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SHELL AND TUBE HEAT EXCHANGER OPTIMIZATION USING METAHEURISTICS

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Abstract. *Optimization techniques have been largely applied in heat exchangers design with several methods available on literature. Many of developed methods have been applied in different types of heat exchangers. Shell and tube heat exchanger, one of the most common configuration applied in industrial processes, involves complex design process, including selection of geometrical parameters and operating conditions. In this context, present study developed a reverse analysis in a shell and tube heat exchanger optimization based on Sinnott et al. (2005), Kern (1950) and Caputo et al. (2008) studies. Minimization of total cost is considered as the objective function with heat transfer performance and working fluid pressure drop restrictions applied. The results obtained through evaluated algorithms are compared with reference original data.*

Keywords: *heat exchanger design, optimization, metaheuristic, PSO and GWO.*

1. NOMENCLATURE

a_1	numerical constant [€]
a_2	numerical constant [€/m²]
a_3	numerical constant
A	heat exchanger surface area [m^2]
A_s	shell side pass area [m^2]
B	baffles spacing [m]
c	acceleration constants
C	numerical constant
C_e	energy cost [€/kWh]
C_i	capital investment [€]
Cl	clearance [m]
C_o	annual operating cost [€/yr]
C_{od}	total discounted operating cost [€]
C_p	specific heat [J/kg K]
C_{tot}	total annual cost [€]
CR	outcrossing rate
d_e	equivalent shell diameter [m]
d_i	tube inside diameter [m]
d_o	tube outside diameter [m]
D	Number of parameters to optimize
D_s	shell inside diameter [m]
F	temperature difference correction factor
f_m	mutation factor
f_s	shell side friction coefficient
f_t	tube side friction coefficient
G	current generation of the optimization
r_0,r_1,r_2	values uniformly distributed in the range [0, 1]
Re_s	shell side Reynolds number
Re_t	tube side Reynolds number
R_{fs}	shell side conductive fouling resistance [m^2 K/W]
R_{ft}	tube side conductive fouling resistance [m^2 K/W]
i	tube side conductive fouling resistance [m^2 K/W]
S_t	tube pitch [m]
T_{is}	shell side inlet fluid temperature [K]
T_{os}	shell side outlet fluid temperature [K]
T_{it}	tube side inlet fluid temperature [K]
T_{ot}	tube side outlet fluid temperature [K]
U	overall heat transfer coefficient [W/m^2 K]
v	particle velocity
$V_{i,G}$	giver vector for the i -th vector target generation G
$V_{j,i,G}$	j- th component of the giver vector
v_s	shell side fluid velocity [m/s]
v_t	tube side fluid velocity [m/s]
w	inertia weight factor
$\vec{X}_{i,G}$	i-th individual of the population in G
$X_{j,i,G}$	j-th individual component of $X_{i,G}$
$\vec{X}_{i,o}$	Individual to the initial population
$\vec{X}$	Limit of the problem of variables to be solved
$X_{j,}$	j-th component of vector $\vec{X}$
$\vec{E}_{i,G}$	experimental vector for the i-th vector target G
$\vec{E}_{j,i,G}$	j-th experimental vector component for the $\vec{E}_{i,G}$

-
process

H	annual operating time [h/yr]
h_s	shell side convective coefficient [W/m^2 K]
h_t	tube side convective coefficient [W/m^2 K]
i	annual discount rate (%); or population particles
j	number of variables to be optimized
k	thermal conductivity [W/m K]; or values uniformly distributed in range [0, 1]
L	tubes length [m]
$\dot{m}_s$	shell side mass flow rate [kg/s]
$\dot{m}_t$	tube side mass flow rate [kg/s]
n	number of tube passes
n_1	numerical constant
ny	equipment life [yr]
NP	dimension of population
N_t	number of tubes
P	pumping power [W]
Pr_s	shell side Prandtl number
P_t	tube pitch [m]
Pr_t	tube side Prandtl number
Q	heat duty [W]
$rand_{j,i}$	Random number generator distributed in range [0,1] for each j-th component of the i-th individual

Greek symbols

β	Contraction/expansion coefficient
Δh	heat transfer difference [W/m^2 K]
ΔP	pressure drop [Pa]
$\Delta P_{tube\ elbow}$	tube elbows pressure drop [Pa]
$\Delta P_{tube\ length}$	tube length pressure drop [Pa]
ΔT_{ML}	mean logarithmic temperature difference [K]
η	overall pumping efficiency
μ	dynamic viscosity [Pa]
ρ	fluid density [kg/m³]

Subscripts

e	equivalent, energy
i	inlet, investment
max	Maximum
min	minimum
o	outlet, operating
od	discounted operating
s	shell side
t	tube side
tot	total
wt	wall

2. INTRODUCTION

With the increasing demand of electrical energy by society, the availability requires constant investments in power sources in the manner of satisfy ordinary consumer and industries. Besides investments in new power suppliers, sustainable or not, the focus turns to energy economy and on its efficient usage, especially in industries, main power consumers.

Most of industrial plants and processes lay out on refrigeration systems to cool or heat process fluids, reject heat or prepare substrates for a determined processing. Refrigeration systems, direct or indirect, uses specific components to transfer thermal energy from one environment or fluid to a second fluid or environment consuming energy through the use of compressors or pumps. According to Mainardes (2007), refrigeration systems are responsible for up to 70% of electrical energy consumption from a power plant. Thus, reach required design parameters using lower levels of energy consumption is essential for a sustainable process.

In this context, a good design of shell-and-tube heat exchangers, shown on Figure 1, improve energy efficiency level, reducing power consumption and minimizing manufacturing, operation and maintenance costs. In this manner, the utilization of optimization methods might be very useful on heat exchanger design process. Many studies treated and used mathematical techniques on the heat exchanger development processes, from empirical methods to metaheuristic optimization algorithms.

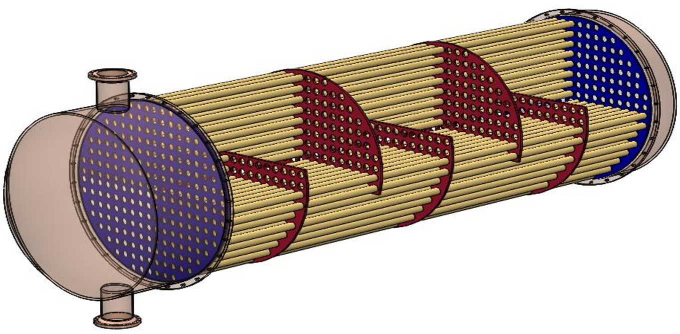

Figure 1. Illustrative representation of a shell-and-tube heat exchanger.

Caputo, Pelagagge and Salini (2008) developed a procedure for shell-and-tube heat exchanger optimal design using genetic algorithm to minimize the equipment total cost including investment capital and the sum of discounted annual energy expenditures related to pumping working fluid. In the evaluated cases, the authors were able to reduce investment capital in up to 7.4% and saves in operating costs up to 93%, decreasing total costs up to 52%.

Fesanhary, Damangir and Soleimani (2009) applied an optimization mechanism called Harmony Search Algorithm or *HSA* as well as Global Sensitivity Analysis or *GSA*. The study presented by the authors aimed to determine the most efficient method using a shell-and-tube heat exchanger design case to perform the evaluation from the economic point of view. The authors showed that the *HSA* method provided converged results faster and with higher accuracy than the results obtained using *GSA*.

Ravagnani *et al.* (2009) obtained an optimum solution for a shell-and-tube heat exchanger applied in a certain condition using as optimization method the Particle Swarm Optimization algorithm or *PSO*. The study aimed to minimize the global cost including heat transfer area and operational costs according to the standards of the Tubular Exchanger Manufacturers Association respecting pressure drops and fouling restrictions. Analyzing literatures cases, the authors concluded that the *PSO* algorithm presented better results when compared with previous studies.

Also using the *PSO* method, Patel and Rao (2010) explored the mentioned algorithm on shell-and-tube heat exchanger from the economic point of view. The authors established three design variables: shell internal and external diameter, baffle spacing and tube layouts. Analyzing four different cases, the study allowed to demonstrate the effectiveness and accuracy of *PSO* algorithm when compared with Genetic Algorithm.

Again, Patel and Rao (2011) applied *PSO* algorithm in shell-and-tube heat exchangers and compared their performance with the Civilized Swarm Optimization or *CSO* from the economic point of view. The authors also took in account the reliability and maintenance due to the fouling formation inside the heat exchanger. Analyzing two applied cases, the authors compared obtained results using both optimization algorithms with *GA* results from previous studies. Patel and Rao (2011) found benefits with both *PSO* and *CSO* algorithms when compared with *GA* data, especially regarding heat exchanger total costs, decreasing values in up to 9.4% with CSO method.

Mariani *et al.* (2012) presented the Quantum Particle Swarm Optimization algorithm or *QPSO* combined with the Zaslavskii Chaotic Map Sequence applying the new method in a shell and tube heat exchanger based on the minimization from the economic point of view. The authors evaluated the proposed method using two cases comparing with results provided by *PSO* method and with classical *QPSO*, showing the best performance with the proposed method.

Fettaka, Thibault and Gupta (2013) tested a multi-objective optimization method called NSGA-II, based on genetic algorithm. Appling the NSGA-II method on shell-and-tube heat exchangers, the authors considered the tube layout pattern, number of tube passes, baffle spacing and cut, tube to baffle and shell to baffle clearance, tube length, external diameter and thickness.

Hadidi and Nazari (2013) developed a new technique called Biogeography Based Optimization or *BBO*. According to the authors the *BBO* technique was applied in order to minimize the total cost of the equipment including capital investment and the sum of expended energy related to pumping. Solving three different cases, Hadidi and Nazari compared obtained results from the proposed algorithm with results obtained with *GA*, *PSO* and *ABC* methods. However, proposed *BBO* algorithm results did not showed significant improvement when compared with evaluated algorithms.

Mirjalili et al. (2014) proposed a new metaheuristic method called Grey Wolf Optimizer (GWO) inspired by grey wolves (*Canis lupus*). This method was proposed based on the hierarchy of leadership and technique, which are factors that serve as artifacts during hunting by grey wolves. This species belongs to the family *Canidae*, classified into groups denominated alpha, beta, delta and omega. In addition, the three main stages of hunting are described in the method, which are: hunting, wrapping and attacking prey.

Available literature related to optimizing techniques applied in shell-and-tube heat exchangers shows that several algorithms were already developed and applied in order to increase heat transfer, decreasing cost and fouling formation as well as cost related to manufacturing and operation. Also, due to the amount of validated results and consolidated methods allow researchers to apply developed optimization techniques in several cases and explore deeply the benefits from one or another method.

In present article, a reverse analysis is performed via metaheuristics algorithms applied heat exchanger originally developed by a commercial software with the parameters defined via genetic algorithm by Caputo *et al*. (2008) considering as an ideal heat exchanger design. The development of this study allows evaluating the performance of optimization algorithms as designing tools of heat exchangers, in this case shell-and-tube type.

3. OPTIMIZATION DESIGN METHODS

In this section first, a brief overview of the classical PSO is provided for formulating solution vectors in which the optimization process is generated, the object function is evaluated, and finally, the proposed GWO is explained. The proposed GWO presents an efficient strategy to improve the search performance in preventing premature convergence to local minima when compared with a GA and classical PSO algorithm.

3.1 Particle swarm optimization.

The stochastic optimization method, PSO (*Particle Swarm Optimization*) consists of a computational strategy for solving global optimization cases, (KENNEDY and EBERHAR, 1995). This concept is based on the ability in which an individual is able to cooperate within a population, stimulating competition within their group. The PSO is characterized as a simple modeling of social situations, being a phenomenon observed in the behavior of birds, fish, bacteria and insects.

The PSO originated in the behavior of animals by the way to manage ideal distances among the other members of their group. Thus, initially the velocity is a ponderable parameter in maintaining optimal spacing. In addition, social behavior was analyzed for a case in which birds sought their food. With this, it was found that birds define their own speed, through their previous best experience and among the other members of the group (MELLO, 2010).

The PSO method presents the strategy of resolution from an initial population, content of random solutions in search of an optimal solution, at the same time as the generations are updated. Currently, the PSO is commonly used in situations that involve spaces with a high degree of complexity, such as non-linear optimizations. Therefore, the PSO approach is quite straightforward in its application, requiring only the parameters that participate in the objective function.

Stochastic methods contain random solutions, which are called particles, with a pair of real-valued vectors, moving in a space and with a definite position and velocity vector. Each particle has the ability to control its coordinate in the region of the problem, following the ideal particles, which have the best value so far. This value is called *pBest*, best personal.

However, there is another better value, which is controlled by the global version of particle swarm optimization, the best overall value, or *gBest*, obtained so far. The PSO strategy is updated with each step of time, changing the speed, that is, accelerating the process of each particle, directing it to its *pBest* value and *gBest* location. The acceleration process is evaluated in relation to its last best value obtained, which will generate the acceleration toward *pBest* and local *gBest*, respectively.

When employing the PSO technique in designs of tube hull heat exchangers, it is recommended to use the stepwise resolution strategies within algorithm. The work of Coelho et al. (2008) a reference regarding the organization of the application stages, where the global version of the PSO method was implemented. The steps to be followed in the PSO application are described below:

Step 1: Initialize population (swarm) containing random positions and velocities. The criterion is that the number of particles in the swarm should be equal to that of variables, in which each particle represents a variable, forming a uniform distribution.

Step 2: Compare all particles with their best personal result by evaluating whether the current value is better than *pbest*, shortly thereafter, the *pbest* value is reset to the current value and the *pbest* location is equal to the location of the current value in space.

Step 3: Confront all particles with their global best by evaluating whether the current value is better than the *gbest*, then redirects the *gbest* to the current particle array index and value.

Step 4: Update the velocity and position on all particles by changing the velocity, *vi*, and position of the particle, *xi*, according to Eqs. (1) and (2). The speed update is performed from the previous speed Eq. (1).

$$v_{i,j}(t+1) = w \cdot v_{i,j}(t) + c_1 \cdot r_1 \cdot \left[p_{i,j}(t) - x_i(t)\right] + c_2 \cdot r_2 \cdot \left[p_{g,j}(t) - x_{i,j}(t)\right] \tag{1}$$

The constant of the particle position Eq. (2), is obtained by adding the previous position and the new velocity, considering $\Delta t = 1$, (COELHO *et al.*, 2008).

$$x_{i,j}(t+1) = x_{i,j}(t) + \Delta t \cdot v_{i,j}(t+1) \tag{2}$$

where $i = 1,2, ..., N$, represents the particles of a population, that is, of the swarm, $t = 1,2, ..., t_{max}$ is the number of iterations, w, is defined as $v_{i,j}(t+1)$, is the velocity of the *i-th* particles, which corresponds to the *ith*-order dimension in the iteration t, and $p_{i,j}(t)$, represents the best previous position of the *i-th* particles with the *jth*-order dimension. The term, $p_{g,j}(t)$, corresponds to the best previous position among the other particles along the *jth* dimension, at iteration t.

The first term on the left-hand side in Eq. (1) defines part of the particle's momentum, where the inertia weight, w, defines the degree of particle momentum. The second term characterizes the independent behavior of the particle itself. The constants, c_1 and c_2, refer to the cognitive and social components, describing the variation of the velocity of the particle in search of *pbest* and *gbest*, respectively. The subscript, g, is the index of the best particle of the swarm. The parameters r_1 and r_2 are the uniformly distributed weights in the interval [0,1].

Step 5: Repeat the evolutionary cycle returning to step 1 until the criterion of stop is met, in the present work is used maximum number of iterations equal to 1000. The velocities of the particles in each of the dimensions are fixed at a maximum speed, v_{max}. If the sum of accelerations causes the velocity in the dimension to exceed v_{max}, which is a parameter specified by the designer, the velocity would then be limited to v_{max}.

3.2 Grey wolf optimizer

Gray wolves are known as predators, once being one of the world's best-distributed animals, able to travel long distances in search of prey. According to Figure 4, the alphas consist of a male wolf and a female, not necessarily the strongest, to engage in hunting, resting, displacement and other tasks. They are recognized when the other members keep their tails lowered before them and mating only between the same pack. The beta class belongs to the subordinates who cooperate in the alphas' decisions. The betas help to discipline the group with feedback to their superior.

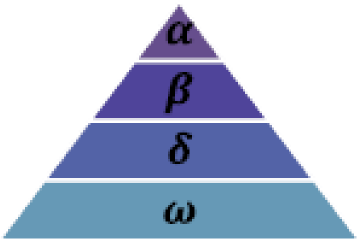

Figure 2. Decreasing hierarchical division among gray wolves. Adapted from Mirjalili *et al*. (2014).

Then there are the delta wolves, submissive to the alphas and betas, they play the role of elders, hunters, caretakers, scouts and sentinels. The elders, experienced wolves with possibilities to become betas and even alphas. The hunters collaborate with alphas and betas in catching prey. Caregivers take responsibility for sick and injured wolves. The Boy Scouts prepare to observe the area's domains and warn of danger. Finally, the sentinels act as security guards, protecting the whole (MIRJALILI *et al.*, 2014).

The social group of omega division play the role of scapegoat and respond to all upper classes. One group may be at risk of losing their omega because of the violence and disappointment of others, as they must satisfy the whole group and maintain the hierarchical distribution. The omegas also act as caretakers of puppies and are the last wolves in the order of feeding.

According to Muro *et al.* (2011), the strategies used in hunting by the gray wolves call as much attention as the way they coexist. The method of hunting of gray wolves can be characterized as follows: (*i*) tracking, chasing and approaching prey; (*ii*) encircling and trapping the prey until it stops moving (*iii*) prey point of attack.

Through the social hierarchy of the lobes, a mathematical optimization model capable of describing the GWO technique was proposed, and alpha (α) was selected as the optimal solution. Thus, betas (β) and deltas (δ) represent the second and third optimal solutions. The hierarchical division omega (ω) represents the other optimal solutions. Thus, the method starts at the moment when the gray wolves surround the prey, such behavior is represented mathematically in Eqs. (3) and (4):

$$\vec{D} = \left|\vec{C} \cdot \vec{X}_p(t) - \vec{X}(t)\right| \tag{3}$$

$$\vec{X}(t+1) = \overrightarrow{X_p}(t) - \vec{A} \cdot \vec{D} \tag{4}$$

where, t represents the current iteration, $\vec{A}$ and $\vec{C}$ are the vector coefficients, $\vec{X}_p$ indicates the position of the prey, and $\vec{X}$ is the position vector of a gray wolf. Since the vectors $\vec{A}$ and $\vec{C}$ are determined as follows:

$$\vec{A} = 2\vec{a} \cdot \vec{r}_1 - \vec{a} \tag{5}$$

$$\vec{C} = 2 \cdot \vec{r}_2 \tag{6}$$

where, the components of $\vec{a}$ decrease linearly from 2 to 0 in relation to the course of iterations. The variables $\vec{r}_1$ and $\vec{r}_2$ represent the random vectors in [0, 1]. Eqs. (3) and (4) can illustratively represent the behavior of lobes through Figure 3, where there is a positioning of the two-dimensional vector (X,Y) for the position of the lobes. This position may change depending on the displacement or not of the dam. With this, it is noticed that there are different places around the best individual, which can be captured in relation to the current point, updating the value of the vectors $\vec{A}$ and $\vec{C}$.

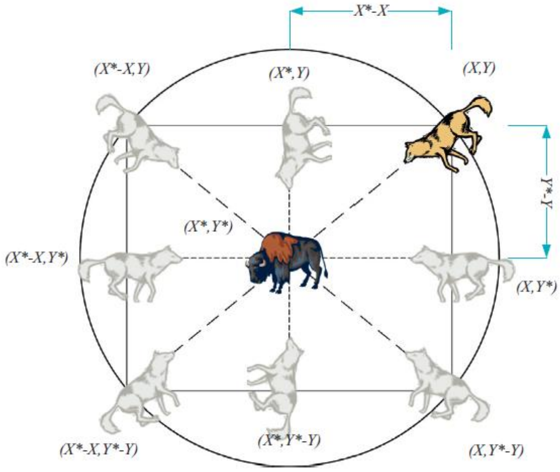

Figure 3. Two-dimensional positioning of the wolf varying according to the position of the prey. Adapted from Mirjalili et al. (2014).

Based on the behavior presented in the previous paragraph, when performing the adjustment of values, there is a new provision for wolves. The vectors $\vec{r}_1$ and $\vec{r}_2$ enable the wolves to reach at any random point around the prey. However, this proposition extends to a space containing *n* dimensions for different positions of the lobes around the best solution. The identification of the position of the prey is made by the alpha, beta and delta participate eventually in the hunting.

Thus, in a search field there is no conception regarding the positioning of the best prey. Mathematically this behavior is considered the hypothesis where the candidate for the best solution beta and delta, presents a greater understanding regarding the optimal point of the prey. Thus, we record the three best solutions and generate a new search, including the omega lobes to fit into a better position Eqs. (7), (8) and (9).

$$\vec{D}_\alpha = \left|\vec{C}_1 \cdot \vec{X}_\alpha - \vec{X}\right|,\ \vec{D}_\beta = \left|\vec{C}_2 \cdot \vec{X}_\beta - \vec{X}\right|,\ \vec{D}_\delta = \left|\vec{C}_3 \cdot \vec{X}_\delta - \vec{X}\right| \tag{7}$$

$$\vec{X}_1 = \overrightarrow{X_\alpha} - \vec{A}_1 \cdot (\vec{D}_\alpha),\ \vec{X}_2 = \overrightarrow{X_\beta} - \vec{A}_2 \cdot (\vec{D}_\beta),\ \vec{X}_3 = \overrightarrow{X_\delta} - \vec{A}_3 \cdot (\vec{D}_\delta) \tag{8}$$

$$\vec{X}(t+1) = \frac{\overrightarrow{X_1} + \overrightarrow{X_2} + \overrightarrow{X_3}}{3} \tag{9}$$

Since the end point is at a random location within the circle established by the alpha, beta and delta arrangement. Soon, the final stage of the hunt is executed with the attack of the wolves, with the prey already immobile. This behavior is expressed mathematically by establishing the approximation to the prey, reducing the value of $\vec{a}$ and its fluctuation $\vec{A}$. However, $\vec{A}$, represents a random value within a range [-2a, 2a], where the elements of $\vec{a}$ decrease from 2 to 0 over the iterations. Thus, the random values of $\vec{A}$ are located in [1,1]. So the subsequent position of an individual can be anywhere between the current point and the point of capture prey.

The search for gray wolves occurs in particular according to the position of alpha, beta and delta. There is a divergence between some wolves in the quest and a convergence during the prey attack. Thus, the above mentioned characteristics describe the divergence, where the random values of $\vec{A}$ are applied, greater than 1 or less than -1, force the element to diverge from the prey and look for another more timely situation. In this way, the analysis is enhanced and characterizes the global search of the GWO algorithm.

Based on Eq. (6), the random values in the range of [0.2] of the vector $\vec{C}$ also allow the exploration during hunting, thus stochastically different random weights for prey are attributed by approaching (C> 1) or C <1) of the dam in order to determine the distance in Eq. (3).

The steps of the GWO search process can be summarized as follows: (*i*) formation of a population of gray wolves; (*ii*) iterations involving the alpha, beta and delta lobes, which evaluate the possible position of the prey; (*iii*) candidate solutions to adjust the distance in relation to prey; (*iv*) reduction of the number of components $\vec{a}$, forcing the search and exploitation, respectively; (*v*) candidate solutions tend to diverge from prey, when $|\vec{A}| > 1$ and converge to the prey when $|\vec{A}| <1$; (*vi*) the GWO algorithm is terminated or not according to the stopping criterion imposed.

4. MATHEMATICAL EQUATIONS

The development of an optimized heat exchanger is more clearly identified by means of the process outline, as shown on Figure 4. The procedure for optimal heat exchanger design includes the following steps: estimation of heat exchanger surface area based on required duty; evaluation of investment capital, operating cost and objective function; utilization of the optimization algorithm in order to select a new set of values for the design parameters and iteration of previous steps until an optimum objective function is found.

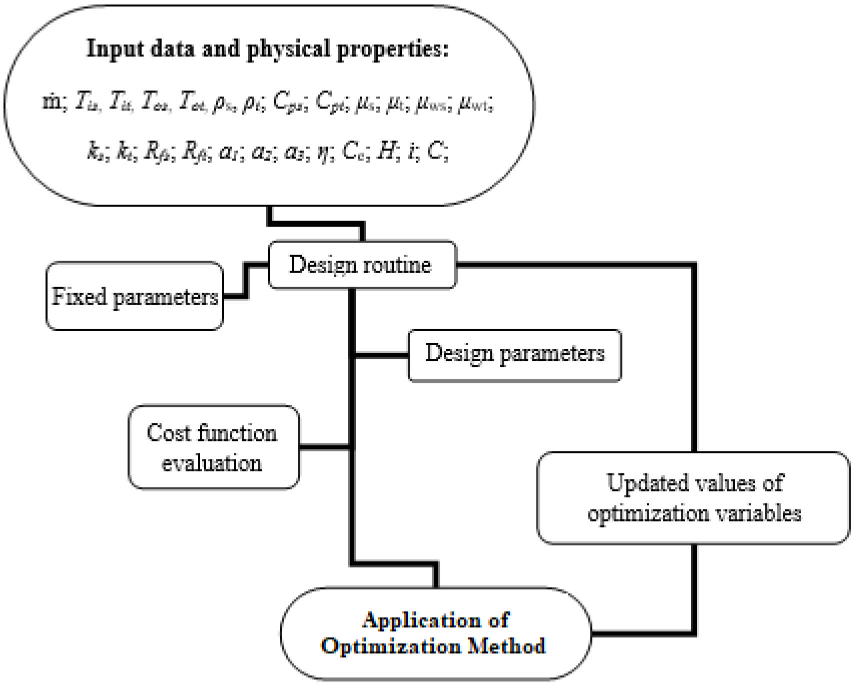

Figure 4. Guidance optimization algorithm.

The specification of the project designates the heat transfer requirement of the heat exchanger, which is obtain through mass flow rate, inlet and outlet fluid temperatures from tube and shell sides, encompassing other parameters, which are determined by the system energy balance. In addition, there are variables that are determined during project design: fluids thermophysical properties, tubes number of passes, fouling resistance and tubes array, as Figure 5 presents.

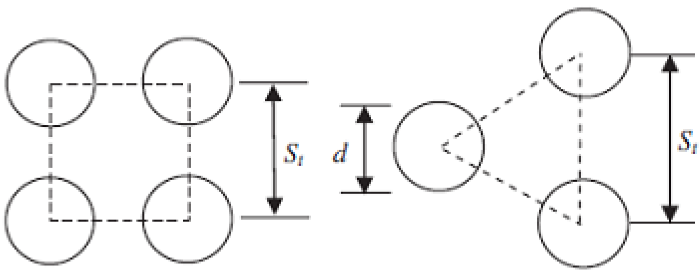

Figure 5. Shell and tube heat exchanger tubes array options.

Tube diameters, internal (d_i) and external (d_o), as well shell diameter (D_s) and baffles spacing (B), values are provided during optimization process iteratively through the implemented algorithm.

Based on the design specification of values on fixed variables and parameters to optimize. The routine exchanger design determines the values of heat exchange coefficients by convection to the shell side, *hs*, and tube side ht, the area of the overall heat exchange A, the number tubes *Nt*, the length *L* of the shell side and tube side and the velocities of the shell side fluid, v_s and tube side v_t. Soon, it defines all constructive terms for the detailed design in order to meet the required thermal specifications.

Applying the values of the velocities of fluids and constructive detailing the exchanger, if possible then measure the objective function. The optimization algorithm is characterize based on the value of the objective function, which may have their values updated through manipulations of optimization parameters, forming then a new exchanger model for the case. The stopping criterion is set to a minimum value of the objective function is obtained or imposed convergence is achieved, evaluating then the new model is physically applicable.

4.1 Heat exchanger design

For the design of the heat exchanger project, heat transfer rate is directly linked to the universal heat transfer coefficient and to heat exchanger surface area, as shown on Eq. (10).

$$Q = A \cdot U \cdot F \cdot \Delta T_{ML} \tag{10}$$

To determine the universal heat transfer coefficient, Eq. (11). Proposed by Selbas (2006) the following correlation is a function of shell coefficients and fouling resistance as well tube-fouling resistance.

$$U = \frac{1}{\left(1/h_s + R_{fs} + (d_o/d_i) \cdot R_{ft} + 1/h_t\right)}, \tag{11}$$

where, the tube inner diameter is defined as:

$$d_i = 0.8 \cdot d_o \tag{12}$$

Following the formulation proposed by Kern (1950), Eq. (13), the convective heat transfer coefficient at the shell side are estimated:

$$h_s = 0{,}36 \cdot \frac{k_s}{d_e} \cdot {Re_s}^{0.55} \cdot {Pr_s}^{1/3} \cdot \left(\frac{\mu_S}{\mu_{ws}}\right)^{0.14}, \tag{13}$$

where, the shell equivalent hydraulic diameter is defined in function of tube array, as shown on Eq. (14a) for square arrangement and (14b) for triangular tubes arrangement, also proposed by Kern (1950).

$$d_e = \frac{4 \cdot \left({S_t}^2 - \left(\frac{\pi \cdot {d_o}^2}{4}\right)\right)}{\pi \cdot d_o} \tag{14a}$$

$$d_e = \frac{4 \cdot \left(0{,}43 \cdot {S_t}^2 - 0{,}5 \cdot \pi \cdot {d_o}^2\right)}{0{,}5 \cdot \pi \cdot d_o} \tag{14b}$$

From the equivalent hydraulic diameter of the shell side, flow

$$v_s = \frac{\dot{m}_s}{A_s \cdot \rho}, \tag{15}$$

where, the shell side pass area is defined by:

$$A_s = \frac{D_s \cdot B \cdot Cl}{P_t}, \tag{16}$$

and the clearance is calculated through Eq. (17):

$$Cl = P_t - d_0 \tag{17}$$

The inside shell diameter is determined by Eq. (18) as proposed on Jacobi *et al.* (2014):

$$D_s = \left(\frac{N_t}{k}\right)^{1/n_1} \cdot d_o \tag{18}$$

Based on Eq. (18), the number of tubes to be used on heat exchanger design is defined by Eq. (19) and (20), also proposed by Jacobi et al. (2014).

$$N_t = \frac{A}{L \cdot \pi \cdot d_o}, \tag{19}$$

$$L = \frac{A}{N_t \cdot \pi \cdot d_o}. \tag{20}$$

Regarding the tube side, convection phenomenon is defined by Caputo et al. (2008) formulation, as shown on Eq. (21a) to (21c).

$$h_t = \frac{k_t}{d_i}\left[3.657 + \frac{0.0677 \cdot \left(Re_t \cdot Pr_t \cdot \left(d_i/_L\right)\right)^{1.33^{1/3}}}{1 + 0.1 \cdot Pr_t \left(Re_t \cdot \left(d_i/_L\right)\right)^{0.3}}\right], \text{Re}_t < 2.300 \tag{21a}$$

$$h_t = \frac{k_t}{d_i}\left[\frac{\left(f_t/_8\right) \cdot (Re_t - 1000) \cdot Pr_t)^{1.33^{1/3}}}{1 + 12.7 \cdot \left(f_t/_8\right)^{1/2} \cdot \left(Pr_t{}^{2/3} - 1\right)} \cdot \left(1 + \frac{d_i}{L}\right)^{0.67}\right], 2.300 < \text{Re}_t < 10.000 \tag{21b}$$

$$h_t = 0.027 \cdot \frac{k_t}{d_i} \cdot Re_t{}^{0.8} \cdot Pr_t{}^{1/3} \cdot \left(\frac{\mu_t}{\mu_{wt}}\right)^{0.14}, \text{Re}_t > 10.000 \tag{21c}$$

On tube side, flow velocity is defined by Eq. (22):

$$v_t = \frac{m_t}{\frac{\pi \cdot d_o}{4} \cdot p_t} \cdot \frac{n}{N_t} \tag{22}$$

Therefore, based on design specifications, setting user-defined parameters, evaluating the tube inside diameter (d_o), tube inside diameter,D_s, and baffles spacing, B, initially delimiting the number of tubes and the equivalent hydraulic diameter of the shell side, applied by Eqs. (14a), (14b) and (19) respectively. Then speeds are ascertained fluid, employing Eqs. (15) and (22). Subsequently determines both the convection coefficient in the shell side and the tube side, Eqs. (13) and (21a) as the overall coefficient of heat transfer, Eq. (11). Thereupon, it defines the area of heat exchange surface by Eq. (10). Finally, one obtains the length L of the tube by Eq. (20).

The process redone in order to obtain new values for A and L, approaching the corresponding exchanger solutions specifications, since the optimization algorithm modifies the values of the design variables in order to minimize the objective function.

3.2 Pressure drop

The tube side pressure drop is a sum of tube length pressure drop parcel and heat exchangers elbows pressure drop parcel, also inlet and outlet nozzles, according to Caputo *et al.* (2008) and shown on Eq. (23).

$$\Delta P_t = \Delta P_{lenght} + \Delta P_{elbows} \tag{23}$$

where, total pressure drop is given by Eq. (24):

$$\Delta P_t = \frac{\rho_t \cdot V_t{}^2}{2} \cdot \left(\frac{L}{d_i} \cdot f_t + p\right) \cdot n \tag{24}$$

In present article, it was used the appropriated value for P. The Darcy friction factor is defined by Hewitt (1998), through Eq. (25):

$$f_t = (1{,}82 \cdot log_{10}(Re_t) - 1{,}64)^{-2} \tag{25}$$

At the shell side, pressure drop is describes as follows:

$$\Delta P_s = f_s \cdot \left(\frac{\rho_t \cdot V_t^{\,2}}{2}\right) \cdot \left(\frac{L}{B}\right) \cdot \left(\frac{D_s}{d_e}\right) \tag{26}$$

The friction factor for shell side is given by Eq. (27):

$$f_s = 2 \cdot b_0 \cdot Re_s^{\,-0{,}15} \tag{27}$$

In the study developed, it was assumed a b_o value equal to 0.72 for the outflow where the Reynolds number is lower than 40,000, as described by Peters and Timmerhaus (1991).

Finally, the pumping power is obtained through Eq. (28).

$$P = \frac{1}{\eta} \cdot \left(\frac{m_t}{\rho_t} \cdot \Delta P_t + \frac{m_s}{\rho_s} \cdot \Delta P_s\right) \tag{28}$$

3.3 Objective function

Following Caputo *et al.* (2008) study, the objective function is represented by the total cost C_{tot}, Eq. (29), which includes the capital investments, C_i, the cost of energy, C_e, the annual cost of operation , C_o, and the total discounted cost operational C_{cod}.

$$C_{tot} = C_i + C_{od}, \tag{29}$$

$$C_i = a_1 + a_2 . A^{a_3}, \tag{30}$$

where, a_1 is equal to 8, a_2 is equal to 259.2 and a_3 has a value of 0.91 for shell and tube heat exchangers made of stainless stell, as proposed by Taal (2003). The total cost of the discounted operational cost is given by:

$$C_{od} = \sum_{j=1}^{ny} \frac{C_o}{(1+i)^j} \tag{31}$$

$$C_0 = P \cdot C_e \cdot H \tag{32}$$

where, C_e is equal to 0.00012, H has a value equal to 7 and i is equal to 0.1.

5. RESULTS

The effectiveness of evaluated algorithm, Differential Evolution, was evaluated through the analysis of three different cases originally analyzed by Caputo et al. (2008) using Genetic Algorithm. Thus, reliable data is created in order to compare optimization algorithms and determine advantages and disadvantages of each method. Evaluated cases represent a considerable range of usual applications using shell and tube heat exchanger. Working conditions and design parameters are presented below:

- Case 1: methanol – brackish water exchanger, duty of 4.34 MW. Study run from Sinnott *et al.* (2005), tube side with two passes and one passage on the shell side;
- Case 2: kerosene – crude oil exchanger, duty of 1.44 MW. Study developed by Kern (1950), four passes on tube side with square arrangement and 1 passage on the shell side;
- Case 3: distilled water – raw water exchanger, duty of 0.46 MW. Study performed by Kern (1950), tube side with two passages with triangular arrangement and a pipe passing through the shell side.

Table 1 presents the original design for each case. Original parameter was used as input parameter in the optimization algorithm in order to obtain optimal solutions.

Table 1. Original design parameter for evaluated cases.

Parameters	***Case 1*** **Shell side**	**Tube side**	***Case 2*** **Shell side**	**Tube side**	***Case 3*** **Shell side**	**Tube side**
ṁ [kg/s]	27.80	68.90	5.52	18.80	22.07	35.31
Te [°C]	95.0	25.0	199.0	37.8	33.9	23.9
Ts [°C]	40.0	40.0	93.3	76.7	29.4	26.7
ρ [kg/m³]	750	995	850	995	995	999
cp [kJ/kg·K]	2.84	4.20	2.47	2.05	4.18	4.18
μ [Pa·s]	0.00034	0.0008	0.0004	0.00358	0.0008	0.00092
k [W/m·K]	0.19	0.59	0.13	0.13	0.62	0.62
Rf [m²·K/W]	0.00033	0.0002	0.00061	0.00061	0.00017	0.00017

For each case and design specifications shown on Table 1, the differential evolution optimization algorithm was applied, resulting optimal heat exchangers configurations. Optimal heat exchanger architectures given by DE algorithm were compared with the optimal heat exchanger configurations given by Caputo et al. (2008) using GA algorithm and also with the original design configuration proposed by the referenced author, as shown on Tables 1 to 3.

Table 2. Optimal results comparison of the heat exchanger design for Case 1.

	Sinnott *et al.* (2005)	**Caputo *et al.* (2008)**	**PSO**	**GWO**
Ds [m]	0.894	0.830	0.861	0.861
L [m]	4.83	3.379	2.478	2.478
B	0.356	0.500	0.461	0.461
do [m]	0.020	0.016	0.015	0.015
Pt [m]	0.025	0.020	0.019	0.019
Cl [m]	0.005	0.004	0.004	0.004
Nt	918	1567	1895.7	1896.2
vt [m/s]	0.75	0.69	0.59	0.59
Ret	14925	10936	9186.4	9184.3
Prt	5.70	5.70	5.69	5.69
ht [W/(m² K)]	3812	3762	7757.6	7755.9
ft	0.028	0.031	0.032	0.032
DPt [Pa]	6251	4298	3532.6	3531
as [m²]	0.032	0.083	0.079	0.079
De [m]	0.014	0.011	0.011	0.011
vs [m/s]	0.58	0.44	0.61	0.47
Res	18,381	11,075	10,999	10,994
Prs	5.1	5.1	5.1	5.2
hs [W/(m² K)]	1573	1740	1810.1	1809.6
fs	0.33	0.357	0.3566	0.3566
DPs [Pa]	35,789	13,267	12,635	12,619
U [W/(m² K)]	615.0	660.0	784.9	784.8
A [m²]	278.6	262.8	221.2	221.3
Ci (€)	51,507	49,259	43,276	43,281
Co (€/yr)	2111	947	748.6	747.8
CoD (€)	12,973	5818	4599.9	4595.2
Ctot (€)	64,480	55,077	47,877	47,876

Table 3. Optimal results comparison of the heat exchanger design for Case 2.

	Kern (1950)	**Caputo *et al.* (2008)**	**PSO**	**GWO**
Ds [m]	0.539	0.630	0.590	0.585
L [m]	4.881	2.153	1.556	1.548
B	0.127	0.127	0.127	0.127
do [m]	0.025	0.020	0.020	0.020
Pt [m]	0.031	0.025	0.019	0.022
Cl [m]	0.006	0.005	0.005	0.005
Nt	158	391	391	391
vt [m/s]	1.44	0.87	0.87	0.89
Ret	8227	4068	4068	4068
Prt	55.2	55.2	55.2	55.2
ht [W/(m² K)]	619	1168	1208	1213
ft	0.033	0.041	0.045	0.045
DPt [Pa]	49,245	14,009	16,899	17,003
as [m²]	0.0137	0.0148	0.0135	0.0119
De [m]	0.0250	0.0190	0.0147	0.0143
vs [m/s]	0.470	0.430	0.492	0.495
Res	25,281	18,327	15,805	16,489
Prs	7.5	7.5	7.6	7.5
hs [W/(m² K)]	920	1034	1283	1302
fs	0.315	0.331	0.335	0.336
DPs [Pa]	24,909	15,717	21,747	22,005
U [W/(m² K)]	317	376	410	410
A [m²]	61.5	52.9	47.8	47.7
Ci (€)	19,007	17,599	16,710	16,705
Co (€/yr)	1304	440	523.3	521.3
CoD (€)	8012	2704	3215.6	3213
Ctot (€)	27,019	20,303	19,926	19,918

Table 4. Optimal results comparison of the heat exchanger design for Case 3.

	Kern (1950)	**Caputo *et al.* (2008)**	**PSO**	**GWO**
Ds [m]	0.387	0.620	0.603	0.603
L [m]	4.88	1.548	1.122	1.122
B	0.305	0.440	0.500	0.500
do [m]	0.019	0.016	0.015	0.015
Pt [m]	0.023	0.020	0.019	0.019
Cl [m]	0.0040	0.0040	0.0037	0.0037
Nt	160	803	865	865
vt [m/s]	1.76	0.68	0.65	0.65
Ret	36,400	9487	8967	8967
Prt	6.2	6.2	6.2	6.2
ht [W/(m² K)]	6558	6043	8136	8136
ft	0.023	0.031	0.032	0.032
DPt [Pa]	6281	3673	24,241	2956
as [m²]	0.0236	0.0541	0.0603	0.0603
De [m]	0.0130	0.0150	0.0107	0.0107
vs [m/s]	0.94	0.41	0.37	0.37
Res	16,200	8039	4882	4882
Prs	5.4	5.4	5.4	5.4
hs [W/(m² K)]	5735	3476	7230	7230
fs	0.337	0.374	0.342	0.403
DPs [Pa]	67,684	4365	28,506	3444
U [W/(m² K)]	1471	1121	1522	1522
A [m²]	46.6	62.5	45.7	45.7
Ci (€)	16,549	19,163	16,405	16,405
Co (€/yr)	4466	272	2872	1897

CoD (€)	27,440	1671	1665	1166
Ctot (€)	43,989	20,834	18,070	17,571

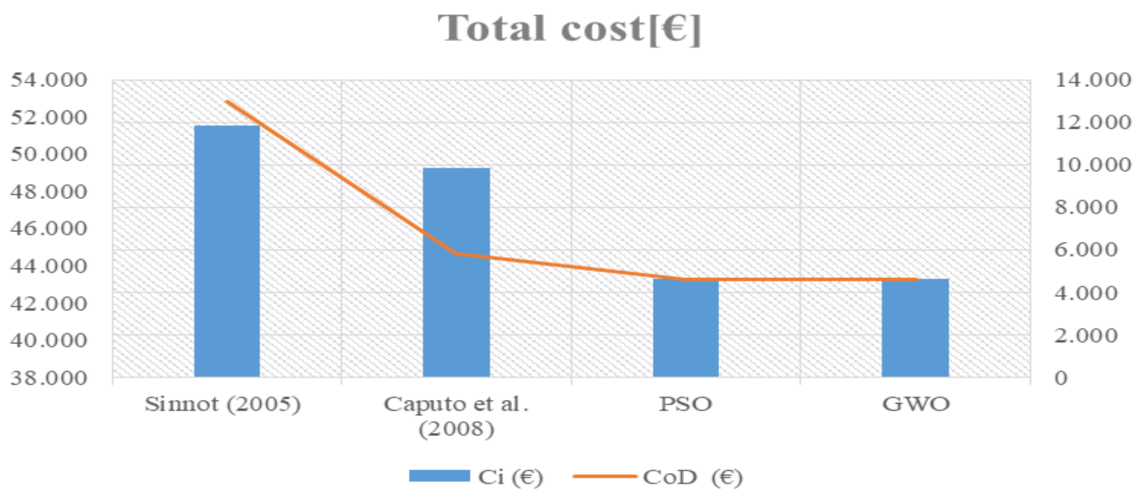

Figure 6. Optimal results comparison of the heat exchanger design for Case 1.

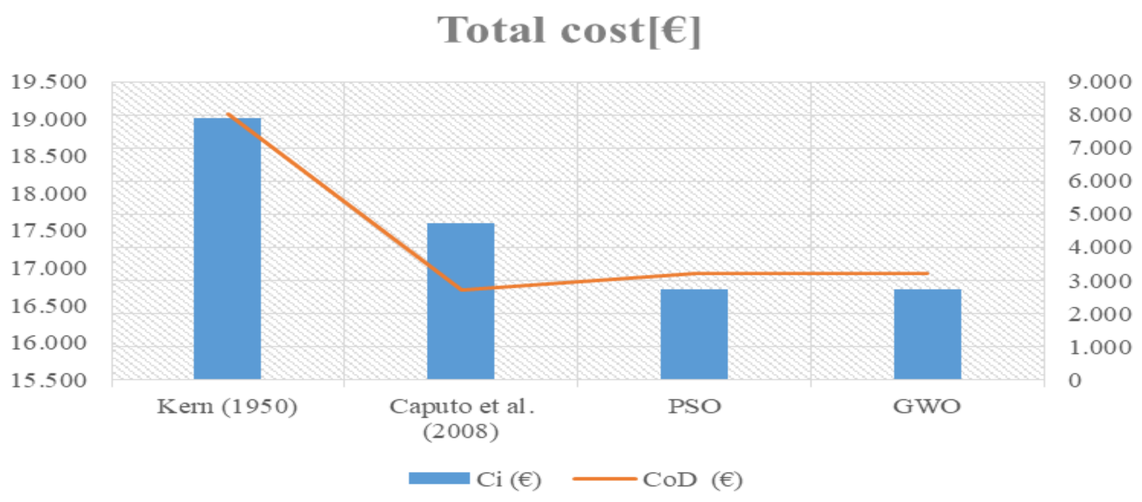

Figure 7. Optimal results comparison of the heat exchanger design for Case 2.

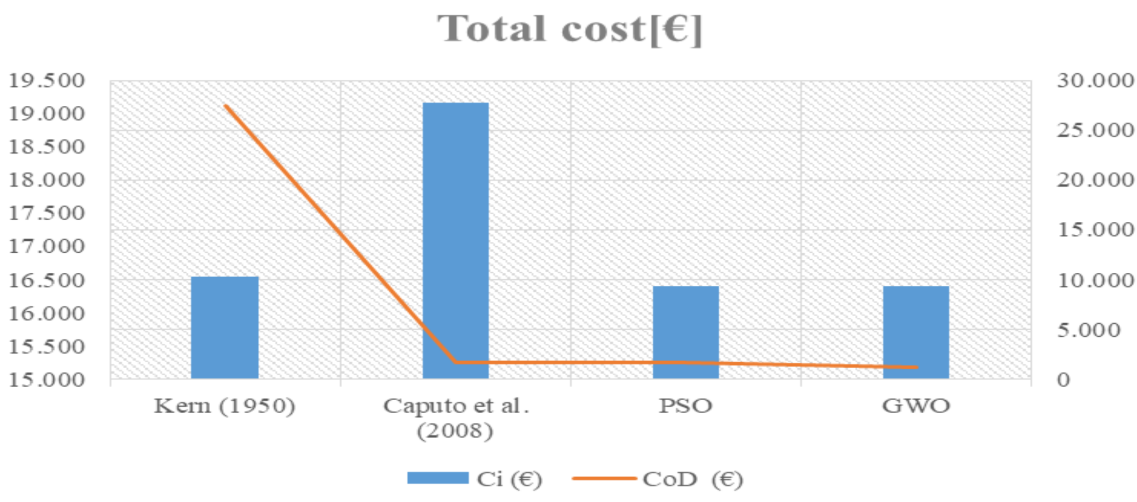

Figure 8. Optimal results comparison of the heat exchanger design for Case 3.

6. CONCLUSIONS

The present work developed a reverse analysis of a shell-and-tube heat exchanger based on Sinnott *et al.* (2005), Kern (1950) and Caputo *et al.* (2008) study that used traditional Genetic Algorithm as optimization technique. Developed analysis used Differential Evolution method and compared results from both techniques, using as objective function total cost taking in account heat transfer and pressure drop. Basing to investigate cases, the reduction of capital investment reached 35 % and 15 % for Sinnott *et al.* (2005) and Caputo *et al.* (2008) of cases 1, respectively.

For Case 2, there was a reduction in the project total cost in almost 36% when compared with Kern (1950) study and 2% compared with Caputo *et al.* (2008). Finally, in case 3, the highest reduction in project total cost was achieved when comparing the results with the work of Kern (1950), where a reduction of 150% and 19% was obtained for Caputo *et al.* (2008), in that order. It is important to note that among the optimization techniques applied in the present work, there were no significant discrepancies between the values achieved, which is due to the similar computational strategies, applied in both techniques.

7. ACKNOWLEDGMENTS

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