the influence of the nonlinear piezoelectric coupling term Θ in works of (Triplett and Quinn, 2009) and (Iliuk *et al.*, 2013), an investigation was performed to check the contribution of this parameter in the adjust of the nonlinear piezoelectric response. The values of this parameter were chosen looking to the bifurcation diagrams to both beams presented in the Fig. 2a and Fig. 2b.



(a) Bifurcation of beam X

(b) Bifurcation of beam Y with mistuning parameter ϵ=1.1

Figure 2: Bifurcation diagrams of the x and y beams against the parameter Θ

As can be seen in both bifurcations in the Fig. 2a and Fig. 2b, the system presents a periodic behavior in the vicinity of $\Theta \simeq 0.67$ and a irregular behavior closeness to $\Theta \simeq 1.0$. To validate the choose of this values the average harvester power was calculated to increments of the parameter Θ. The result was depicted in the Fig. 3 were is clear that the picked values represents two maxima points in the graphic.

The values assigned to Θ, were: $\Theta = 0.67$ and $\Theta = 1.0$ respectively.

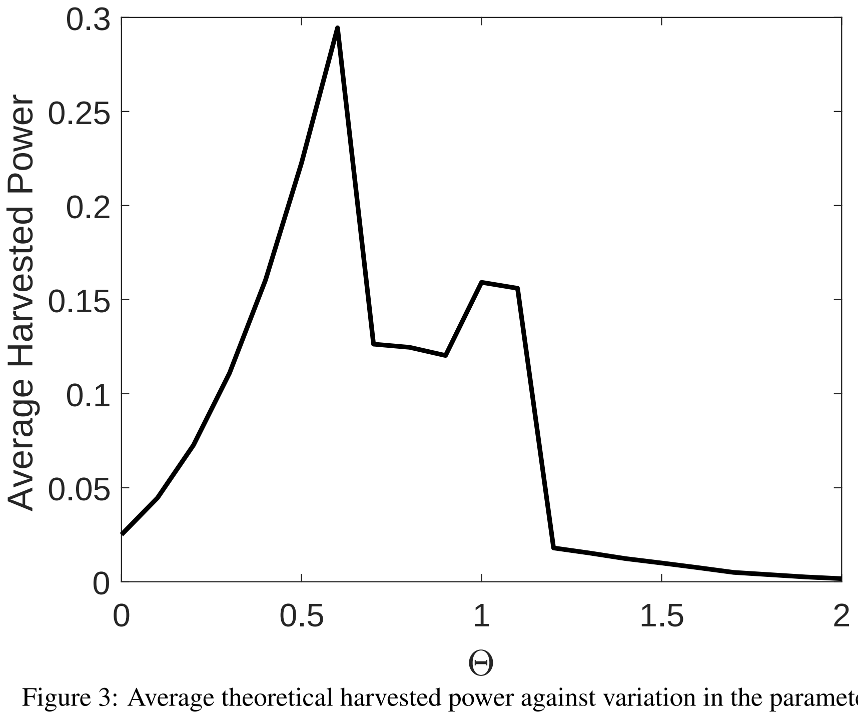


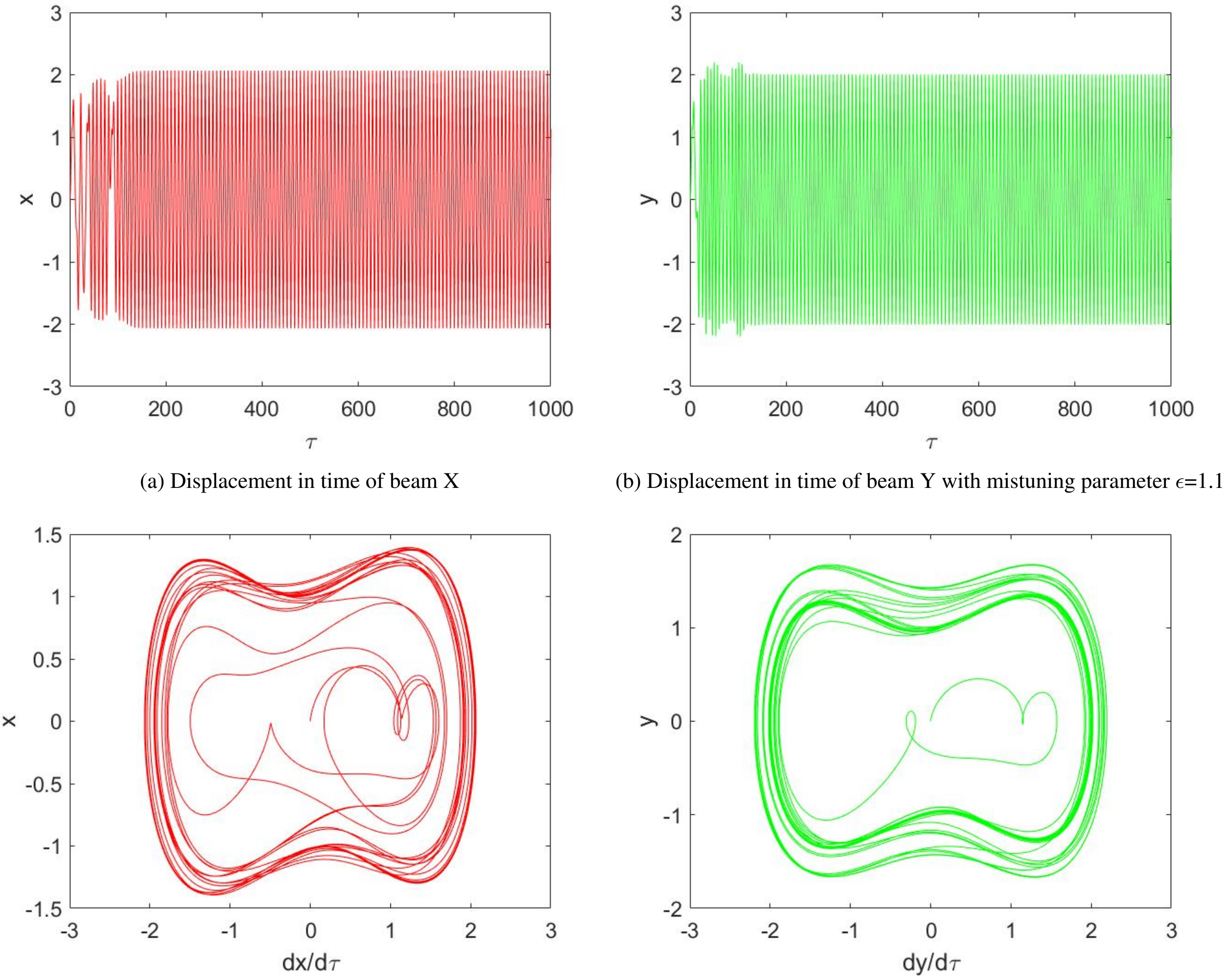
Figure 3: Average theoretical harvested power against variation in the parameter Θ

In order to characterize quantitatively the attractors of the system the 0-1 test proposed by (Gottwald and Melbourne, 2004) was computed. The 0-1 test is applied mainly to characterize the attractor of the dynamical system that presents difficulties in the Lyapunov exponents technique. A concise description about theory and application of the 0-1 test is found in (Bernardini and Litak, 2016).

The 0-1 test receives a time series as input and returns a single scalar value. A value approximately equals to 0 is returned for periodic attractors and 1 for chaotic attractors.

The Fig. **??** show the displacement of two cantilever beams along the time with average angular velocity of the DC motor close to the resonance frequency of the model Fig. 4a and Fig. 4b. The phase portrait for the dissipation force is depicted int he Fig. 4c and Fig. 4d respectively.

A periodic attractor can be seen with the nonlinear piezolectric coupling parameter $\Theta = 0.67$. The values of the 0-1 test for the beam x and y were 0.04366 and 0.0295 respectively, characterizing the periodic attractors.



(a) Displacement in time of beam X

(b) Displacement in time of beam Y with mistuning parameter ϵ=1.1

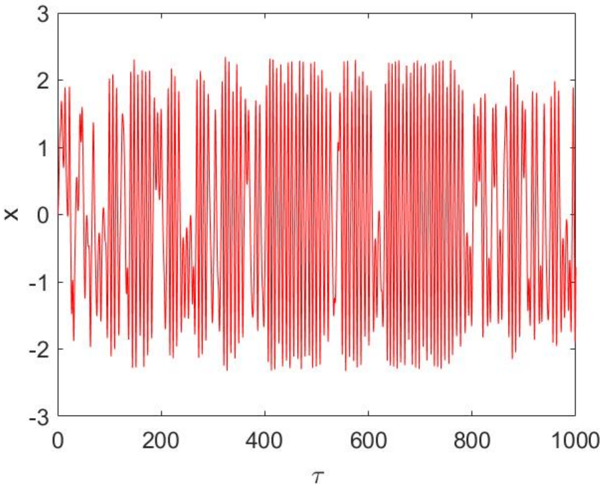
(c) Phase portrait beam X

(d) Phase portrait beam Y

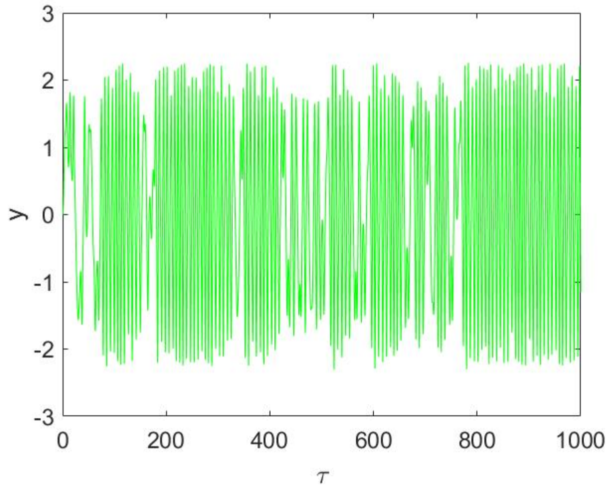
Figure 4: Results of the dynamical behavior of the system with $\Theta = 0.67$

The Fig. **??** show the displacement of two cantilever beams along the time with average angular velocity of the DC motor close to the resonance frequency of the model $\rho_2 \approx \rho_3$.

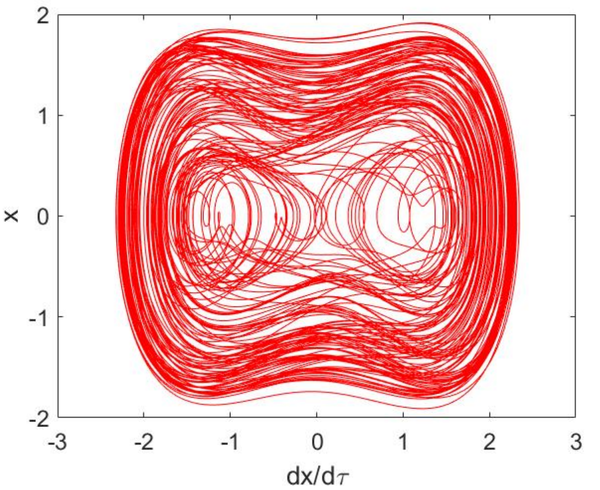
A chaotic attractor was found with the nonlinear piezolectric coupling parameter $\Theta = 1.0$ Fig. 5c and Fig. 5d. The values of the 0-1 test for the beam x and y were 0.9125 and 0.9508 respectively, characterizing the chaos of the attractors.



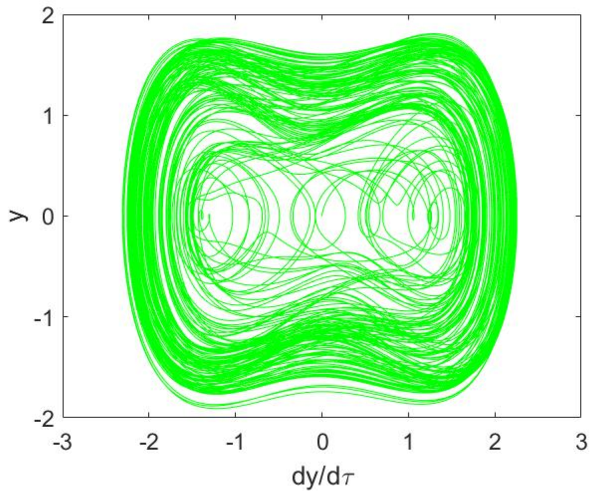
(a) Displacement in time of beam X



(b) Displacement in time of beam Y with mistuning parameter ϵ=1.1



(c) Phase portrait beam X



(d) Phase portrait beam Y

Figure 5: Results of the dynamical behavior of the system with $\Theta = 1.0$

4. CONCLUSIONS

This paper shows the investigation of a non-ideal magnetopiezoelastic energy harvesting with nonlinear piezoelectric coupling and driven by a non-ideal energy source. The model includes a mistuned stiffness parameter. The piezoelectric patches are connected in a parallel way by an electrical circuit. The maximum harvested power was obtained when the angular frequency of the DC motor was approximately with the typical resonance curve $\rho_2 \approx \rho_3$. Due to the mistuning stiffness, the system changes its behavior among synchronized and unsynchronized regime, presenting a strong influence on the harvested power. We found a chaotic solution in vicinity of the resonance region of the system operation with the influence of the nonlinear piezoelectric coupling parameter $\Theta = 1.0$. The maximum dimensionless harvested power over the piezo patches is depicted in the Fig. 3, with the value of the nonlinear piezoelectric coupling term $\Theta = 0.67$ could be a point to setup a passive control strategy like a Nonlinear Energy Sink (NES).

5. ACKNOWLEDGEMENTS

The authors acknowledge support by FAPESP, CNPq and CAPES, all Brazilian research funding agencies.

6. REFERENCES

Bernardini, D. and Litak, G., 2016. "An overview of 0–1 test for chaos". *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, Vol. 38, No. 5, pp. 1433–1450. ISSN 1806-3691. doi:10.1007/s40430-015-0453-y. URL `https://doi.org/10.1007/s40430-015-0453-y`.

Erturk, A. and Inman, D.J., 2011. *Piezoelectric energy harvesting*. John Wiley & Sons Ltd. doi:10.1002/9781119991151.

Gottwald, G.A. and Melbourne, I., 2004. "A new test for chaos in deterministic systems". *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, Vol. 460, No. 2042, pp. 603–611. ISSN 1364-5021. doi:10.1098/rspa.2003.1183. URL http://rspa.royalsocietypublishing.org/content/460/2042/603.

Iliuk, I., Balthazar, J.M., Tusset, A.M., Piqueira, J.R., Pontes, B.R., Felix, J.L. and Bueno, A.M., 2013. "A non-ideal portal frame energy harvester controlled using a pendulum". *The European Physical Journal Special Topics*, Vol. 222, No. 7, pp. 1575–1586.

Litak, G., 2014. "Energy harvesting by coupled oscillators forced by excitations with harmonic and random components". *PAMM*, Vol. 14, No. 1, pp. 289–290. ISSN 1617-7061. doi:10.1002/pamm.201410132. URL http://dx.doi.org/10.1002/pamm.201410132.

Triplett, A. and Quinn, D.D., 2009. "The effect of non-linear piezoelectric coupling on vibration-based energy harvesting". *Journal of Intelligent Material Systems and Structures*, Vol. 20, No. 16, pp. 1959–1967. doi:10.1177/1045389X09343218. URL http://jim.sagepub.com/content/20/16/1959.abstract.

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