24th COBEM - 2017

24th ABCM International Congress of Mechanical Engineering
December 3-8, 2017, Curitiba, PR, Brazil

COBEM-2017-0223
THREE-DIMENSIONAL STATIC BEHAVIOR OF RISERS SUBMITTED TO TRACTION AND TORSIONAL LOADS AT THE TOP

Santiago Jordan Granados Sánchez[1]
Juan Pablo Julca Avila[2]
Centro de Engenharia, Modelagem e Ciências Sociais, UFABC, Santo André, SP, Brazil
[1]santiago.granados@ufabc.edu.br, [2] juan.avila@ufabc.edu.br

Abstract. *The purpose of this paper is to present a computational code for three-dimensional static analysis of slender beams subjected to torsional and tensile loads, as the case of risers used as components in system for offshore hydrocarbon production. This kind of analysis involves large geometry changes in the configuration of the riser. For this, an Euler space beam element with small deformations was developed using the corrotational approach. A scheme for the finite rotations treatment was implemented based on pseudovectors and quaternions. The equilibrium configurations were obtained based on two iterative incremental schemes: the Full Newton Raphson Method and the Cylindrical Arc Length Method with load and displacement control. Convergence criteria based on displacements, strain energy and residual force were taken into account. Straight spatial beams with geometric imperfections subjected to combined loading of torsion and compression were simulated in order to obtain satisfactory comparative results. Simulations with imposition of displacements on top of a curved catenary beam were also performed and compared satisfactorily. This computational code is able to construct slender beam equilibrium configurations until the beginning of the stability loss.*

Keywords: *Spatial Beams, Corotational Approach, Finite Rotations, Arc Length Method, Structural Stability.*

1. INTRODUCTION

Different kind of slender structures may be found in oil and gas offshore facilities. One of those structures is the flexible riser. Riser is a tube used for transporting hydrocarbons from the seabed up to sea surface. Along its extension, the riser interacts with different types of loads whose shape its changing geometrical configuration. Although is it desirable to conclude in a dynamic analysis due to the dynamic nature of the acting loads, the author considers a good approach to start with a static analysis. A structural analysis helps to ensure that this kind of structures must be capable of keep working besides of suffering significant changes in its configuration. Because of the significant changes in the configuration, we must consider geometric nonlinearity when working with Finite Element Method. That means that the equilibrium equations must be built from the deformed configuration. The change in the direction of the internal forces are significant to the analysis because they can cause the collapse of the structure. One of the possible acting loads in a riser is the torsion load. The main purpose of this paper is evaluating a slender structure under the effect of torsion loads.

2. MATHEMATICAL MODELING

The Finite Element Method is used in this case to solve the boundary value problem associated to the risers elastic problem. The basic assumption is that the riser undergoes large transverse displacements and rotations while strains remain small and elastic. Based on these assumptions the corotational formulation was implemented to solve the geometrically nonlinear problem.

2.1 Corotational Formulation

The corotational approach in three dimensions was first introduced in the early 80's by authors like Simo &Vu-Quoc (1986) and it was based on the theory of finite rotations studied by Argyris (1982). Recently, corotational formulation has been successfully applied by authors like Neto (2013), Silva (2013), Yaw (2009), Goya (2006) and Crisfield (1998).

In the corotational formulation, beam elements are submitted to translation, rotation and deformations. Translations and rotations are called motions of rigid body. Internal forces depend exclusively on the deformations. In order to separate motions of rigid body and deformations in the element, it is introduced a referential system that accompanies the motions of rigid body while another referential systems accompany the deformational rotations that occur at the ends of the element. The corotational formulation presented in this paper was based on the work developed by Crisfield (1990).

In Figure 1, it is shown the reference system **E** fixed to the beam element. Reference system **E** follows the rotations and translations of the element. In Figure 2, there is a representation of the reference systems **T** and **U**, both are attached to the ends of the beam element. Reference system **E** with unitary vectors $\mathbf{e_1}$**,** $\mathbf{e_2}$ and $\mathbf{e_3}$ is rotated in relation to the global reference system XYZ, as it can be seen in Figure 1. The global coordinates of the node 1 and node 2 define the direction of the unitary vector $\mathbf{e_1}$**.** Meanwhile, vectors $\mathbf{e_2}$ and $\mathbf{e_3}$ are calculated from nodal reference systems **T** and **U**.

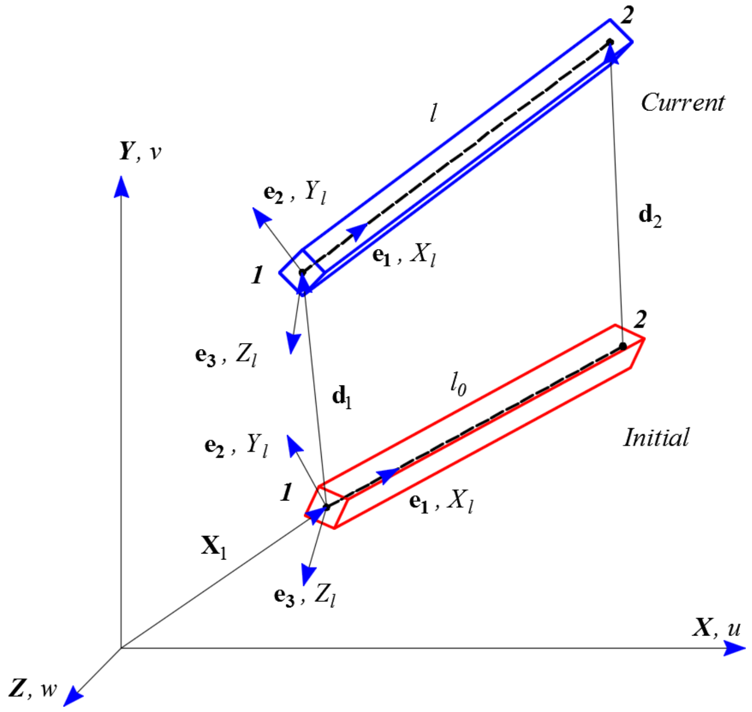

Figure 1: Initial and current configurations of the beam element.

Based on the Figure 2, we could imagine how the triads **T, U** and **E** worked together to characterized the deformational rotations that occur in the beam element. Based on those deformations, the model is able to compute the internal forces for each beam element. One of the disadvantages of this model is that does not take into account the effect of shearing forces due to cross-section rotation as a Geometrically Exact beams do. This model is more similar to a Euler beam in the dimensions.

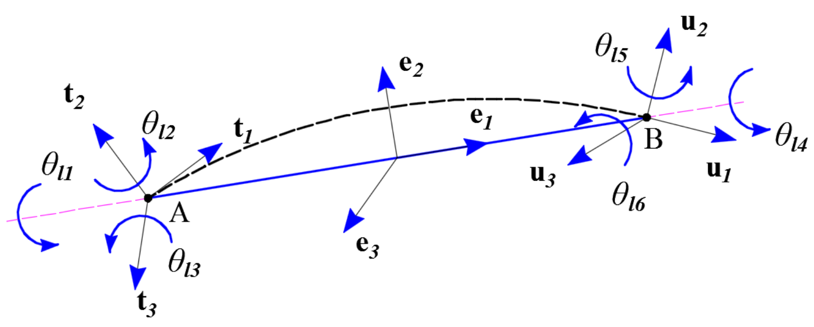

Figure 2: Tridimensional beam's triads.

It is shown in the Figure 3 the internal moments (M_1, M_2, M_3, M_4, M_5 and M_6) and the normal force N in local coordinates. It is also shown the internal moments (M_{x1}, M_{y1}, M_{z1}, M_{x2}, M_{y2} and M_{z2}) and forces (F_{x1}, F_{y1}, F_{z1}, F_{x2}, F_{y2}

and F_{z2}) in global coordinates XYZ. Those internal forces come from the constitutive relations between deformations and internal forces. To compute the internal force vector in global coordinates from the internal force vector in local coordinates, the corotational formulation uses the geometrical relations due to rotations between the reference systems **E**, **T** and **U** in relation to the global reference system XYZ. Those rotations may be characterized by pseudovectors and Rodrigues equation (Argyris, 1982). In Figure 3, the pseudovector **α** characterized the accumulated rotation between the triad **T** and the global reference system and the pseudovector **β** characterized the accumulated rotation between the triad **U** and the global reference system.

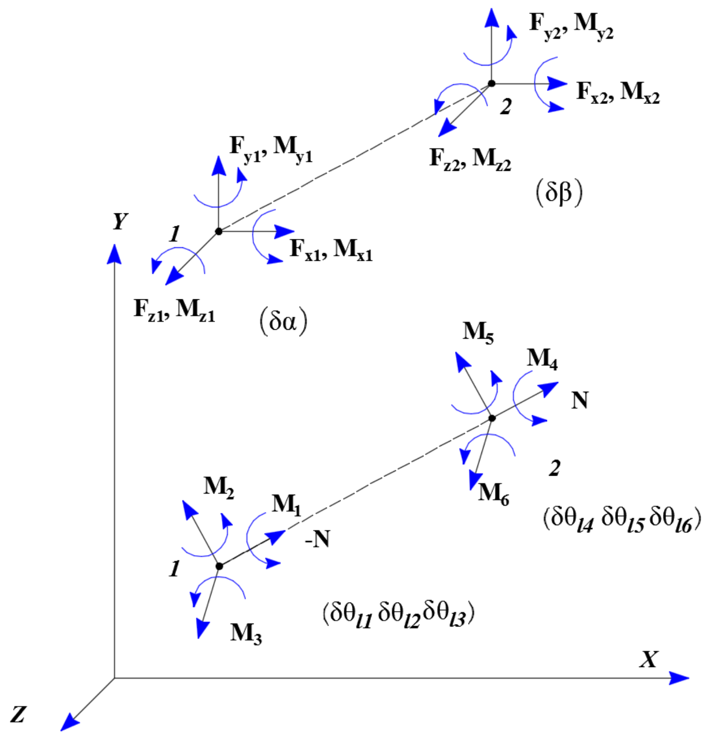

Figure 3: Components of internal forces and rotations in the global and local coordinate systems.

2.2 Numerical Methods

Nonlinear programming techniques are used with a finite element discretization of the flexible beam in order to obtain the displaced shapes corresponding to the stable equilibrium positions of the system. The incremental finite element coefficient matrices are solved using a Full Newton Raphson Method and Arc-Length Method algorithm (Crisfield, 1998). Those methods try to solve the equilibrium presented in Equation 1.

$$\mathbf{g}_i = \mathbf{f}_i(\mathbf{p}) - \mathbf{q} = \mathbf{0} \tag{1}$$

where **g** are the residual forces, **f** are the internal forces and **q** are the external forces. The internal forces **f** depend on the displacements **p**.

3. STATIC ANALYSIS

In order to validate the code, it was tested with classical applications of combined loads. The first application consist on a straight beam with initial imperfection submitted to shortening and then torsion as presented by Miyazaki (1997). It is a beam fixed in one end while the other end is loaded sequentially to compression and then to torsion. The simulation characteristics and shown in the Table 1.

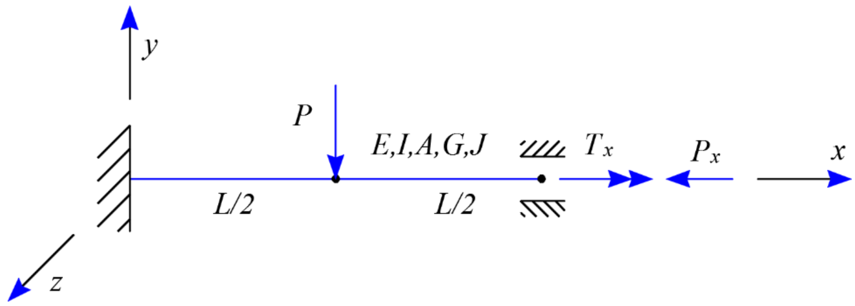

Figure 4**:** Straight beam with initial imperfection submitted to shortening and then torsion. Miyasaki (1997).

Table 1: Simulation parameters of the straight beam.

Parameter	Value
Length, *L* (m)	400
External diameter, D (m)	0.4
Axial stiffness, *EA* (N)	7000000
Bending stiffness, *EI* (N.m^2)	120000
Torsional stiffness, *GJ* (N.m^2)	100000
Shear stiffness, *GA* (N.m^2)	2000000

The results shown in Figure 5 and Figure 6 exemplify how the beam changes its configuration. In Figure 7, it is shown the equilibrium path for this application. It is shown how it is related the applied torsion and the torsion angle in the end of the beam. The aim of representing the phenomena was successfully achieved in this application.

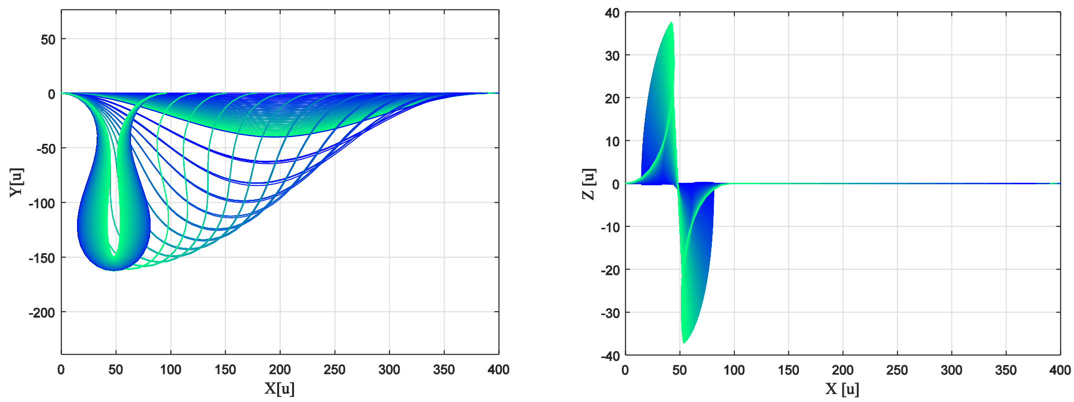

Figure 5: Equilibrium configurations for the beam submitted to torsion at the end. View of plane XY and plane XZ.

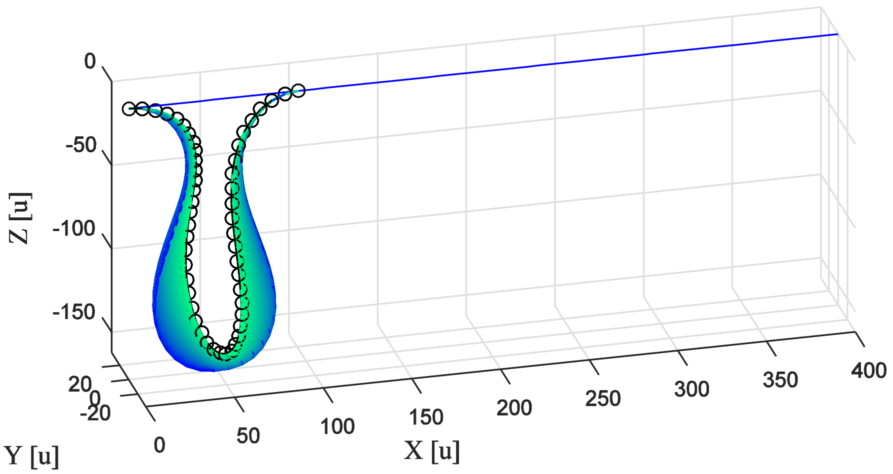

Figure 6: Equilibrium configurations for the beam submitted to torsion at the end. Tridimensional view.

In Figure 7, it is represented the nonlinear relation between the torsion load and the torsion angle of the end of the beam. At the top of the curve, the application present snap-through and there was the necessity for controlling force and displacement at the same time with a method like Arc-Length Method.

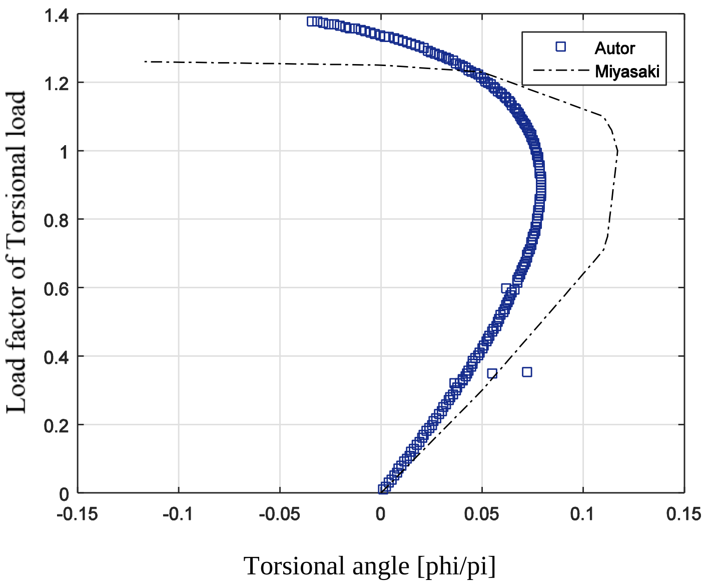

Figure 7: Equilibrium path for the shortened beam submitted to torsion.

It was studied the behavior of a flexible riser in catenary configuration under effect of torsional load. The strategy followed for simulating this case was the following: 1) It was simulated the riser as a horizontal beam under the effect of its own weight and supported on the ends which are free to rotate but no displacement is permitted. 2) It was imposed a displacement of 150m in the vertical and 150 in the horizontal to one free end of the beam while the other end remains free to rotate but not to displace. With this step, the catenary configuration is obtained. 3) Finally, a torsional load is applied to its free end while the other end is fixed. The catenary configuration is shown in Figure 8 and the simulation parameters are shown in the Table 2.

Table 2: Simulation parameters of the flexible riser.

Parameter	Value
Total length, L (m)	350
External diameter (m)	0.26
Internal diameter (m)	0.20
Bending stiffness EI (Nm^2)	20.96 . 10^3
Axial stiffness EA (N)	15.38 . 10^8
Aparent weight of riser (N/m)	346.11

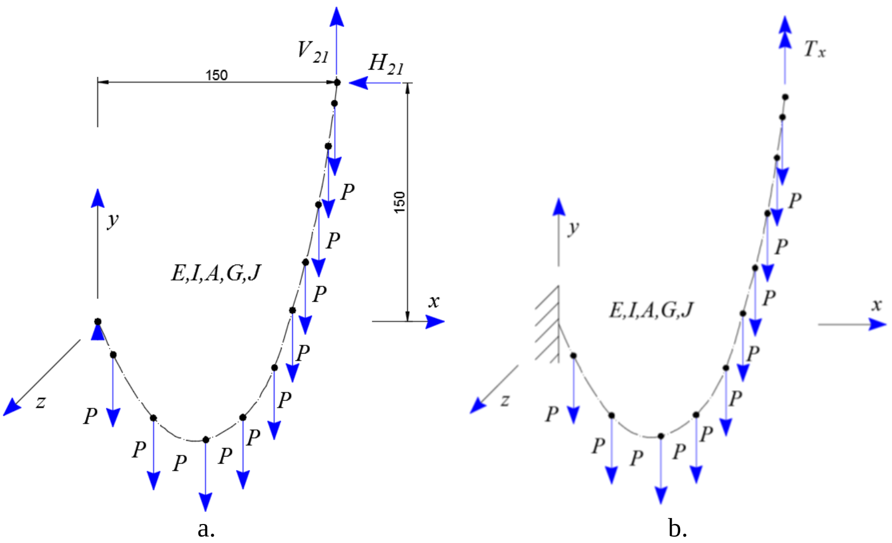

Figure 8: Riser in catenary configuration submitted to torsion. a) Riser after the imposition of displacements. b) Riser at the begging of the torsion load application.

In Figure 9, we can see how the riser changes its configuration before losing stability. So far, it is difficult to represent the loss of stability, but it was possible to capture how the riser forms a hockling.

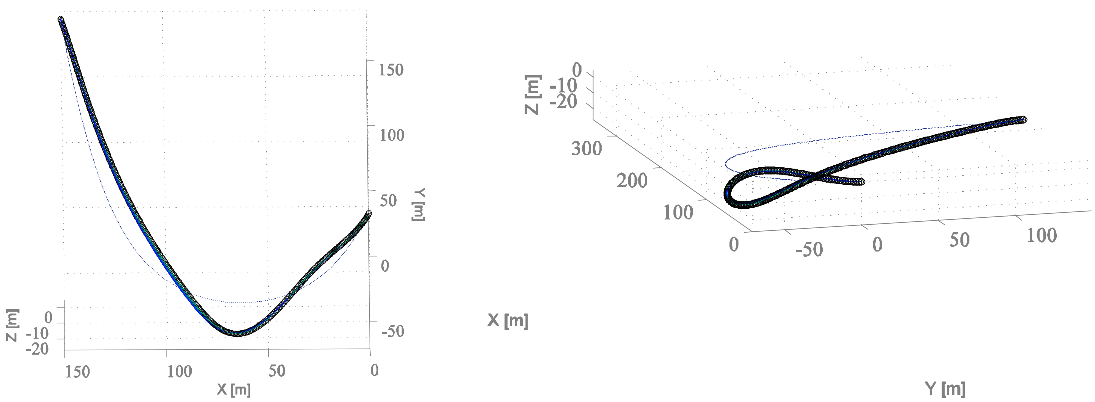

Figure 9: Equilibrium configuration of the catenary riser submitted to twist at the top.

4. FINDINGS

Although it was possible to simulate the classic example of Miyasaki (1997), there is still a work to understand better nonlinear solution methods. Convergence is a key issue to pay attention in this kind of analysis and scaling from the Newton Raphson Method to Arc-length Method presented some challenges in the treatment of the rotation of the reference systems. To reach convergence, the external load was applied with small increments.

5. CONCLUSIONS

It was implemented a computational code for evaluating the tridimensional static behavior of beams submitted to combined loads using corotational approach. This was tested with problems that consider the effects of geometric nonlinearity with large translations and rotations. Rotations were described using pseudovectors. The corotational formulation in three dimensions was characterized using additional reference systems along the beam. With these reference systems was possible to separate the motion of rigid body and deformations. A local reference system accompanies the rotation of the rigid body while nodal reference systems accompanies the rotation of each end of the beam. The use of the Rodrigues formulation for pseudovectors brought imprecisions with the computation of the rotation of reference systems. The general recommendation was to proceed with small increments in order to reach convergence in a few iterations. For validating this code was used a classical example of Miyasaki and it was seen the necessity of having control of load and displacements, so it was implemented the Arc-Length Method. In the case of the analysis of the flexible riser, the author found the necessity to conduct a deep study of the effect of the convergence criteria and it was possible to study the behavior of riser when it suffer large changes in its configuration. A downside was the computational cost for this kind of applications.

6. ACKNOWLEDGEMENTS

We acknowledge Centro de Engenharia, Modelagem e Ciências Sociais da Universidade Federal do ABC (CECS-UFABC) for support of this work.

7. REFERENCES

Argyris, J., 1982. “An excursion into large rotations”. *Computer Methods in Applied Mechanics and Engineering*, Volume 32, Issues 1–3, Pages 85-155.

Crisfield, M. A., 1998. “Consistent Co-Rotational Formulation for Non-Linear, Three Dimensional, Beam Elements”, *Computer Methods in Applied Mechanics and Engineering*, 81, 131-150.

Crisfield, M. A., 1998. *Nonlinear Finite Element Analysis of Solids and Structures*, John Wiley & and Sons, Chichester, Vol. 2.

Goyal, S., 2006. A dynamic rod model to simulate mechanics of cables and DNA, Ph.D. Dissertation in Mechanical Engineering, University of Michigan, Ann Arbor, 2006.

Miyasaki, Y. and Kondo, K., 1997. Analytical solution of spatial elastica and its application to kinking problem, *International Journal of Solids and Structures*, Volume 34, Issue 27, Pages 3619-3636.

Neto, A. and Martins, C. A., 2013. “Structural stability of flexible lines in catenary configuration under torsion”, Marine Structures, Volume 34, Pages 16–40.

Silva, W. A, 2013. “Análise dinâmica não-linear de pórticos espaciais utilizando a formulação corrotacional”, Ph.D. Dissertation - Universidade de Brasília, Brasília.

Simo, J. C. and Vu-Quoc, L., 1986. “A Three-dimensional Finite-Straing Rod Model. Part II: Computational Aspects”, *Computer Methods in Applied Mechanics and Engineering*, Vol. 58, 79-116.

Yaw L.L., 2009. “2D Corotational Beam Formulation”, Walla Walla University.

8. RESPONSIBILITY NOTICE

The author is the only responsible for the printed material included in this paper.