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CONVECTION IN A LID-DRIVEN CAVITY SUBJECTED TO A GRAVITATIONAL STABLE CONDITION: EFFECTS OF ASPECT RATIO

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Abstract. *The lid-driven cavity subjected to a gravitational stable condition is used to model transport phenomena in various complex engineering systems. In this work, the cavity verticals walls are adiabatic while the base is kept at a temperature lower than the top sliding-lid, which moves with constant velocity. The action of the gravity gives rise to a condition where buoyancy acts tending to cease the flow and stagnate the fluid. Since the problem is well known for square cavities in terms of the influence of Reynolds (Re) and Grashof (Gr) numbers, a more detailed numerical study regarding the aspect ratio (A=H/L) variation is conduced. By using the finite volume method with the SIMPLE and the QUICK schemes and having a unitary Prandtl number (Pr), simulations are performed in the range from a shallow to a tall enclosure,* $0.5 \leq A \leq 2$*, as well as the values for Re and Gr that characterize laminar flow,* $100 \leq Re \leq 2500$ *and* $10^3 \leq Gr \leq 10^5$*. As expected, the aspect ratio variation affects the flow as if the tall cavity (A>1) favors the occurrence of quiescent fluid resulting in a conductive regime. Alternatively, the shallow enclosure (A<1) benefits the establishment of a convective-dominant configuration since the flow is able to reach regions closer to the enclosure base.*

Keywords: *convection; lid-driven cavity; aspect ratio; gravitational stable condition; numerical simulation*

1. INTRODUCTION

The convection within a lid-driven enclosure saturated with a stable stratified fluid is ubiquitous to a series of engineering applications as pointed out by Khanafer *et al*. (2007) and also for modeling meteorological and oceanic flows, as discussed by Ramakrishna *et al*. (2012). The stable stratified fluid is characterized to be subjected to a gravitational stable condition where the temperature gradient alignment with the gravity is not capable of promoting spontaneously the fluid flow. The gravitational stable is one of the many conditions the fluid within the cavity may experience due to the alignment of the temperature gradient with the gravity. Cheng (2010) studied the effects of the temperature gradient orientation over the lid-driven flow within a square cavity. Works have been reported in which the buoyant-induced flow may effectively assist the flow promoted by the lid movement, increasing the fluid circulation and enhancing the heat transfer. Such configuration is called gravitational unstable and may be accomplished either by horizontal heating (Franco and Ganzarolli, 1995) or keeping the cavity base temperature lower than the one at the lid (Cheng, 2011). Curiously, the numerical results of Cheng (2011) show that the surface-averaged Nusselt number, Nu_{av}, increases with the Reynolds number, *Re*. As for the Grashof number (*Gr*), which is associated with buoyancy, its variation affects the flow in the sense of creating a clock-wise circulation in the enclosure upper half and a counter clock-wise one in the lower half.

In the gravitational stable condition, the flow is solely promoted by the lid-movement and the buoyancy hinders the fluid flow promoting an interesting competitive effect. On one hand, the fluid momentum tends to make the flow penetrate toward the cavity base. On the other hand, there is a stagnant-prone condition due to the action of buoyant forces which tends to restrict the flow to the cavity upper half, leaving the lower half filled with quiescent fluid. Depending on the balance between the Reynolds and the Grashof number, two flow patterns can be observed (Poletto *et al*., 2016). When the *Gr* scale suppresses the *Re*, $O(Gr/Re^2)>1$, the fluid tends to stagnate on the vicinity of the enclosure base and the flow is restrained close to the cavity lid. Such condition is known as conduction-dominant since the transfer of heat across such a quiescent fluid is by diffusion. If the *Re* scale is higher than the *Gr*, $O(Gr/Re^2)<1$, the flow spreads across the enclosure establishing a convection-dominant situation.

The lid-driven flow has been the focus of a number of authors including Iwatsu *et al*. (1993) that studied the relation between the *Gr* and *Re* that grants the fluid to penetrate toward de cavity base in a square cavity with a stable vertical temperature gradient. Such study was widened by the numerical results provided by Poletto *et al*. (2015) and Poletto *et al*. (2016). The effects of the fluid over the convection were numerically investigated by Moallemi and Jang (1992) with respect to the Prandtl number variation. The effects of double lid-driven cavity, where both the top and the bottom walls move with the same constant velocity, was studied by Ouertatani *et al* (2009). Sivakumar *et al.* (2010) analyzed the influence of the position and the extension of the heated portion of the wall over the mixed convection.

Given the importance of the lid-driven flow, a number of studies has been conducted in a sense that the phenomenon is quite understandable in terms of the variation of the Reynolds, the Grashof and the Prandtl number. However, the question concerning the influence of the aspect ratio over the convection remains not fully investigated. Indeed, Mansour and Viskanta (1994) conduced numerical and experimental analysis for a tall cavity with mixed convection. The buoyant forces tried to make the flow occur in anti-clockwise direction, while the shearing of the lid tried to impose a clockwise circulation. Prasad and Koseff (1996) studied experimentally the mixed convection in a tall lid-driven cavity where both the shear and buoyant forces established the flow in the same direction. Mohamad and Viskanta (1995) numerically simulated the flow in a lid-driven shallow cavity subjected to a gravitational stable condition.

In this work, the aspect ratio effect in a lid driven cavity problem subjected to a gravitational stable condition is investigated numerically. Effects over the heat transfer process either on convection or conduction-dominant conditions are presented. While the Prandtl number is kept constant, Pr=1, the flow parameters range are $100 \leq Re \leq 2500$ and $10^3 \leq Gr \leq 10^5$. The geometries considered vary from shallow to tall enclosures.

2. PROBLEM FORMULATION

The problem geometry is displayed in Figure 1, where the vertical walls are adiabatic and the lid is kept at a temperature T_H higher than the bottom wall T_C. The flow is promoted as the lid moves with constant velocity U_H. The cavity has length L and height H with Cartesian coordinate origin set at the bottom left corner, where u and v are the velocity components in the direction x and y, respectively. The gravity is oriented in the y-axis negative direction. The cavity aspect ratio is defined as $A=H/L$ in a way that $A>1$ configures a tall enclosure and $A<1$ a shallow one.

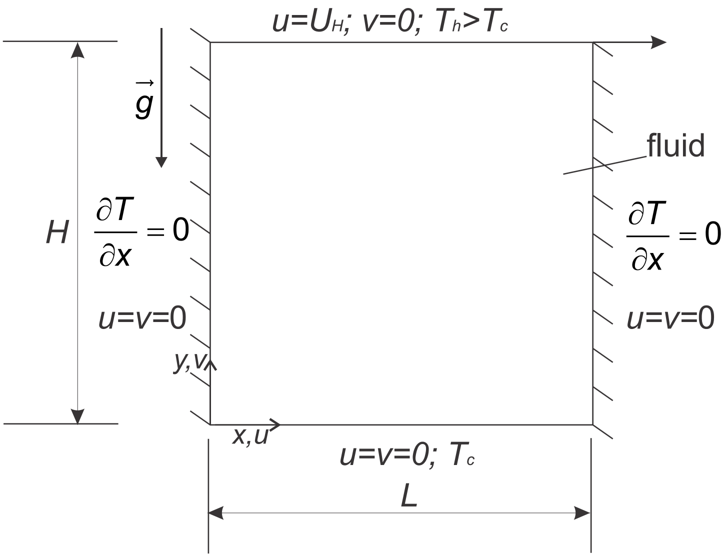

Figure 1. Problem geometry and boundary conditions

The flow is considered laminar, steady and two-dimensional. The fluid is Newtonian and with constant properties, which also includes constant specific mass ρ [kg/m^3], in a sense that it is not necessary to consider compressible flow equations to model the flow. Buoyancy effects are incorporated in the Navier Stokes equations for an incompressible flow, through a source term. The Boussinesq-Oberbeck approximation $\rho_c - \rho \approx \rho_c \beta (T - T_c)$ is employed to couple variation of temperature with the specific mass (Bejan, 2013), with the subscript c denoting the specific mass ρ_c and the base temperature T_c [K], g being the gravity and β [1/K] the fluid isobaric volume expansion coefficient. Such linear approximation between $\Delta\rho$ and ΔT is valid for mild-temperature variations (Gray and Giordini, 1976).

Since the viscous heat dissipation and thermal radiation are not considered, the non-dimensional variables can be put as: $X=x/L$, $Y=y/H$, $(U,V)=(u,v)/U_H$ and $\theta=(T-T_C)/(T_H-T_C)$ and $P=p/\rho_c U_H^2$. Thus, the balance equations can be written as follow.

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \quad (1)$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{1}{Re}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 V}{\partial X^2}\right) \quad (2)$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{Re}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 V}{\partial X^2}\right)+\frac{Gr}{Re^2}\theta \quad (3)$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{Re\,Pr}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial X^2}\right) \quad (4)$$

The non-dimensional groups utilized in the equations are the Reynolds, the Grashof and the Prandtl number, respectively.

$$Re=\frac{LU_H}{\upsilon};\ Gr=\frac{g\beta\left(T-T_c\right)L^3}{\upsilon^2};\ Pr=\frac{\upsilon}{\alpha} \quad (5)$$

A non-slip condition is valid for both the vertical and the bottom walls as the top-lid is at constant velocity U=1.The vertical walls are adiabatic; the cavity base and the sliding-lid are kept at a constant temperatures θ=0 and θ=1, respectively.

$$X=0: U=V=0; \frac{\partial \theta}{\partial X}=0 \quad (6)$$

$$X=1: U=V=0; \frac{\partial \theta}{\partial X}=0 \quad (7)$$

$$Y=0: U=V=0; \theta=0 \quad (8)$$

$$Y=1; U=1; V=0; \theta=1 \quad (9)$$

The Equations (1) - (4) are non-linear with respect to the velocity and are also strongly coupled with respect to the temperature and velocity fields. Therefore, a numerical solution is pursuit via the Finite Volume methods (Patankar, 1980). The method demands the discretization of the domain in a number of finite control volumes following an integration of the fore mentioned equations on each one of the control volumes. A linearization of the equations is accomplished allocating the variables defined on the control surface on the center of the respective control volume through the QUICK interpolating scheme (Leonard, 1982) for the advection terms and the least-square cell-based for the gradients. The solution is based on the pressure, whose values are interpolated and allocated on a staggered-grid. The linearized equations are solved via a multigrid method (Hutchinson and Raithby, 1986) in a segregated form (Chorin, 1968). The pressure is computed through the SIMPLE coupling scheme (Patankar and Spalding, 1972) with the employment of under relaxation factors to further stabilization and convergence of the method.

The flow results are shown using the streamlines, calculated according to Equation (10) for a discrete domain (Kimura and Bejan, 1983). The values Ψ_{ij} and $\Psi_{i,j-1}$ are considered, respectively, at the center of the control volumes situated in the positions i,j-th and i,j-1, respectively.

$$\Psi=\Psi_{i,j}=\Psi_{i,j-1}+\int_0^1 U dY \quad (10)$$

The heat transfer results are evaluated through the surface-averaged Nusselt number computed over the sliding-lid. The Nu_{av} can be related with the surface-averaged heat transfer coefficient, h_{av}.

$$Nu_{av}=\int_0^1 -\left.\frac{\partial \theta}{\partial Y}\right|_{Y=1} dX \Leftrightarrow Nu_{av}=\frac{h_{av}L}{k} \quad (11)$$

3. RESULTS

Results are presented here with the aim to vary the parameters of Table 1. The aspect ratio ($A=H/L$) ranges from shallow to tall enclosure ($0.5 \leq A \leq 2$) for a fluid with a unitary Prandtl number and a range of *Re* and *Gr* that characterizes laminar flow, such as $10^3 \leq Gr \leq 10^5$ and $100 \leq Re \leq 2500$ (Ghia *et al.*, 1982).

Table 1. Summary of problem parameters.

Re	100; 250; 500; 750; 1000; 1500; 2000; 2500
Gr	10^3; 10^4' 10^5
A	0.5; 1; 2
Pr	1

The reproduction of results found in the literature is represented in Table 2 by the means of the Nu_{av} and the quadratic error function, Equation (12). Results obtained by Poletto *et al.*, (2016) for the convection within a square lid-driven cavity subjected to a gravitational stable condition and by Junqueira *et al.*, (2013) for the aspect ratio variation effects are verified simulating the natural convection in a horizontally-heated enclosure. Notably, the *Err* function for both cases is not higher than 1% giving credibility for the current work results.

$$Err = \left(1 - \frac{\Theta_{cal}}{\Theta_{ref}}\right)^2 \tag{12}$$

Table 2. Verification results of Nu_{av} : comparison with literature.

Pr	*A*	*Ra*	*Gr*	*Re*	*Nu*	Poletto *et al.*(2016)	Junqueira *et al.*(2013)	*Err* [%] Eq. (12)
1	1	-	10^3	750	6,3379	6,373	-	0,003
		-		1500	9,0603	9,137	-	0,007
		-	10^5	1000	6.662	6.673	-	0,000
		-		2000	10,273	9,424	-	0,812
		-		2500	11,599	12,059	-	0,146
0.71	0.5	10^5	-	-	4,104	-	4,158	0,017%
		10^6	-	-	8,726	-	8,506	0,066%
		10^7	-	-	16,281	-	16,331	0,001%
		10^8	-	-	30,770	-	30,265	0,028%
	2	10^5	-	-	5,032	-	5,033	0,000%
		10^6	-	-	9,232	-	9,253	0,001%
		10^7	-	-	17,312	-	17,143	0,010%
		10^8	-	-	31,380	-	31,079	0,009%

A grid sensitivity test is carried out to identify the grid size able to produce accurate results in a feasible simulation time. Initially, it is necessary to address the most difficult case to be simulated with respect either to the geometry and the flow parameters. The latter are attained with the calculation of the gradients within the boundary layers. The higher the *Re*, the thinner the boundary layers (Bejan, 2003), so $Re=2500$ configuration is chosen for the mesh test. As increasing the *Gr* yields in flow hindrance, the lower the *Gr* the higher the Ψ and, therefore, $Gr=10^3$ is also selected. As the geometry is a key parameter in the problem, the mesh capable of simulating the most complex geometry is expected to simulate the others. According to Table 1, the enclosures considered can be devised replicating a square lattice ($A=1$) either in the *x*-direction, which gives a shallow enclosure, or in the *y*-direction to obtain a tall one. The shallow cavity has a larger area of contact between the fluid and the sliding-lid aiding the momentum transfer from the lid movement to the fluid. Thus, the mesh capable of solving the $A=0.5$ case would also solve the other configurations of *A*.

The results show an error lower than 1% between the meshes 100x200 and 200x400. For instance, if a grid of 300x600 is considered the error gets even smaller, but the simulation becomes longer. Therefore, the 200x400 grid is chosen for all the simulations. The mesh generation for the shallow enclosure can be considered as the replication of the mesh of square lattice containing 200x200 volumes. Therefore, for $A=1$ and $A=2$ it is considered the uniform grids 200x200 and 400x200, respectively.

The results for the conduction-dominant regime are displayed in Figure 2 (a) and (b) for $Re=100$ and $Gr=10^5$ in terms of the isotherms and streamlines, respectively. As for the square cavity, the streamlines are concentrated near the sliding-lid indicating that the fluid is confined to the upper half of the enclosure. The lower half of the cavity is filled with stagnant fluid and, therefore, diffusion is the only mean of heat transport. Such fact is corroborated by the vertical stratification of the isotherms and the unitary value of the Nu_{av}. For the conduction-dominant condition, the variation of the aspect ratio is not able to reverse the fluid stagnation and consequently the flow is promoted throughout the enclosure. However, the decreasing of *A* enhances the fluid circulation and increases the Nu_{av}, and the fluid remains halted in vicinity of the cavity base. Conversely, incrementing *A* results in a lower value of Ψ and, curiously, a Nu_{av} that even lower than the unity is achieved. The Nu_{av} variation can be explained in terms of Equation (11) that states the functional dependence with the length *L*. As for *A*=1 it yields $Nu_{av}\approx 1$, A=0.5 and A=2 can be achieved, respectively, by reducing in half and doubling the length *L*. such behavior is reflected in the Nu_{av} scale.

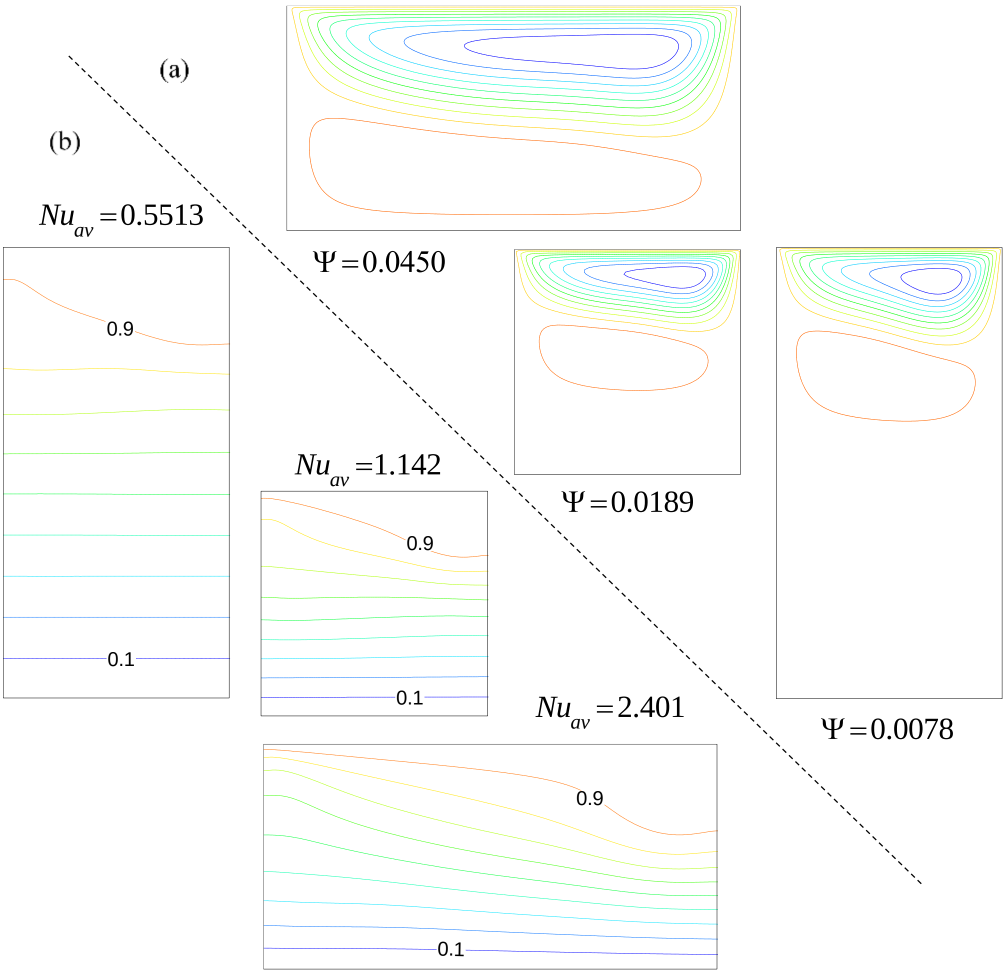

Figure 2. Results for Re=100 and Gr=10^5: (a) Streamlines and (b) Isotherms.

For *Re*=1000 and *Gr*=10^3, the flow in the square cavity (*A*=1) is convective-dominant, as the streamlines of Figure 3 (a), spread across the enclosure. The isotherms, in Figure 3 (b), show a tendency to uniformize the temperature at the center of the enclosure and accumulate the isotherms over its base, indicating a thermal gradient intensification that is corroborated by the increase in the Nu_{av}. Remarkably, an intensification of Ψ and an increasing in Nu_{av} is expected as *Re* increases.

The variation of the aspect ratio plays an important role as the decrease in *A* (shallow enclosure) establishes a convection-prone situation that favors the heat transfer enhancement since the flow easily reaches the bottom of the cavity. Additionally, in this configuration one can observe a larger area to exchange heat both with the lid and the enclosure base. The shallow cavity also shows a higher value for Ψ indicating that the fluid finds less resistance to flow and, therefore, a higher value for Nu_{av} is obtained. On the other hand, in the tall cavity the fluid near the base remains quiescent and the heat has to be diffused by approximately half of the height of the domain. Consequently, a unitary Nu_{av} that is typical of conduction-dominant situation is observed.

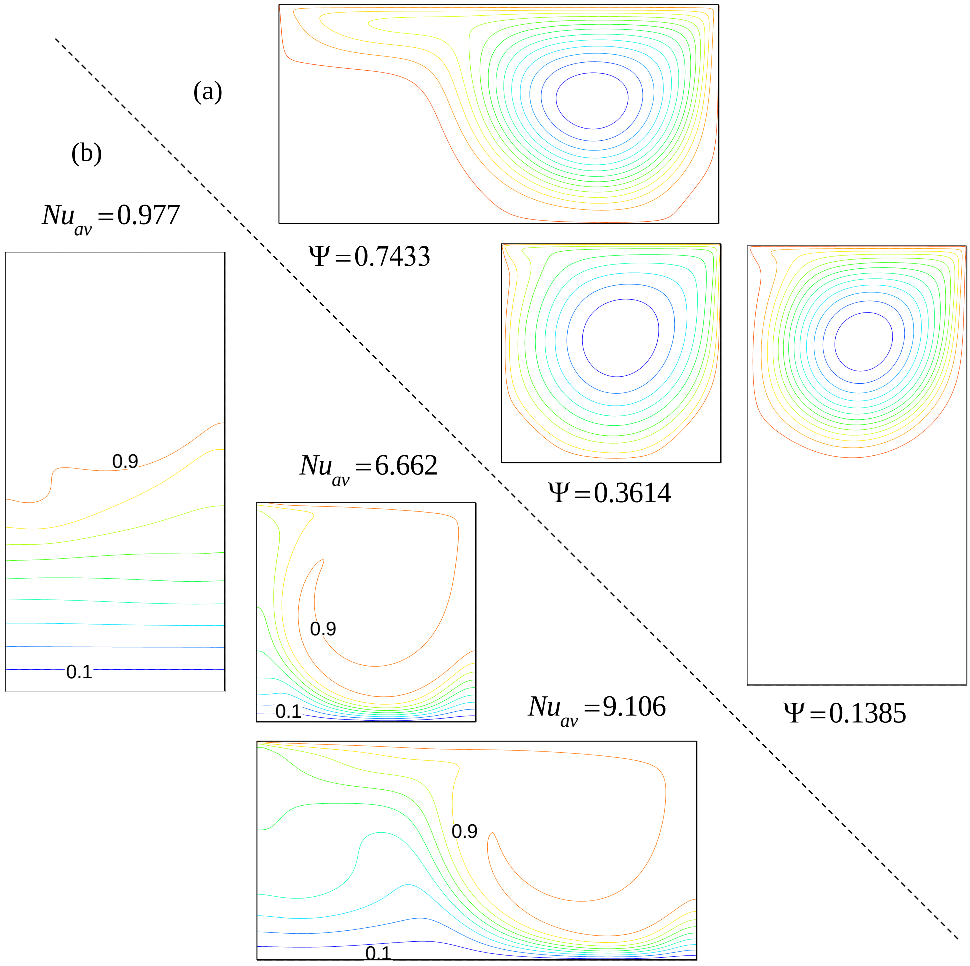

Figure 3. Results for *Re*=1000 and *Gr*=10^3: (a) Streamline and (b) Isotherms.

A summary of the results for the surface-averaged Nusselt versus *Re* is presented in Figure 4 for a number of values of *A* and *Gr*. One can be observe that the highest values for Nu_{av} are found for the shallow cavities (A<1) in which the predominance of convection is verified. The tall cavity (A>1) shows the lowest values for average Nusselt, since the fluid finds more resistance to penetrate the regions toward the base. Additionally, for low values of *Re* the aspect ratio shows a mild difference in the value of Nu_{av}, once there is predominance of the conduction in all the cavities, but as the values of *Re* increases the aspect ratio has a more significant effect in the heat transfer mechanism. In the shallow cavity it is easier to establish a convection heat transfer than for tall one.

4. CONCLUSION

In this work, the convection in a lid-driven cavity under the restraining effect of buoyancy is numerically simulated. Noticeably, there is a competitive effect between the fluid momentum and the buoyancy which results in two possible heat transfer patterns: convection-dominant and conduction-dominant. In the latter, the flow is restricted to the adjacency of the lid and in the lower half of the enclosure the fluid is stagnant. In such situation, the variation of the aspect ratio does not influence significantly the heat transfer as there would always be stagnated fluid near the cavity base. Conversely, for the convection-dominant configuration, the shallow enclosure increases the Nu_{av}, while in the tall enclosure there is quiescent fluid near the base.

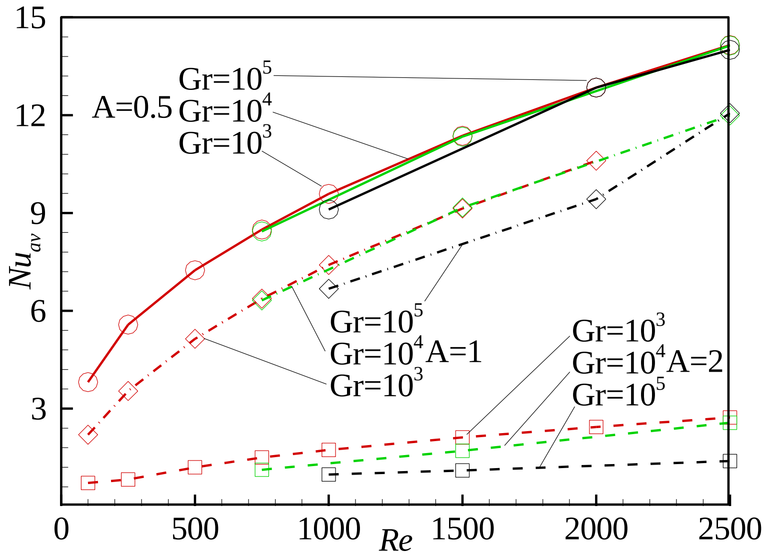

Figure 4. Effect of the aspect ratio and the flow parameters over the Nu_{av}.

5. ACKNOWLEDGEMENTS

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