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STRUCTURAL ANALYSIS OF BEAMS UNDER ALTERNATING AXIAL LOADING

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Abstract. Amusement rides and roller coasters are the main attractions of amusement parks and fairs worldwide. In 2012, Walt Disney's theme parks received over 126 million people. Accidents in these kinds of attractions are unusual, however get a lot of attention from the media and society when they happen. The use of redundant safety systems and computational analysis have become increasingly frequent, reducing failure risks and allowing designers to develop more thrilling attractions. It is appropriate, therefore, to study the impact of possible buckling failures that can occur in these devices, as well as possible solutions to bypass them. The object of this research is to calculate, by means of analytical equations available in literature and finite element analysis, the effective buckling length coefficient for a fixed end and a pinned end column, and compare them to the expected values given by differential equation solutions.

Keywords: Entertainment equipment, Buckling, Finite element analysis.

1. INTRODUCTION

Amusement parks industry in Brazil attracts over 20 million people annually and invoices R\$ 1 billion. In 2012, North America industry's growth was 7.0%, while Brazil had the largest increase in the number of visitors of a single park in the world: Beto Carrero World park has received 1.5 million visitors, growth of 42.9% compared to the previous year, according to the Theme Index Global Attraction Presence Report 2012 (AECOM, 2013). The main amusement parks in Latin America are represented in Fig. 1.



Figure 1. Amusement parks in Latin America (AECOM, 2013)

The object of this work is to investigate, through analytical equations available in literature and finite element analysis, how different types of end connections and geometries of an amusement ride's support structure can contribute for their buckling. In addition, it is intended to compare the effective buckling length coefficient for a fixed end and a pinned end column to the expected values given by differential equation solutions.

2. EQUIPMENT CONFIGURATION

Popularly known as Frisbee in Europe and the United States, this amusement park ride consists of seats fixed to a ring, which is linked to a beam. This beam freely balances as a pendulum, powered by electric or hydraulic motors, with total angle amplitude of 240 degrees. The pendulum is supported by four inclined columns.

The columns may have circular, square, rectangular or prismatic cross sections, and the area moment of inertia may or not vary along the column length. In general, when columns are fixed (and not pinned) to the floor, the supports have circular and constant cross section along the length. Different cross sections are shown in Fig. 2.

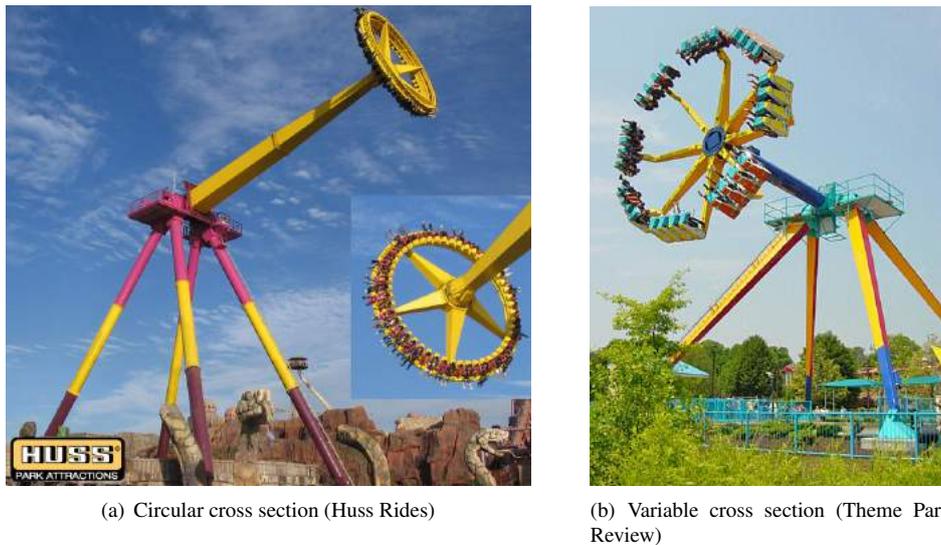


Figure 2. Different cross section configurations

There are versions of these rides whose columns' ends are fixed to the floor, as shown in Fig. 3(a). Other rides have free rotation columns' ends, as shown in Fig. 3(b).



Figure 3. Different columns' ends configurations (Luna Rides)

3. THEORY INTRODUCTION

3.1 Pendular dynamics

The centripetal force F_c caused by the normal acceleration a_n when the ring's mass m travels the circular path at high speed can be calculated by Newton's second law:

$$F_c = m \cdot a_n \tag{1}$$

Centripetal acceleration can be calculated by Eq. 2.

$$a_n = v_\alpha^2 / r \quad (2)$$

v_α corresponds to the speed of the rotating set for each angle α , from 120 to 0 degrees:

$$v_\alpha = \sqrt{2 \cdot g \cdot \Delta h} \quad (3)$$

Δh is the drop height from maximum ring's height to the current ring's α position, as show in Fig. 4.

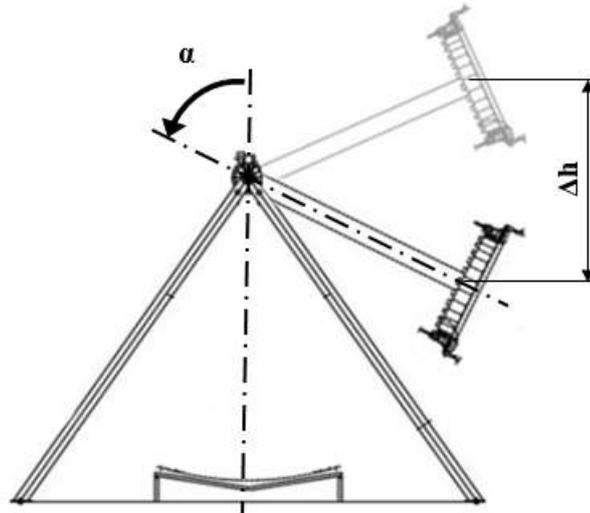


Figure 4. Drop height Δh

3.2 Buckling

Buckling can be defined as the sudden deformation of a structure due to a small increase in the compression load under which the structure exhibited little, if any, deformation before the load was increased. If the compression load is removed, the column returns to its original straight format. The structural failure of buckling is not, therefore, a failure of the material, but rather of stability (Philpot, 2011).

The differential equation of the axis of a beam with only axial load P is given by Eq 4 (Timoshenko, 1961):

$$\frac{d^4 w}{dx^4} + \frac{P}{E \cdot I} \cdot \frac{d^2 w}{dx^2} = 0 \quad (4)$$

Where $w(x)$ is the lateral displacement along the length of the beam, E is the modulus of elasticity of material and I is the area moment of inertia of the cross section. This homogeneous fourth-order differential equation has a general solution given by Eq. 5.

$$w(x) = A \sin(\lambda x) + B \cos(\lambda x) + Cx + D \quad (5)$$

The four constants A, B, C, D are determined by the boundary conditions (the beam's constraints at each end). For a pinned-pinned column, with no displacement and moment at $x = 0$, the solution of Eq. 4 is Euler formula in Eq. 6.

$$P_{cr} = \frac{\pi^2 \cdot E \cdot I}{L^2} \quad (6)$$

To consider the effect of different boundary conditions, a more general equation is proposed by Euler (Eq. 7).

$$P_{cr} = \frac{\pi^2 \cdot E \cdot I}{(K \cdot L)^2} \quad (7)$$

K is the column effective length factor, as shown in Fig. 5. L is unsupported length of column and I is the minimum area moment of inertia.

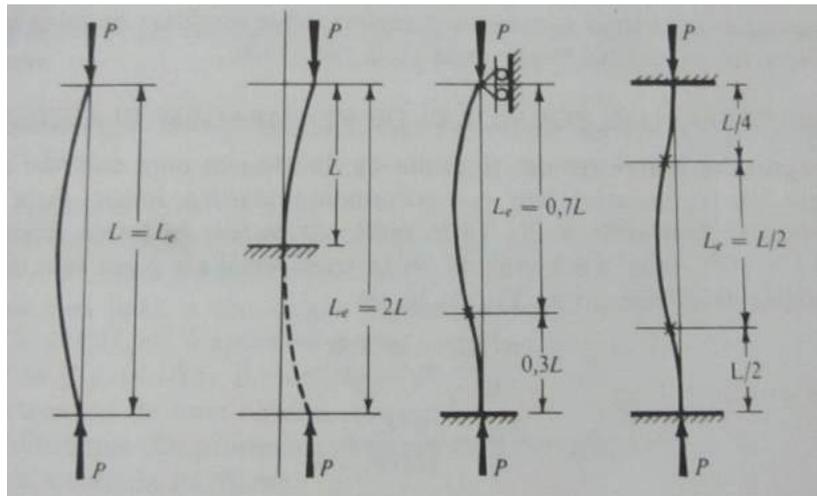


Figure 5. Buckling effective length coefficient for different end conditions (Popov - 1978)

3.3 Computer Aided Engineering

Computer Aided Engineering (CAE) is a tool that uses the computer to support engineering by assisting it in the development of projects through predefined analyzes, which includes among other Finite Element Analysis (FEA) and Multibody Dynamics (MBD) (McGraw-Hill Concise Encyclopedia of Engineering, 2002). The main purpose of computational engineering is to obtain information about the response of physical systems to the conditions imposed, usually called loading. This information is subsequently used to justify or make engineering decisions (Szabo and Babuska, 1991).

Through the Finite Element Method (MEF) it is possible to determine tensions, deformations, heat transfer, fluid flows and buckling loads for real problems. Theoretically, it is possible to calculate as many modes of buckling (or critical buckling loads) as there are degrees of freedom in the FEA model. In most of the analysis, however, only the first mode is calculated. This is because higher buckling modes can not occur - buckling most often causes a catastrophic failure or makes the structure unusable (Kurowski, 2011).

Multibody system technique solve kinematics, static, quasi-static and dynamic equations of systems with multiple bodies and many degrees of freedom. Using the multibody methodology, the dynamic solution of systems with large displacements, nonlinear links and shock can be obtained.

4. BUCKLING ANALYSIS

The model proposed for solving the buckling analysis is shown in Fig. 6.

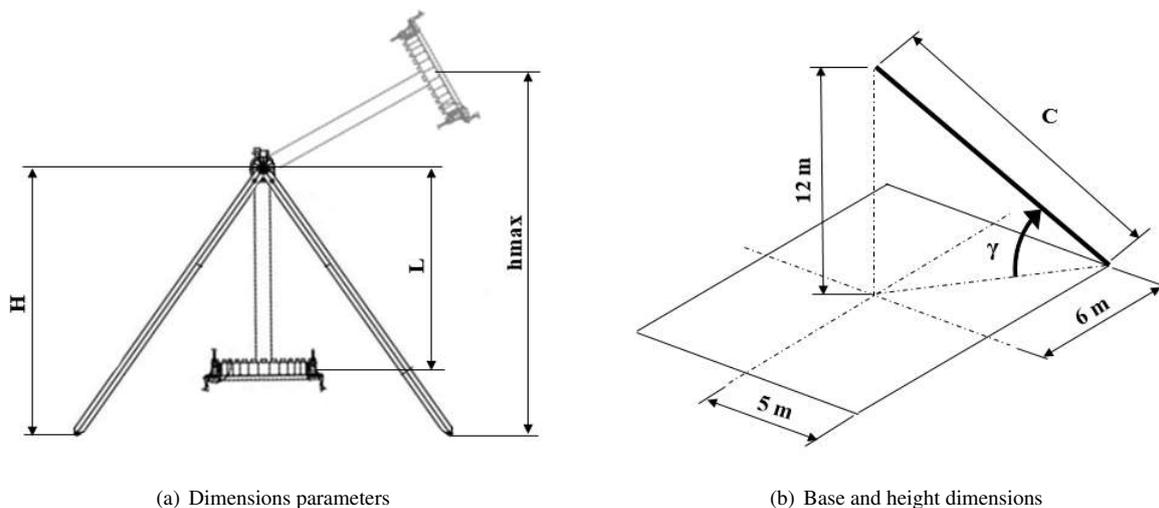


Figure 6. Model dimensions

The data is presented in Tab. 1.

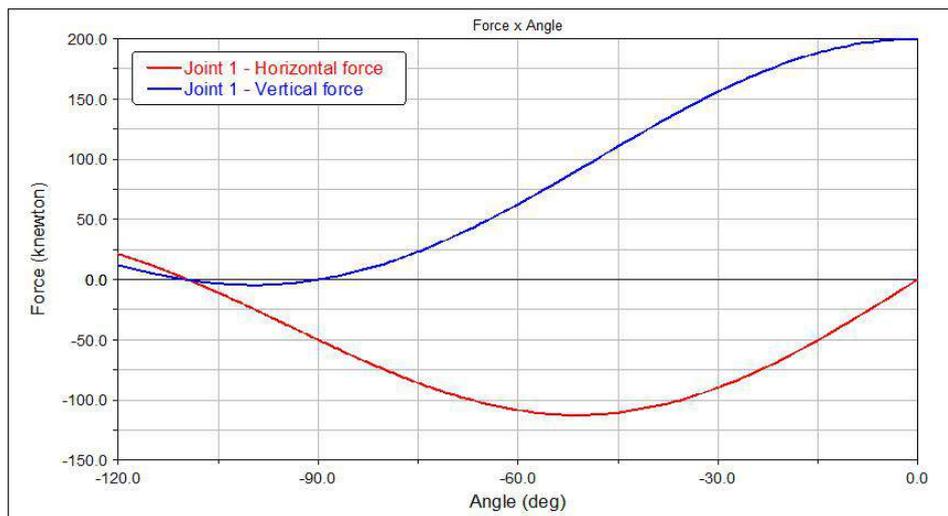
Table 1. Model data

Item	Symbol	Value
Riders number	n	12
Mass of rotating set with riders	m	5000 kg
Length of rotating beam	L	10 m
Length of column	C	14.320 m
Maximum balance height	h_{max}	17 m
Rotation axis height	H	12 m
Angle of inclination of columns	γ	57 degrees
Earth's gravity	a_g	10 m/s^2
Steel modulus of elasticity	E	200000 MPa

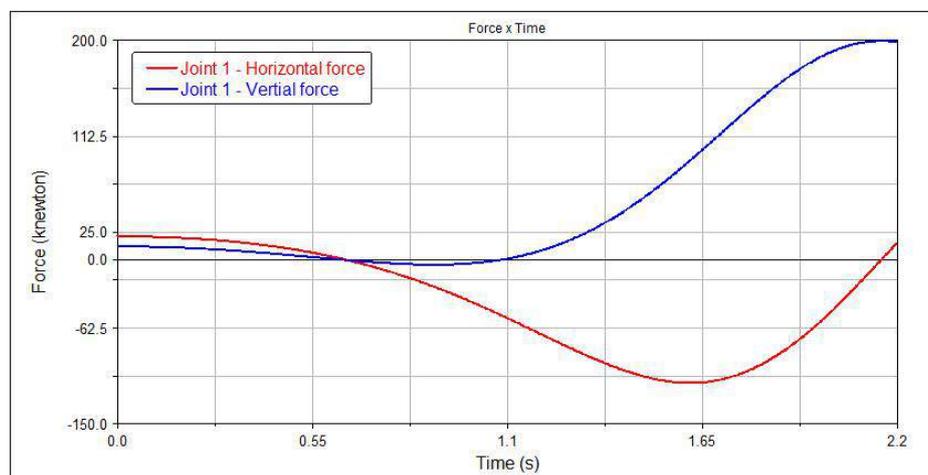
4.1 Centripetal force calculation

For each angle α , from 120 to 0 degrees, Δh can be calculated. For $\alpha = 0$, $\Delta h = 15m$. Using equation Eq. 3 and Eq. 2 it is possible to calculate the normal acceleration for $\alpha = 0$. Earth's gravity acceleration must be added to centripetal acceleration by vector sum in order to get the total force acting on axle joint.

Using Adams MBD software it is possible to get the centripetal force as a function of α and also as a function of time. Fig. 7 shows these functions.



(a) Force as a function of angle α



(b) Force as a function of time

Figure 7. Horizontal and vertical projections of centripetal force

4.2 Case 1 - Constant cross section with fixed end

In the first case of analysis, it is used a constant cross section column with fixed ends (no rotation allowed). The column data for case 1 is shown in Tab. 2.

Table 2. Column data for Case 1

Item	Symbol	Value
Width of column	a	250 mm
Depth of column	b	250 mm
Wall thickness	e	5 mm
Area moment of inertia	I	49040833 mm^4

Using Eq. 7 it is possible to calculate the effective length coefficient (Eq. 8). The critical load used in this equation will be calculated using finite element analysis.

$$K = \sqrt{\frac{\pi^2 \cdot 200000 \cdot 49040833}{P_{cr} \cdot 14320^2}} \quad (8)$$

4.3 Case 2 - Variable cross section with pinned end

In the second case of analysis, it is used a variable cross section column with pinned ends (rotation allowed). The dimensions of columns ends are chosen in order to keep the same mass of the column used in Case 1. The column data for Case 2 is shown in Tab. 3.

Table 3. Column data for Case 2

Item	Symbol	Value
Width of column (pinned end)	a	170 mm
Depth of column (pinned end)	b	170 mm
Width of column (superior end)	c	330 mm
Depth of column (superior end)	d	330 mm
Wall thickness	e	5 mm
Minimum area moment of inertia	I	14987500 mm^4

Using Eq. 7 in order to find the effective length coefficient leads to Eq. 9.

$$K = \sqrt{\frac{\pi^2 \cdot 200000 \cdot 14987500}{P_{cr} \cdot 14320^2}} \quad (9)$$

5. FINITE ELEMENT ANALYSIS

Finite element analysis were conducted using Ansys Workbench 18.1. Eigenvalue buckling analysis is done based on the solution of a static structural analysis, as seen if Fig. 8. The result is a factor, which multiplied by the force applied on the structure gives the force which will make the structure buckle.

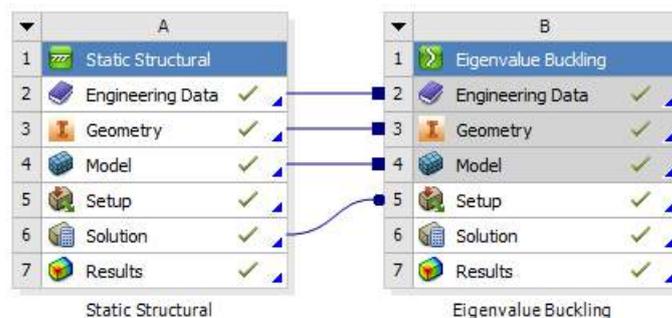


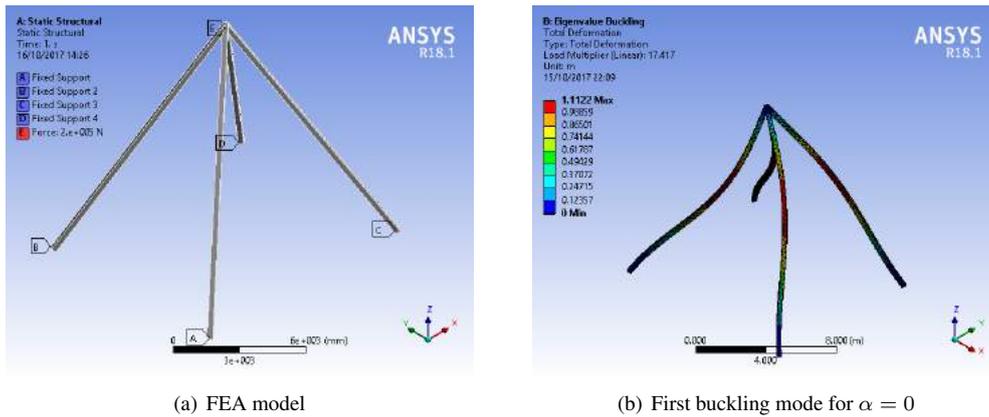
Figure 8. Static structural and buckling analysis on Ansys Workbench 18.1

Simulations were performed with shell-like triangle elements with sides up to 0.3 m and thickness of 5 mm. Shell elements can save a great deal of simulation time, since a model of thin plates has much less elements than a solid model.

Shell meshes are also simpler to perform and are less prone to negative Jacobian errors that can occur when using extremely thin solid elements. Low order, Lagrangian-based, finite elements are often affected by spurious strains or stresses which lead to an overestimation of the stiffness matrix. As a consequence, an underestimation of the nodal variables appears, this behavior being referred to as locking (Caseiro et al, 2014).

5.1 Case 1 - Constant cross section with fixed end

Fig. 9(a) shows first case model. The columns' ends were fixed with no rotation allowed. The first buckling mode for $\alpha = 0$ is shown in Fig. 9(b).



(a) FEA model (b) First buckling mode for $\alpha = 0$
 Figure 9. Case 1 finite element model and results for $\alpha = 0$

The forces of columns under compression were calculated. Multiplying them by the load factor leads to the forces that buckle these columns, as shown in Fig. 10. Replacing each of these results into Eq. 8 leads to graphic showed in Fig. 11.

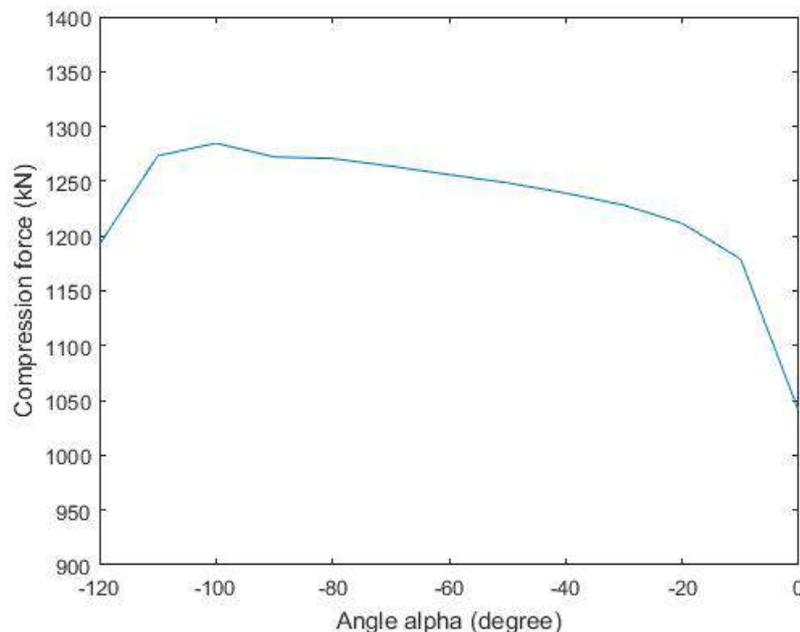


Figure 10. Critical load for each column under compression for Case 1

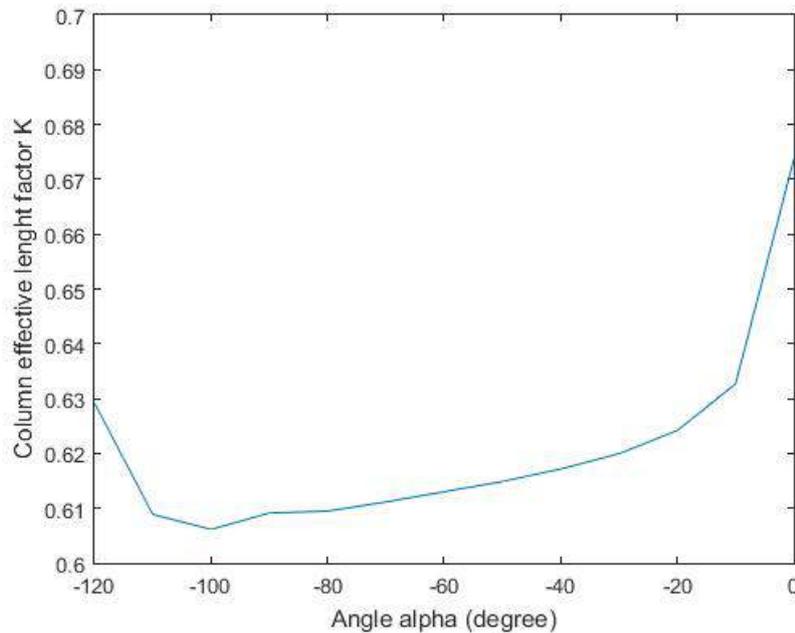


Figure 11. Effective length coefficient \times angle α for Case 1

The average effective length coefficient for Case 1 is 0.6208.

5.2 Case 2 - Variable cross section with pinned end

Second case model is presented in Fig. 12(a). The columns' ends were pinned, with rotation allowed. The first buckling mode for $\alpha = 0$ is shown in Fig. 12(b).

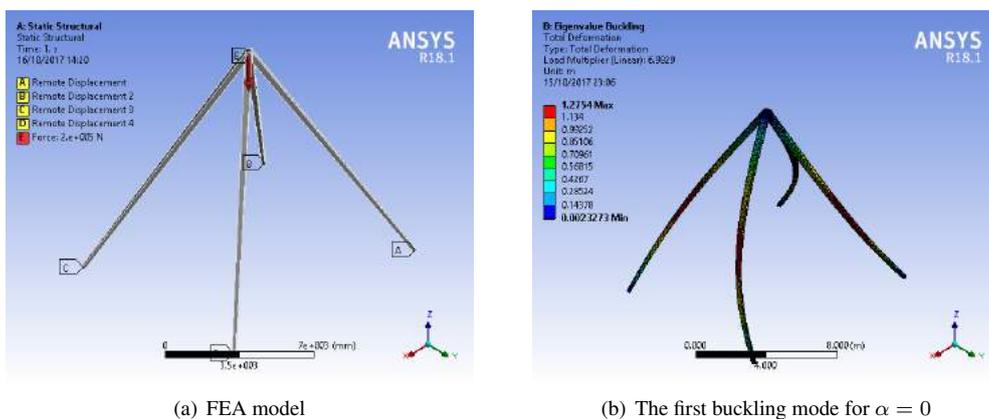


Figure 12. Case 2 finite element model and results for $\alpha = 0$

The forces of the columns under compression were calculated. Multiplying them by the load factor leads to the forces that buckle these columns, as shown in Fig. 13. Replacing each of these results into Eq. 9 leads to graphic showed in Fig. 14.

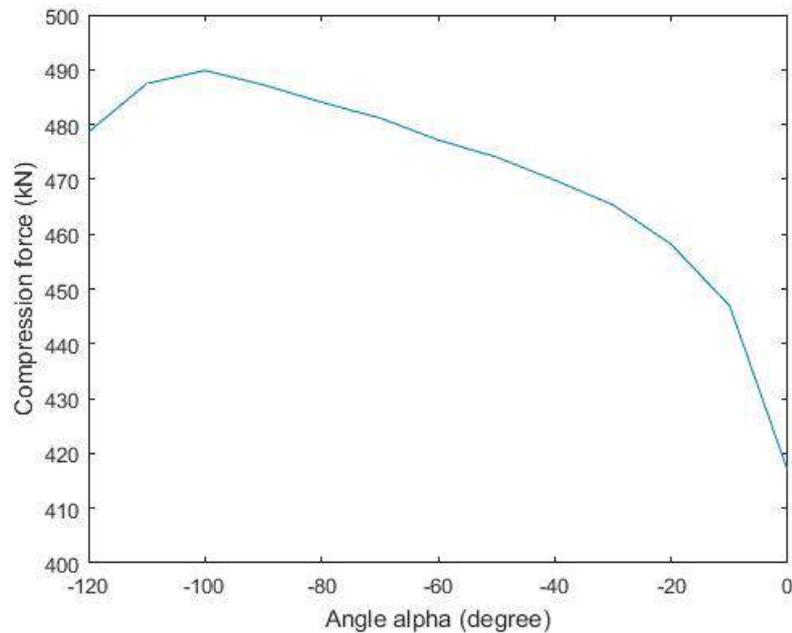


Figure 13. Critical load for each column under compression for Case 2

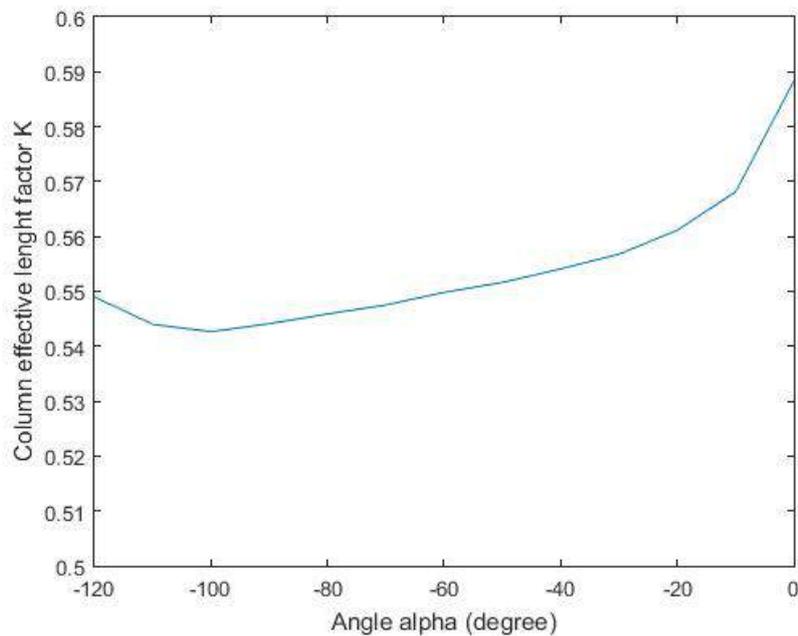


Figure 14. Effective length coefficient \times angle α for Case 2

The average effective length coefficient for Case 2 is 0.5541.

6. CONCLUSIONS

The calculation of forces that result from pendulum movement of the rotating set suggests that, in general, it is indicated to do a computational analysis of multibody dynamics, especially if there are frictional forces involved.

For the constant cross section column with fixed ends to the floor (Case 1), it was found an average effective length coefficient of 0.6208. Inasmuch as the rotational stiffness of the 4-column connection joint is smaller than a fixed end connection and greater than a pinned end connection, the average value is expected to lie between 0.5 (proposed K for a fixed-fixed column) and 0.7 (proposed K for a fixed-pinned column).

For the variable cross section column with pinned ends to the floor (Case 2), it was found an average effective length

coefficient of 0.5541. In this case, it is expected K value to lie between 0.7 and 1.0 (for a pinned-fixed column and a pinned-pinned column, respectively). The value of K lower than expected is due to the fact that in Eq 9, it is used the minimal area moment of inertial of the column. This is a conservative way of finding the critical load of a column with variable cross section.

It has been shown that the finite element analysis for this problem is fundamental, since the equations proposed by Euler are limited and do not contemplate variations of area moment of inertia, for example. Therefore, beams with variable sections would hardly be well analyzed by this Euler's method.

In practical terms, manufacturers' choice for using pinned end columns occur when the equipment is done for traveling parks, in order to facilitate their assembly and disassembly. Since a free rotation end does not transmit moment, the resulting bending moment increases along its length moving away from the pinned end. A variable cross section may support a variable bending moment and optimize the column weight.

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