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STRESSES MEASUREMENTS IN SIMPLE SHEAR TEST FOR A POLYMER UNDER LARGE DEFORMATIONS

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Abstract. *Simple shear apparatus (SSA) is proposed to measure the normal stress component of a thin sheet of solid polymer at large deformations. Displacement fields of the test specimen were estimated by the Digital Image Correlation (DIC) method. The measured forces and the displacement were used to evaluate the shear stress, normal stresses and the amount of shear. Hence, a nonlinear stress-strain response was achieved. The measured normal stress was compared with those calculated from the experimental data by assuming two hypotheses: the first one was based on a plane stress condition, while, on the second, the normal component of the traction on the inclined surfaces was assumed to be zero. Finally, the initial shear modulus of the material was estimated using the Lopez-Pamies strain energy function. The stresses results indicate that there are no agreement between measured values and the ones calculated by both hypotheses. Moreover, the initial shear modulus obtained is accordance with that found in the literature. In addition, the principal advantages of the proposed apparatus shear test are a simple assembly and a wide range of shear strains is achieved.*

Keywords: *large deformation, polymer, simple shear apparatus, stresses components*

1. INTRODUCTION

Recently, there has been growing interest in the state of simple shear due to its importance to characterize mechanical properties of rubber-like material as soft biological tissue (Mihai *et al*., 2015; Destrade *et al*., 2012, 2008). This is also reported in many textbooks on nonlinear theory (Gurtin *et al*., 2010; Ogden, 1997; Holzapfel, 2008; Truesdell and Noll, 1965), however, some points concerning stress components remain unclear. Simple shear test is not easy to perform experimentally (Brown, 2006). Therefore, few experimental works on simple shear have been reported in the literature. Mooney (1944) reported a series of measurements on soft rubber using a thin walled hollow cylinder. More recently, Nunes (2011, 2010) studied the nonlinear mechanical behavior of a hyperelastic material under small and large simple shear deformations by a single lap shear test. Moreover, Nunes and Moreira (2013) determined the normal stress components from the experimental data by assuming two hypotheses: the first one was based on a plane stress condition, while, on the second, the normal component of the traction on the inclined surfaces was assumed to be zero. Some points concerning mechanical behavior on shear state are still open to discussion. Despite all previous contributions, it is evident that further investigation based on experimental and theoretical studies is essential in order to understand simple shear. In this context, the present work develops a simple shear apparatus in order to measure the stresses components of the specimen manufactured with a hyperelastic material. In this test, the full-field displacements were evaluated employing a powerful and noncontacting method, known as Digital Image Correlation (DIC). The applied force and the measured angular distortion were used to evaluate the shear stress and the amount of shear.

2. EXPERIMENTAL PROCEDURE

In this section, information about the proposed simple shear apparatus (SSA) configuration is presented. Fig. 1 shows a schematic representation of the apparatus. One load cell is fixed axially (direction $\mathbf{X_1}$) on an aluminium plate support while the second load cell is fixed perpendicular (direction $\mathbf{X_2}$) to another plate support that is clamped on the testing machine. Besides, the supports have "U" shape in order to avoid rotation e lateral displacement of the load cells. Due to its simplicity, the SSA is an attractive approach, however, it is necessary perform tests with a range of materials. For the first test, a rectangular specimen of pesilox silicone adhesive, that presents high flexibility and elasticity and had a length of 70 mm, width of 55 mm and thicknesses equal to 2.5 mm was used. The experimental arrangement was composed of the simple shear apparatus that was mounted on motorized testing machine (model Impac AEV-5000) and the DIC system, as illustrated in Fig. 1. It is necessary to ensure that two plates move parallel towards the applied load, keeping the distance 4.5 mm between of them. Accordingly, the left plate was kept fixed, while the right plate, mounted on testing machine, could move in the down vertical direction. A CCD camera (Sony XCD-SX910) set perpendicularly to the specimen was used for capturing the images. All images were acquired using a 10 Zoom C-Mount lens. It is important to remember that the experiments were carried out in low-velocity of 8 mm/mim and at room temperature, i.e., 25 º C. The Digital image correlation (DIC) method was employed to measure full-field displacements of the specimen in order to evaluate the amount of shear.

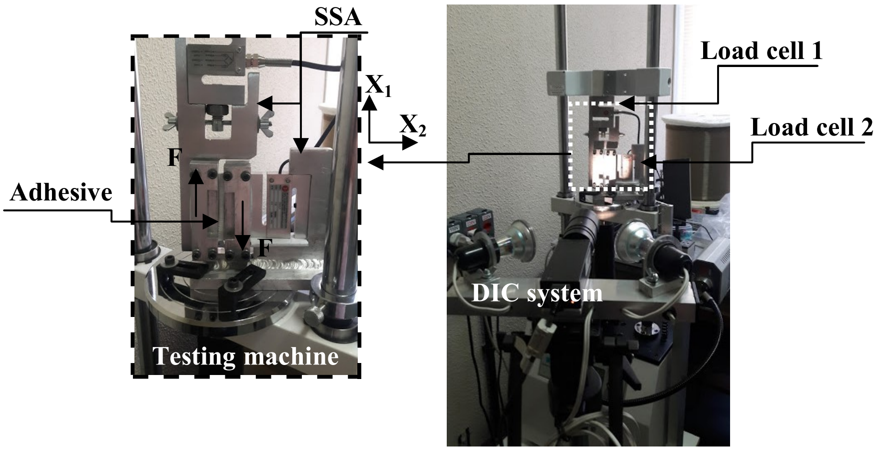

Figure 1. Experimental setup with detailed SSA specimen.

3. STRESS COMPONENTS FOR SIMPLE SHEAR

Fig. 2 illustrates schematically stress components for simple shear specimen. A rectangular block of specimen under applied load **F** is associated to a small rectangle on the surface at the central region of the sample (adhesive).

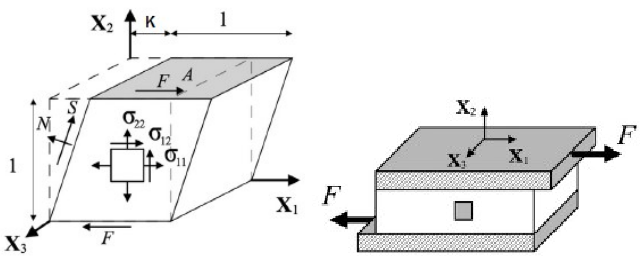

Figure 2. The stress components for simple shear deformation.

In the present investigation, the Cauchy shear stress is defined by σ_{12} = F/A, where F and A are the applied load and the unstrained cross-sectional area, respectively. According to Nunes and Moreira (2013), two approaches are commonly used in order to solve the problem of simple shear: the first one is based on a plane stress condition, in this case $\sigma_{33} = 0$; the second hypothesis assumes that the normal component of the traction on inclined surfaces is equal to zero, i.e. $N=0$ (Rivlin,1948; Gent et *al.*, 2007; Horgan and Murphy, 2010). The tangential S and normal N components on inclined surfaces are illustrated in Fig. 2. As pointed out by Rivlin (1948) and used by Horgan and Murphy (2010), these tractions can be expressed by

$$S = \frac{\sigma_{12}}{1+k^2} \text{ and } N = \sigma_{22} - kS \tag{1}$$

Assuming the hypothesis of plane stress condition, the Cauchy stress tensor components are given by:

$$\begin{aligned} \sigma_{11} &= k\sigma_{12} \\ \sigma_{22} &= \sigma_{33} = 0 \\ \sigma_{12} &= \frac{F}{A} \end{aligned} \tag{2}$$

In contrast with the previous approach, if one considers that the normal component of the traction on inclined surfaces is equal to zero (*N=0*), the equations for the Cauchy stresses are:

$$\begin{aligned} \sigma_{11} &= \frac{k(2+k^2)\sigma_{12}}{1+k^2} \\ \sigma_{22} &= \sigma_{33} = \frac{k}{1+k^2}\sigma_{12} \\ \sigma_{12} &= \frac{F}{A} \end{aligned} \tag{3}$$

It should be mentioned that a class of universal relations for isotropic elastic solid may be described by the coaxiality of the left Cauchy-Green strain tensor and Cauchy stress tensor, i.e. $\boldsymbol{\sigma}\mathbf{B} = \mathbf{B}\boldsymbol{\sigma}$, as presented by Beatty (1987). Thus, the trivial relations are given by $\boldsymbol{\sigma}_{13} = \boldsymbol{\sigma}_{23} = \boldsymbol{\sigma}_{31} = \boldsymbol{\sigma}_{32} = 0$ and the usual one

$$\sigma_{11} - \sigma_{22} = k\sigma_{12} \tag{4}$$

An important property that not depends of the type of experimental test is the initial shear modulus. For simplicity, the elastic behavior is assumed in terms of a strain-energy function that depends only on first invariant. It was considered an incompressible, homogeneous, isotropic material and time-dependent effects were neglected. There are in the literature several models, based on strain-energy function, to describe the mechanical behavior of hyperelastic material. However, Lopez-Pamies model presented good results (Lopez-Pamies, 2010). The shear stress can be evaluated, considering this model defined as

$$\sigma_{12} = \mu k\left(\frac{k^2+3}{3}\right)^{\alpha-1} \tag{5}$$

where μ is the initial shear modulus and α a material parameter, with $\mu > 0$ and $\alpha > 0.5$.

4. RESULTS AND DISCUSSION

Full-field displacements of a small rectangle on the surface at the central region of the sample (see Fig. 2) were estimated using the DIC method. The $\mathbf{X_1}$ and $\mathbf{X_2}$ displacement fields of the selected region, related with vertical and horizontal directions, for an applied load of 50 N, are illustrated in Fig. 3 (a) and (b).

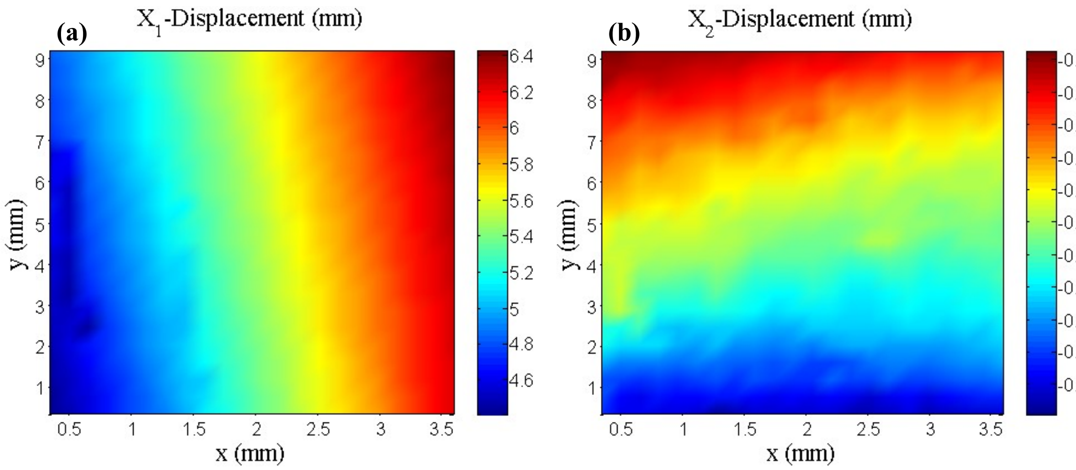

Figure 3. Full-field displacements: (a) vertical and (b) horizontal displacements.

It can be noted that the $\mathbf{X_1}$-displacement (vertical) field varies linearly along the horizontal direction (X_2), while the $\mathbf{X_2}$-displacement field does not present significant variation. This indicates that an angular distortion was generated and the horizontal distribution remained almost underformed. Displacement fields data were used to determine the amount of shear *k*.

Shear stress was calculating considering the applied loads per cross-sectional area of each specimen. The mean values of shear stress and standard deviation from two repeated measurements versus amount of shear are shown in Fig. 4. One can notice that the relation between shear stress and amount of shear is nonlinear, as reported in the literature (Janmey *et all*, 2007; Nunes, 2010, 2011; Moreira and Nunes, 2017). According to the results, the sample is softening in shear.

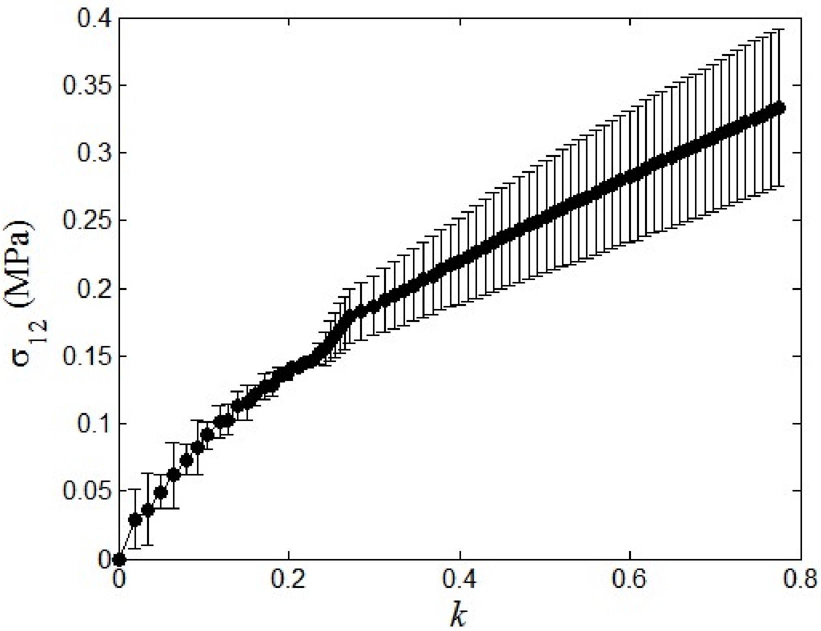

Figure 4. Shear stress versus amount of shear.

As previously described in Section 3, the normal stress components can be expressed as functions of shear stress assuming two approaches, i.e. N = 0 or plane stress ($\sigma_{33} = 0$). Thus, by substituting the experimental data of shear stress into Equations (2) and (3), the normal stress components were achieved. The force component created during the test on direction $\mathbf{X_2}$ was measured by load cell 2 (Fig.1), positioned perpendicular to direction of applied load **F**. Results of normal stress components σ_{22} and σ_{11} as functions of the amount of shear are presented in Fig. 5 and 6, respectively. The lateral stress σ_{22} measured assumes zero for small values of *k*. However, for large values of *k,* σ_{22} assumes small negative values, achieving -0.02 MPa. Therefore, the second normal stress measured from SSA is closer to those values predicted by the hypothesis of plane stress state. On the other hand, the values for σ_{22} estimated considering the hypothesis of *N=0* (Eq. 3.b) present significant differences with experimental data. Thus, the hypothesis of the normal component of the traction on the inclined surfaces was assumed to be zero is not adequate for this case. It is known that most materials have a tendency to expand or contract in the perpendicular direction to the applied shear stress, yielding a positive or negative normal stress (Mihai and Goriely, 2013; Janmey, *et all*, 2007). It can be seen from Equations (2.b) and (3.b) that $\sigma_{33} = \sigma_{22}$ for both hypotheses.

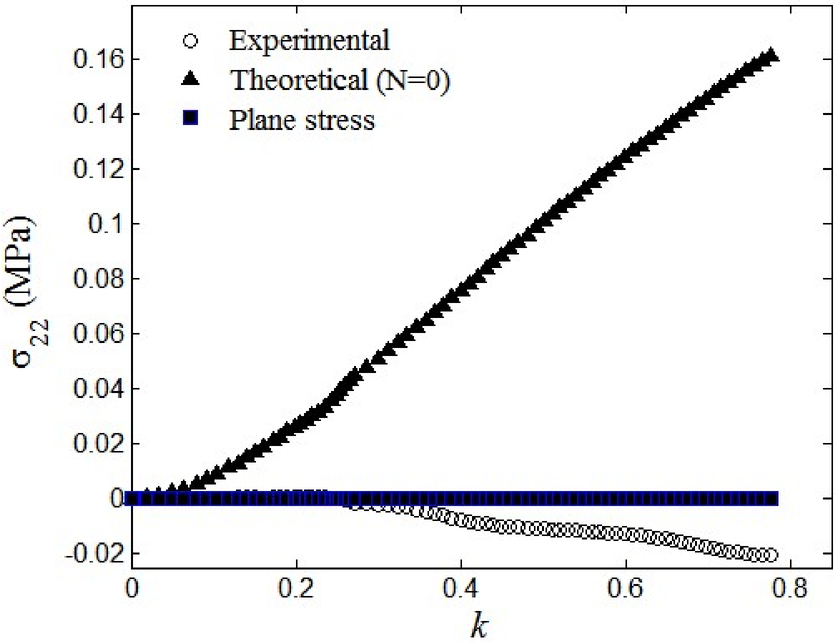

Figure 5. The normal stress component σ_{22} for simple shear deformation.

One formulation for stress component σ_{11} was obtained using the universal relation given by Eq. (4), considering measured values of the component σ_{22} and shear stress σ_{12}. Is important to emphasize that σ_{11} determined by Eq. (4) coincides with those values estimated by hypothesis of plane stress Eq. (2), due to the component σ_{22} appears small values as discussed previously. Although the qualitative features of all three plots for σ_{11} are the same, it can be seen from Fig. 6 that the hypothesis of plane stress and universal relation form yields values about two-thirds the values given by the hypothesis of *N=0*.

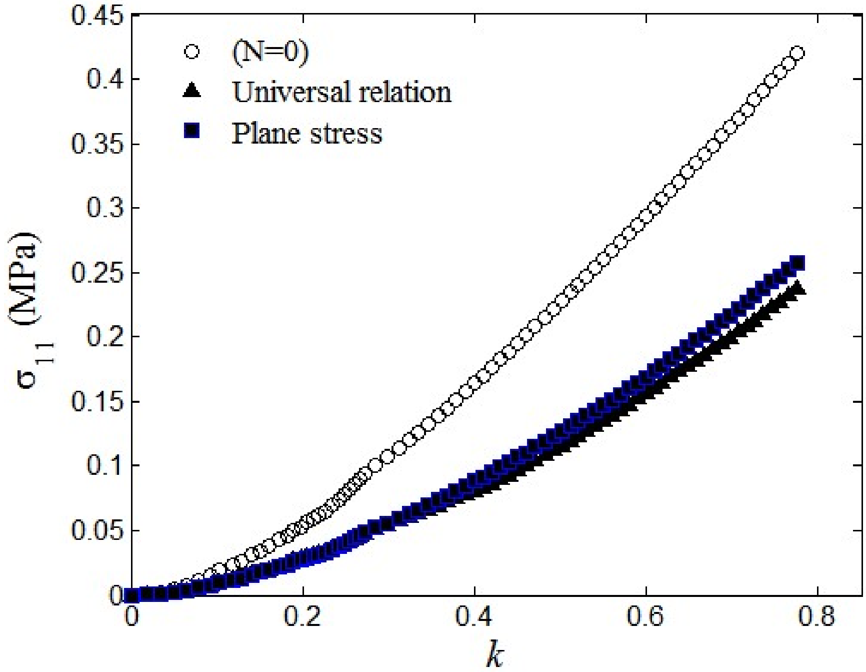

Figure 6. The normal stress component σ_{11} for simple shear deformation.

Fig. 7 illustrates the components tangential S and normal N on the inclined faces as functions of amount of shear obtained using Eq. (1). It is possible to notice that the normal and tangential traction have positive and negative values, respectively. It can see that $S < \sigma_{12}$ and $\sigma_{22} > N$, as σ_{22} is negative so is N (Atikin and Fox, 1980). Moreover, for the range of k in this study, both components have the same order, showing that the deformation is homogeneous (Destrade *et al.*, 2012).

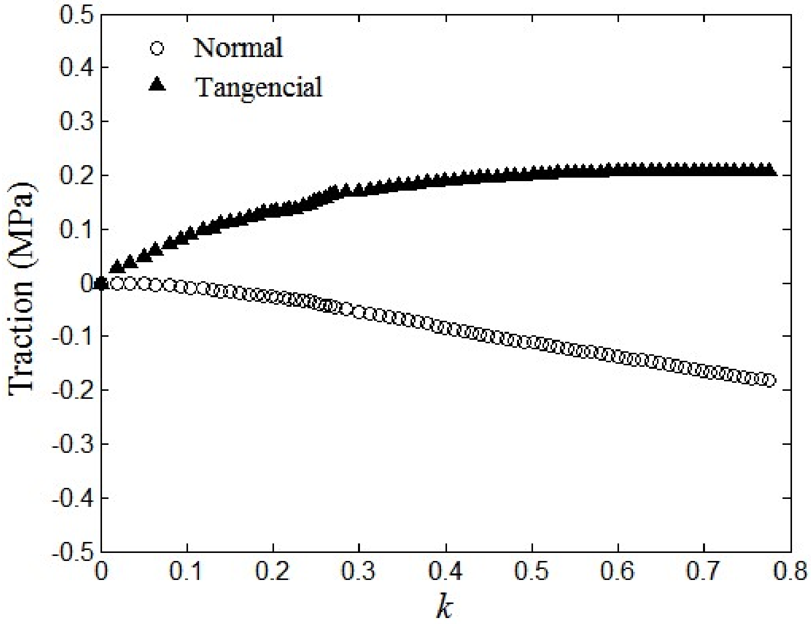

Figure 7. Normal and tangential components of the traction on the inclined faces versus amount of shear.

The Lopez-Pamies model (see Eq. (5)) was used to describe the mechanical behavior of adhesive under simple shear test using the SSA. The obtained material parameters values were $\mu = 0.518$ and $\alpha = 0.5$ (fixed at bound) for the model. Consequently, the value of initial shear modulus was ≈ 0.5 MPa. The results of experimental data and fitted model for simple shear are illustrated in Fig. 8. Moreira and Nunes (2017) investigated the mechanical behavior of fiber-reinforced incompressible nonlinearly elastic solids under large simple shear deformations. They estimated the initial shear modulus of the polymeric matrix (pesilox) using single lap shear test and obtained 0.5 MPa. Therefore, the SSA is able to perform simple shear test of materials under large deformations.

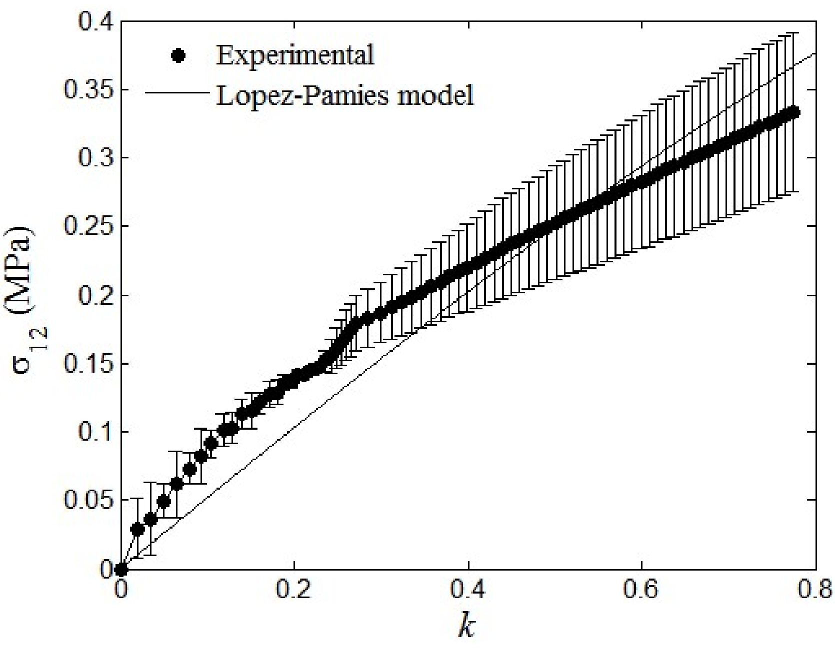

Figure 8. Comparison between experimental data and Lopez-Pamies model.

5. CONCLUSIONS

In this study, a modified apparatus shear test is proposed and used to obtain the shear stress and normal stresses response of a polymer under large deformations. For range of deformations evaluated, there are no agreement between stress measured values and the ones calculated by both hypotheses: plane stress and normal component of the traction on the inclined surfaces null. Besides, the initial shear modulus obtained is close to that achieved using a well known single lap shear test. Finally, the proposed apparatus for shear test was demonstrated to be effective in the present case, in which a simple assembly and wide range of shear strains was achieved.

6. ACKNOWLEDGEMENTS

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