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## A NOTE ON CONTROL IN MICRO ELETROMACHANICAL SYSTEMS WITH HYSTERETIC DAMPING

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**Abstract.** *In this work, the hysteretic damping in a micro electromechanical system (MEMS) represented by a micro accelerometer is investigated. This analysis is carried out through mathematical modelling and numerical simulation. Damping due to a cyclical stress shows that a damping force is independent of frequency. This internal damping is nominated hysteretic damping. It is to perform a test between two control techniques testing its uncontrolled efficiency of a micro electromechanical system with chaotic behavior and hysteretic damping, therefore, two control strategies are used to obtain the trajectory of the system, which are Sliding Mode Control and State Dependent Riccati Equation (SDRE).*

**Keywords:** *Nonlinear Dynamics, Micro electromechanical system (MEMS), Control SDRE, Sliding Mode Control, Hysteretic Damping.*

## 1. INTRODUCTION

Micro electromechanical systems arouse great interest and have been the subject of several studies, as can be seen in Tusset *et. al.* 2012 and Luo *et. al.* 2011., whose authors combined electrical components and mechanical coupling.

The investigation of the hysteretic damping occurs during the internal movement inside the molecules when deformed, causing an energy dissipated by the material. The measure of structural damping is given by the amplitude of the stress during deformation. A response of a system with hysteretic damping is similar to a system with viscous damping.

The micro electromechanical system studied here is represented as an electrostatically driven micro beam, as illustrated in Fig. 1. Micro beams have a wide range of applications due to their simple geometry, easy production, durability and compact area of motion. The system shown in Fig. 1 is composed of two fixed plates and a movable plate between the fixed ones, whose system is applied a voltage  $V(t)$  which is composed of a bias voltage  $DC$ ,  $V_p$  and an alternating voltage  $AC$ ,  $V_i \sin(\omega t)$ . The  $DC$  voltage applies an electrostatic force on the beam and generally changes the equilibrium position. The plates have the function of supplying electrodes to form a capacitor of storing electrical energy, and provide elasticity or mechanical stiffness.

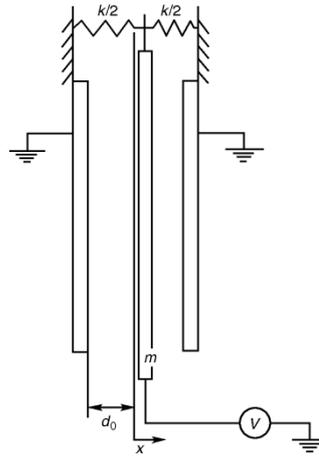


Figure 1 - Micro electromechanical systems [Moon, 1998].

The equation of motion can be represented by:

$$\ddot{u} + c\dot{u} + a_1u + a_3u^3 = \beta V^2 \frac{u}{(d_2^2 - u^2)^2} \quad (1)$$

where  $V^2$ , can be represented by:

$$V_p^2 + \frac{1}{2}V_i^2 + 2V_pV_i \sin(\omega T) - \frac{1}{2}V_i^2 \cos(2\omega T) \quad (2)$$

The constant damping in this case can be represented by:

$$c = \frac{h}{\omega} \quad (3)$$

which is valid approximation for the harmonic motion of frequency  $\omega$ . This situation is represented in Fig. 2.

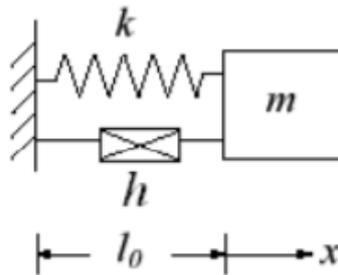


Figure 2 - Model of a damped harmonic oscillator (hysteretic damper).

The equation describing hysteretic damping is:

$$F = -c\dot{x} \quad (4)$$

Replacing in the governing equation of the movement for the micro electromechanical system, the equations of motions are given by Eqs. (5).

$$\dot{u}_2 = -cu_2 - a_1u_1 - a_3u_1^3 + \beta \frac{u}{(d_2^2 - u^2)^2} \left[ \dot{u}_1 = u_2 \right] \left[ V_p^2 + \frac{1}{2}V_i^2 + 2V_pV_i \sin(\omega T) - \frac{1}{2}V_i^2 \cos(2\omega T) \right] \quad (5)$$

Considering the parameters:  $\alpha = 1, \alpha_3 = 0.4, \beta = 69141.6, c = 1, d = 25, \omega = 6.28, V_p = 2, V_i = 10, u_0 = 1$  and  $\dot{u}_0 = 0.5$ . The displacement and plane phase can be observed in Fig. 3.

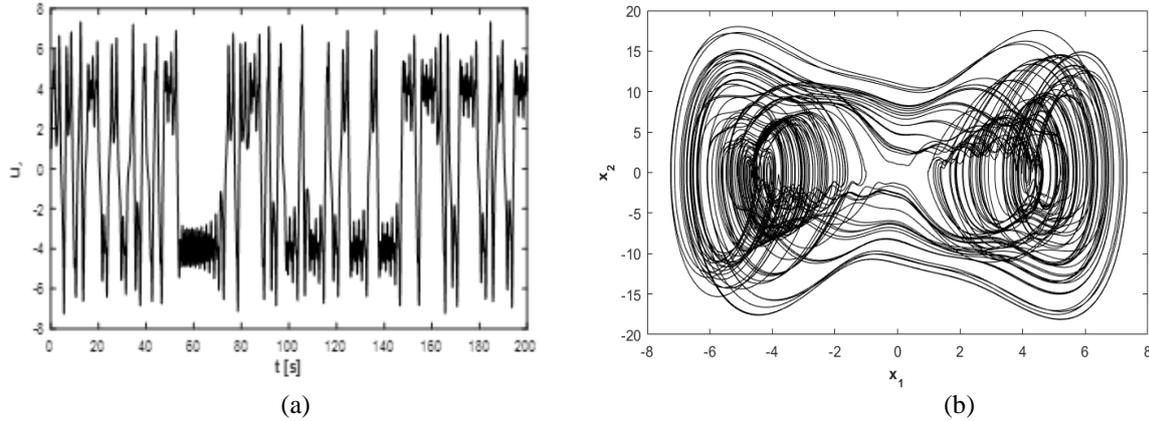


Figure 3 - (a) displacement and (b) plane phase.

## 2. CONTROL AND RESULTS

Two control strategies are used to obtain the trajectory of the system that are Sliding Mode Control and State Dependent Riccati Equation (SDRE).

The SDRE method is used to determine the control signal applied to the system to be controlled in a desired orbit.

The matrix  $A$  and matrix  $B$  are represented by:

$$A = \begin{bmatrix} 0 & 1 \\ -a_1 u_1 - a_3 u_1^3 + \frac{\beta V^2}{(d^2 - u_1^2)^2} & -c \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (6)$$

and defining

$$Q = \begin{bmatrix} 2000 & 0 \\ 0 & 2 \end{bmatrix}, R = [1] \quad (7)$$

The Riccati equation was solved using the LQR method, defining the desired  $u^*(t)$  trajectory.

The displacement is shown in Fig. 4a, and the phase plane in Fig. 4b, considering the application of the control and  $u_1^*(T) = 1.1180 \cos(T)$ .

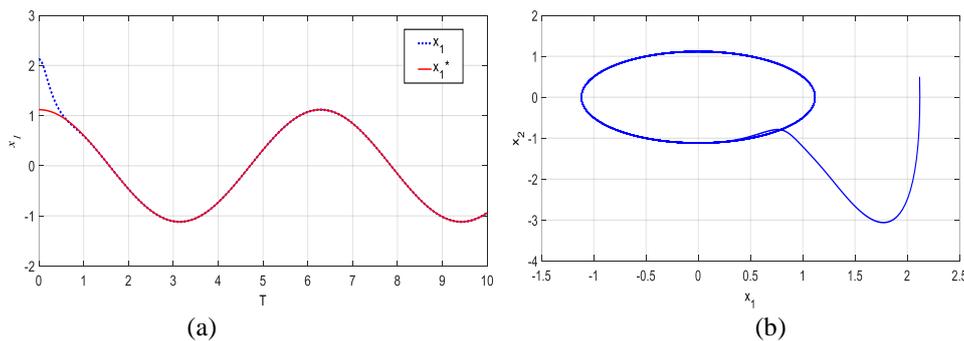


Figure 4 - System with control SDRE. (a) Displacement. (b) Phase portrait

The sliding mode control was applied aiming to avoid chaotic movements in order to improve or change the behavior of the system.

Consider Eq. (8) in the following form of Eq. (9).

$$\begin{aligned} \dot{u}_1 &= u_2 \\ \dot{u}_2 &= -c u_2 - a_1 u_1 - a_3 u_1^3 + \beta V^2 \frac{u_1}{(d^2 - u_1^2)^2} + U \end{aligned} \quad (9)$$

where  $U$  is the sliding mode control.

Moreover, it is defined the desired trajectory errors as:

$$e = \begin{bmatrix} u_1 - u_1^* \\ u_2 - u_2^* \end{bmatrix} \quad (10)$$

where  $u_1^*$  is a desired trajectory.

The sliding mode control field, the sliding surface is generally taken to be:

$$s = e_2 + \lambda e_1 \quad (11)$$

The existence of the sliding mode requires the following conditions:

$$\begin{cases} s = e_2 + \lambda e_1 = 0 \\ \dot{s} = \dot{e}_2 + \lambda \dot{e}_1 = 0 \end{cases} \quad (12)$$

where:  $\lambda$  represents a real number.

Equation (11) defines the output of the sliding mode controller, and while the reaching law is given by (Yau et al. 2011) and (Utkin, 1992)

:

$$U = \begin{cases} U_{\max} & \text{if } \frac{s}{\phi} < -1 \\ -U_{\max} & \text{if } \frac{s}{\phi} > 1 \\ K_c s & \text{if } -1 < \frac{s}{\phi} < 1 \end{cases} \quad (13)$$

where  $\phi$  layer thickness of the control,  $K$  is a proportional gain, and  $U_{\max}$  is control the saturation value.

For the determination of  $K_c$  is considered the Lyapunov function of the system, which is defined as:

$$V = \frac{1}{2} s^2 \quad (14)$$

The first derivative of this system related to time can be expressed as:

$$\begin{aligned} \dot{V} = s\dot{s} = s[\dot{e}_2 + \lambda \dot{e}_1] = \\ s \left[ \alpha_1 (e_1 + u_1^*) - \alpha_3 (e_1 + u_1^*)^3 - b(e_2 + u_2^*) + \right. \\ \left. + \beta V^2 \frac{(e_1 + u_1^*)}{(d^2 - (e_1 + u_1^*))^2} + KU + \lambda e_2 \right] = \\ s[K_c U] \leq -K_c |s| \end{aligned} \quad (15)$$

If  $K_c > 0$  is set, then the reaching condition ( $s\dot{s} < 0$ ) is always satisfied. Therefore, the system of Eq. (10) can be stabilized to a desired trajectory  $u_1^*(T)$ .

According to Tusset *et al.* 2012,  $u_1^*(T)$  can be defined as  $u_1^*(T) = 1.1180 \cos(T)$ , and defining  $\lambda = 4$ ,  $K_c = 50$ ,  $\phi = 10^{-4}$  and  $U_{\max} = 500$ , Figs. 5a and 5b show the time history of the controlled system and its phase plane, respectively.

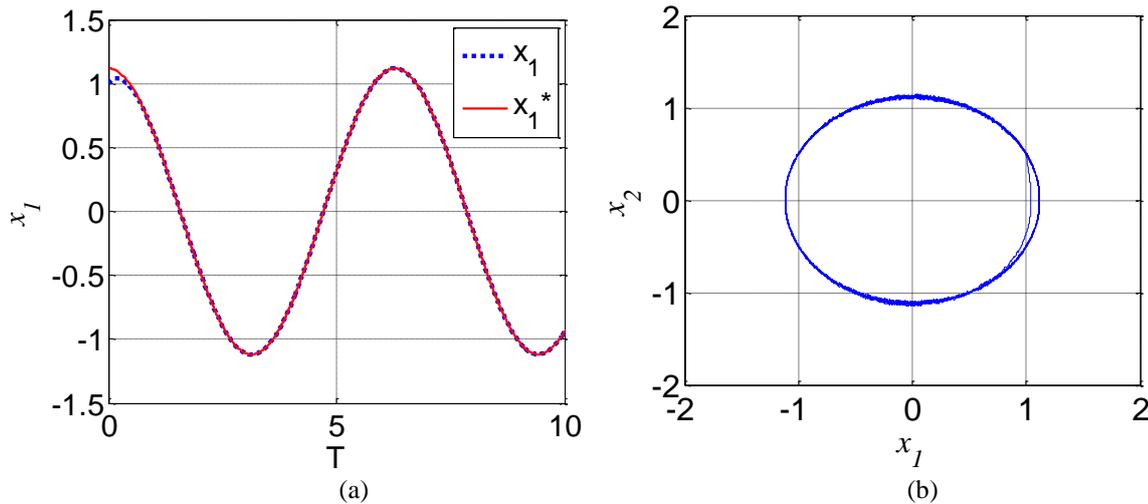


Figure 5 - System with sliding mode control. (a) Displacement. (b) Phase portrait

### 3. CONCLUSIONS

As observed in Figs. 4a, 4b, 5a and 5b, both sliding mode control and SDRE control were efficient in controlling the oscillations of the system, leading the system to periodic orbit.

### 4. ACKNOWLEDGMENTS

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