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THE INFLUENCE OF GEOMETRY ON THE YOUNG-LAPLACE METHOD

Alexandre Zabot
Federal University of Santa Catarina, Mobility Engineering Departament, Joinville, Brazil
alexandre.zabot@ufsc.br

Abstract. *The simulation of two-phase flow of fluids in porous media is a problem with very important practical applications. However the simulation time is so high for most computational methods available that it becomes prohibitive when the geometry is too large. The Young-Laplace Method (YLM) is a numerical method that predicts approximate results with much lower computational cost. Although YLM is indicated in the literature as a method with great predictive potential, it is still unclear in which situations it can be used with confidence. The geometry of the porous medium can be a crucial factor in determining when the method works well or not. In this work an investigation of the influence of the geometry of the porous medium on the results of the YLM is made. For this, several geometries, regular and irregular, are created, following ordering criteria in spatial distribution. For each geometry a simulation of two-phase flow is made with the YLM. Fluids saturation during the invasion are compared with expected results and with each other. Finally, it is shown that the geometry does not play a crucial role in the results of the YLM. The most important factor is that the Digital Rock is larger than the Representative Elementary Volume regardless of whether the geometry is organized or not.*

Keywords: *Young-Laplace Method, Mathematical Morphology, Two-phase flow, Porous medium.*

1. INTRODUCTION

A porous medium is a material composed of both solid and porous regions and with a complex geometry. The pores may have similar or varying sizes. Figure 1 shows a slice of a typical rock with its porous geometry.

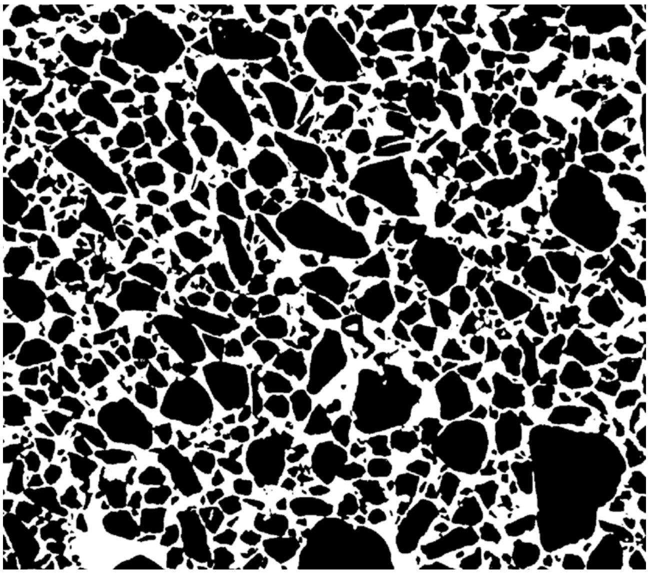

Figure 1. Adaptation of Fig. 4b of Sheppard *et al.* (2006). It is shown the geometry of a typical porous rock medium. Black represents the solid parts and white pores.

Understanding the flow of fluids in these porous media is a very important physical problem because several applications depend on this knowledge. Examples include the responsible use of aquifer reserves as well as the problem of their contamination by liquid pollutants. The recovery of rock oil or even medical issues related to bone physiology are other examples.

To understand the physics of these processes it is necessary to have a proper representation of the system geometry and a numerical technique to simulate the flow. The geometry can be generated artificially based on hypotheses about the formation of each system or obtained in the laboratory with, for example, x-ray tomographers.

The flow simulation, however, is more complicated because there are a myriad of techniques available. Of course, each technique has an application scope and degree of accuracy and a particular computational cost.

In this paper we will study the conditions under which the Young-Laplace Method (YLM) is able to reproduce the two-phase flow of fluids in porous media. The YLM is based on the Young-Laplace's equation to identify flow equilibrium states for a given fluid geometry, pressure, and saturation. Since it does not determine the velocity field, but predicts the various equilibrium steps, it becomes a very fast numerical technique. The method is a simplification of the physics of the two-phase flow when C_a tends to zero. That is, when fluid inertia is negligible. The application of the method outside this scheme leads to inaccuracies. In particular, the YLM is not able to predict dynamic variables of the flow process, such as flow velocities, permeabilities, etc. But the YLM is able to predict the saturation of the two fluids for a given capillar pressure.

The objective of the YLM is to consider in a realistic manner the geometry and complex topology of a porous system to simulate two-phase flow of immiscible fluids. The microtomography technique applied to reservoir rocks allows to obtain very realistic 3D images. It was aiming to analyze this microCT images that Hazlett (1995) and Magnani *et al.* (2000) created this algorithm.

The central idea of the YLM is to map equilibrium states of a fluid invasion into a porous medium previously occupied by another fluid. In these equilibrium states, the invasion front have a boundary described by the Young-Laplace equation (Magnani *et al.*, 2000),

$$r \propto \frac{1}{|P_V - P_A|} \tag{1}$$

where r is the radius of the meniscus of the invasion front and is related to the pressure difference between the two fluids (P_V and P_A), as shown in Fig. 2.

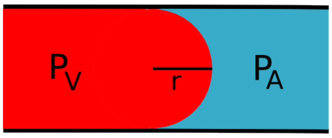

Figure 2. Invasion front in a capillary tube, according to Eq. (1)

Equation (1) associates the meniscus radius with a pressure difference between two immiscible fluids. Starting from the premise that the invasion of a channel only happens when the meniscus radius is less than or equal to the radius of the channel, one can think of successive stages of invasion happening every time the pressure difference between the fluids increases. This successive invasion is shown in Fig. 3.

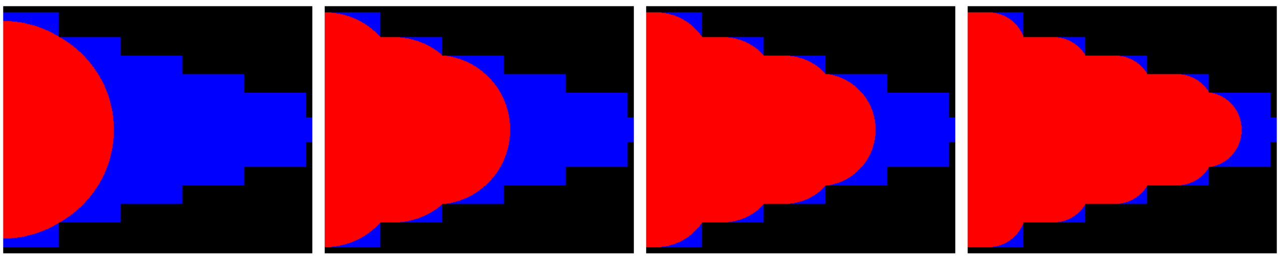
Figure 3. Invasion of the red fluid as the pressure difference between the two fluids is increased, causing the meniscus radius to decrease and to be able to access narrower channels.

The steps of the simulation are described by Hazlett (1995), Magnani *et al.* (2000) and Hilpert and Miller (2001), with an important correction of the algorithm being indicated by the latter.

Experience with YLM simulations shows that it does not work well in very regular geometries (Fig. 4). The invasion takes place in a single (or few) pressure step. However, it was not possible to identify exactly which are the relevant geometric criteria.

Of course it is not possible to use YLM as an alternative to slower methods (ex LBM) if one does not know in more detail which criteria determine whether it works well or not. For this, it is needed to test the YLM with many different geometries. Fortunately, the YLM is fast enough to allow to investigate various geometries in a feasible computational time.

It was possible to obtain pressure saturation curves for digital rocks of oil reservoirs with YLM (Zabot *et al.*, 2014). This information is very important for the oil industry to characterize the oil reservoirs. In this way, it is interesting to test

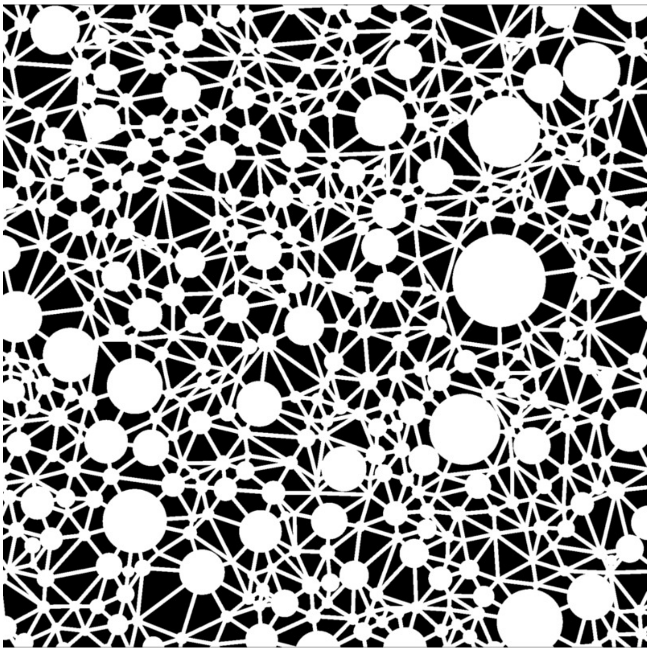

Figure 4. The YLM often fails in very regular geometries.

with several geometries if the YLM provides good results for this problem, following a procedure that allows to identify in which one (and why) the method succeeds.

To perform this investigation, it is necessary which identify the important parameters to characterize a porous medium geometry. Thus it will be possible to develop a clear criteria of which geometric parameters influence the quality of YLM results.

2. COMPUTATIONAL PROCEDURE

In this work the relavance of the geometry disorder to the saturation calculated by the YLM is investigated. In order to test these effects, a perfectly ordered geometry was obtained and then mixed up. A given number of circles were initially regularly disposed and then these circles were gradually moved, without overlapping, according to the Fig. 5.

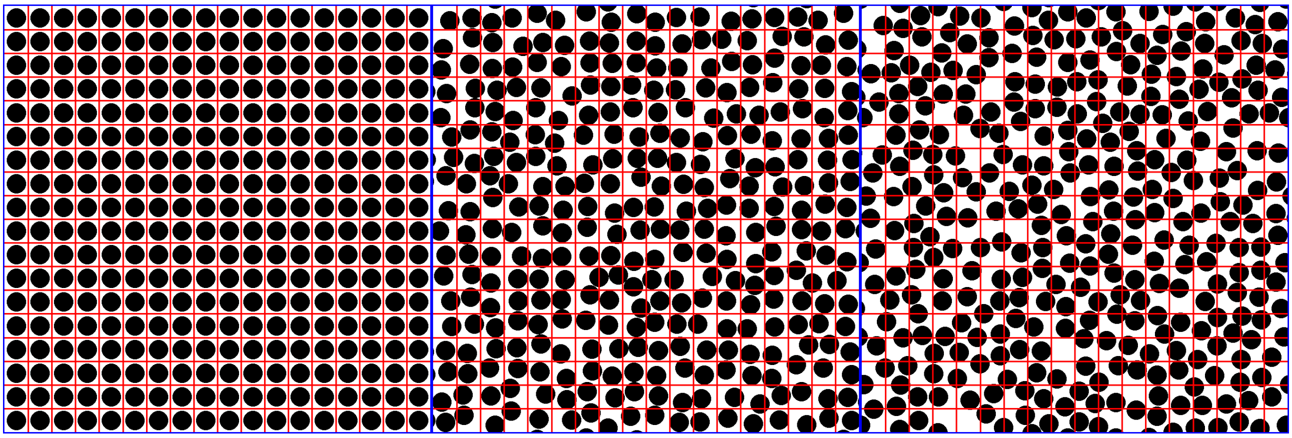

Figure 5. Measurement of the disorder in the geometry by the quadrat method. The quadrats are shown in red.

Because the circles do not overlaps, the porosity of the geometry remains constant in every steps, given by:

$$\phi = 1 - \frac{\pi}{4}\left(\frac{N_c}{S/D}\right)^2 \tag{2}$$

where N_c is the number of circles per line and S the square's edge, in units of circle's diameter (therefore $S \leq N_c$).

The image disorder was quantified with the Quadrat Method (Krebs, 1999). The disorder is the standard deviation of the number of pixels in the squares (Fig. 5). Figure 6 shows the normalized standard deviation values for each step of moving the circles. The number of squares was set equal to the number of circles. It was chosen three geometries ($\sigma/\sigma_{max} = \{0, 0.5, 1\}$), representative of three states of disorder.

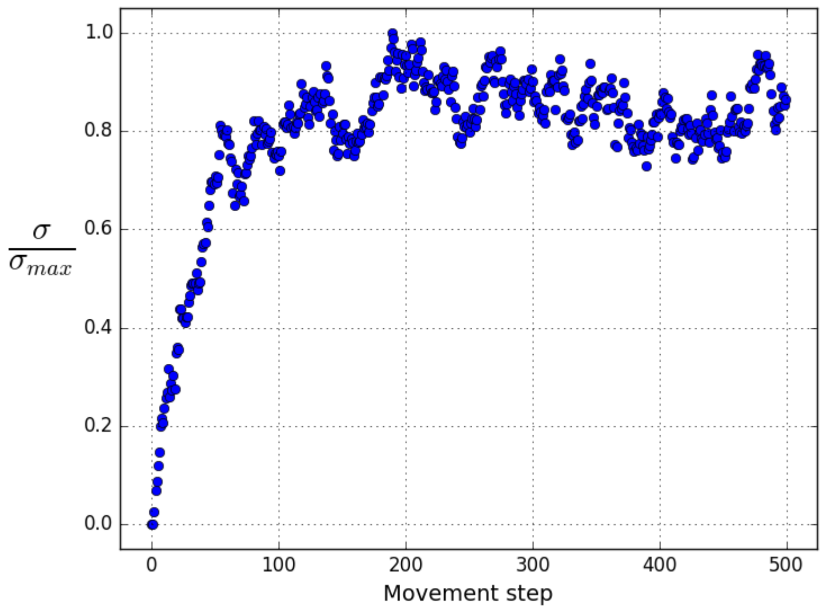

Figure 6. The standard deviation of the number of pixels in the squares (σ) gives the disorder of a given geometry. As the circles are randomly moved, the disorder increases.

To ensure that only the geometry effects were being tested on the saturation given by the YLM, it was needed to check if the artificial geometries were above the Representative Elementary Volume (REV). The REV is a very important quantity in digital rock physics simulations. A given physical quantity measurement or simulation can not depend on the size of the rock sample, otherwise the measured value can not represent the geometry as a whole.. Ideally one could always get a large enough rock sample in which this dependence do not exist. This minimal size of the rock sample is the REV. It is clear that the REV will vary with the physical quantity studied. To test if the geometries were above the REV, it was used five image sizes: S, $2S$, $4S$, $8S$, $16S$. The saturation curves should be similar for all the geometries above the REV.

In addition, it was also tested if the results change with the geometry porosity. Two values of porosities were tested: 0.47 and 0.72. Tables 1 and 2 list the parameters of the geometries and Fig. 7 show examples for a given disorder step.

Table 1. List of parameters for the geometry with $\phi = 0.47$.

N_c	$S(px)$	$D(px)$	S/D
18	594	27	22
36	1188	27	44
72	2376	27	88
144	4752	27	176
288	9504	27	352

Table 2. List of parameters for the geometry with $\phi = 0.72$.

N_c	$S(px)$	$D(px)$	S/D
6	570	57	10
12	1140	57	20
24	2280	57	40
48	4560	57	80
96	9120	57	160

Therefore, for each porosity, five geometry sizes were created (Tab. 1 and 2) and the circles were moved randomly until σ/σ_{max} reached a plateau (Fig. 6). Then the saturation curve was obtained with YLM for two geometry realizations

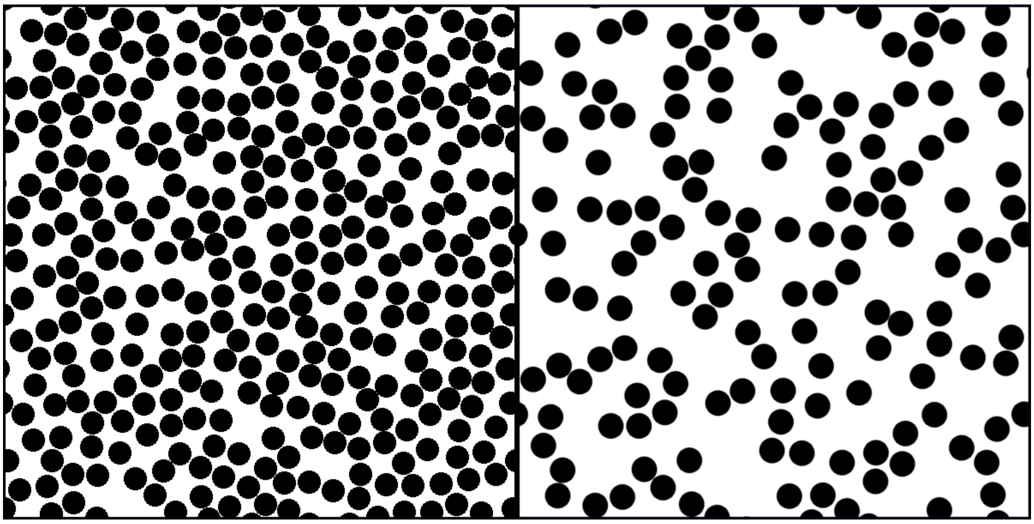

Figure 7. Example of geometries with porosities 0.72 (left) and 0.47 (right).

(σ/σ_{max} = 0.5 and 1). To get stronger results and avoid effects of the randomness of the geometries, each simulation was repeated five times.

3. RESULTS AND DISCUSSION

Figures 8 and 9 show the saturation curves for the different porosities, geometries size and disorder steps. The errors bars were calculated based on five realizations of the same simulation. Figure 10 shows five realizations of invasion for porosity 0.47 for each N_c of Tab. 1. Figure 11 shows the equivalent for porosity 0.72 and Tab. 2.

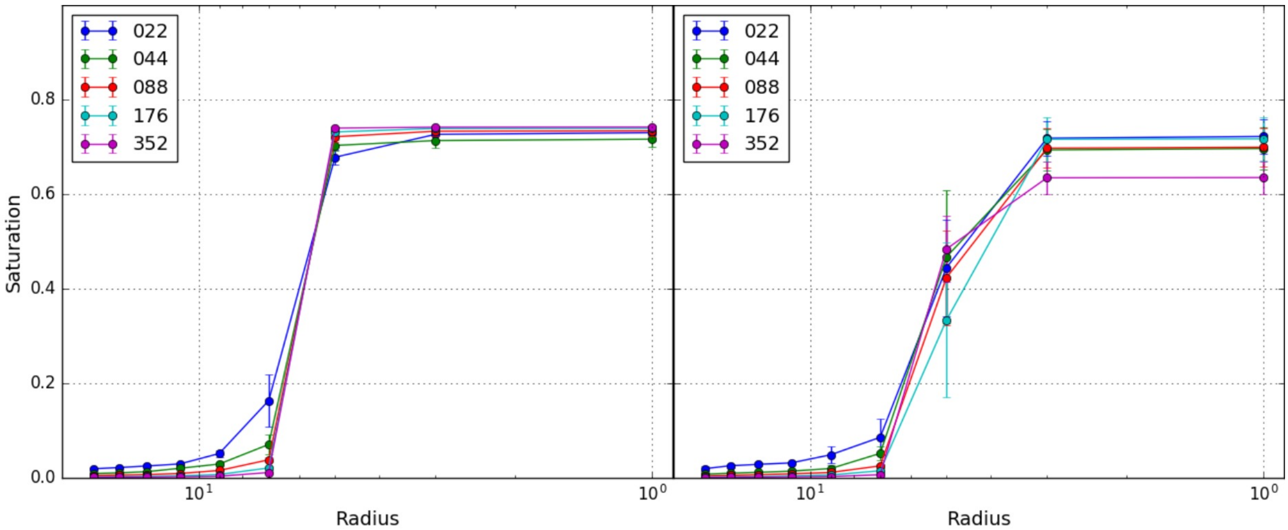

Figure 8. Saturation curves with YLM for $\phi = 0.47$. On the left for $\sigma = 0.5$ and on the right for $\sigma = 1$. Different curves on each image represent geometry sizes.

For each porosity and disorder state, the saturation curves for different geometry sizes are similar: the percolation occurs at the same radius and the maximum saturation is approximately constant. Therefore it may be concluded that all the geometries are above the REV and that the results obtained are not affected by the geometry size.

For both studied porosities, the saturation curves were not affected by the disorder parameter. Some small differences can be identified in the curves but the two relevant invasion parameters seems not to be affected: the percolation radius and the maximum saturation.

One may therefore conclude that the disorder parameter is not important in the saturation curve obtained by the YLM. This result contradicted the motivation of the present study, which indicates that the relevant parameter is not the geometry disorder, but the pore sizes distribution. If the pore size distribution is wide (Fig. 1), the YLM is able to provide a good saturation curve prediction. On the other hand, if the pore size distribution is narrow, or have only some values (Fig. 4), YLM is not able to provide a good saturation curve because the invasion occurs in few steps. The disorder does not

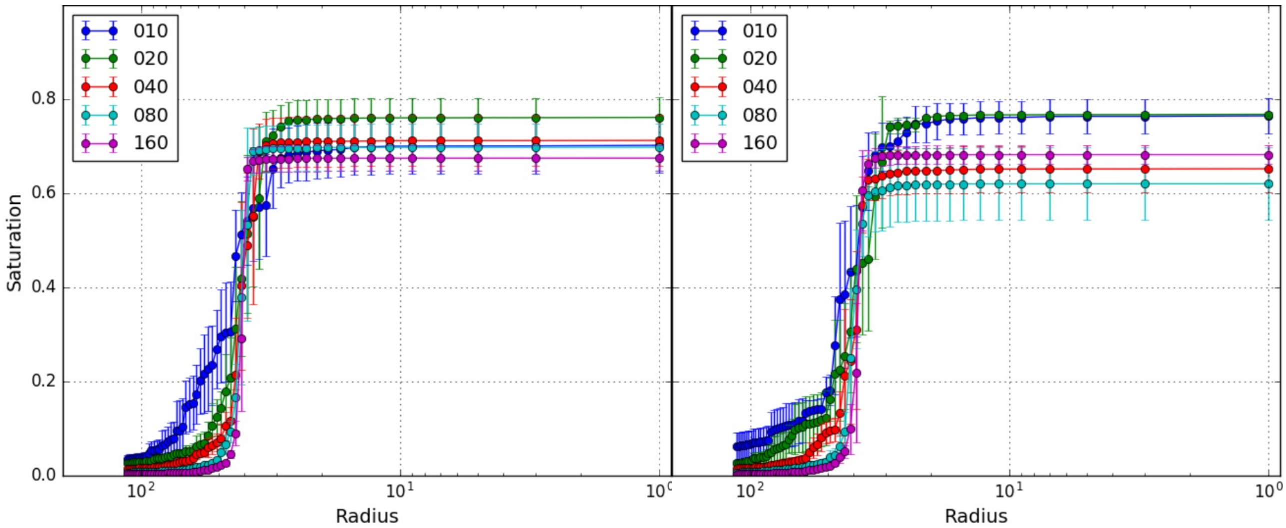

Figure 9. Saturation curves with YLM for $\phi = 0.72$. On the left for $\sigma/\sigma_{max} = 0.5$ and on the right for $\sigma/\sigma_{max} = 1$. Different curves on each image represent geometry sizes.

change the pore size significantly.

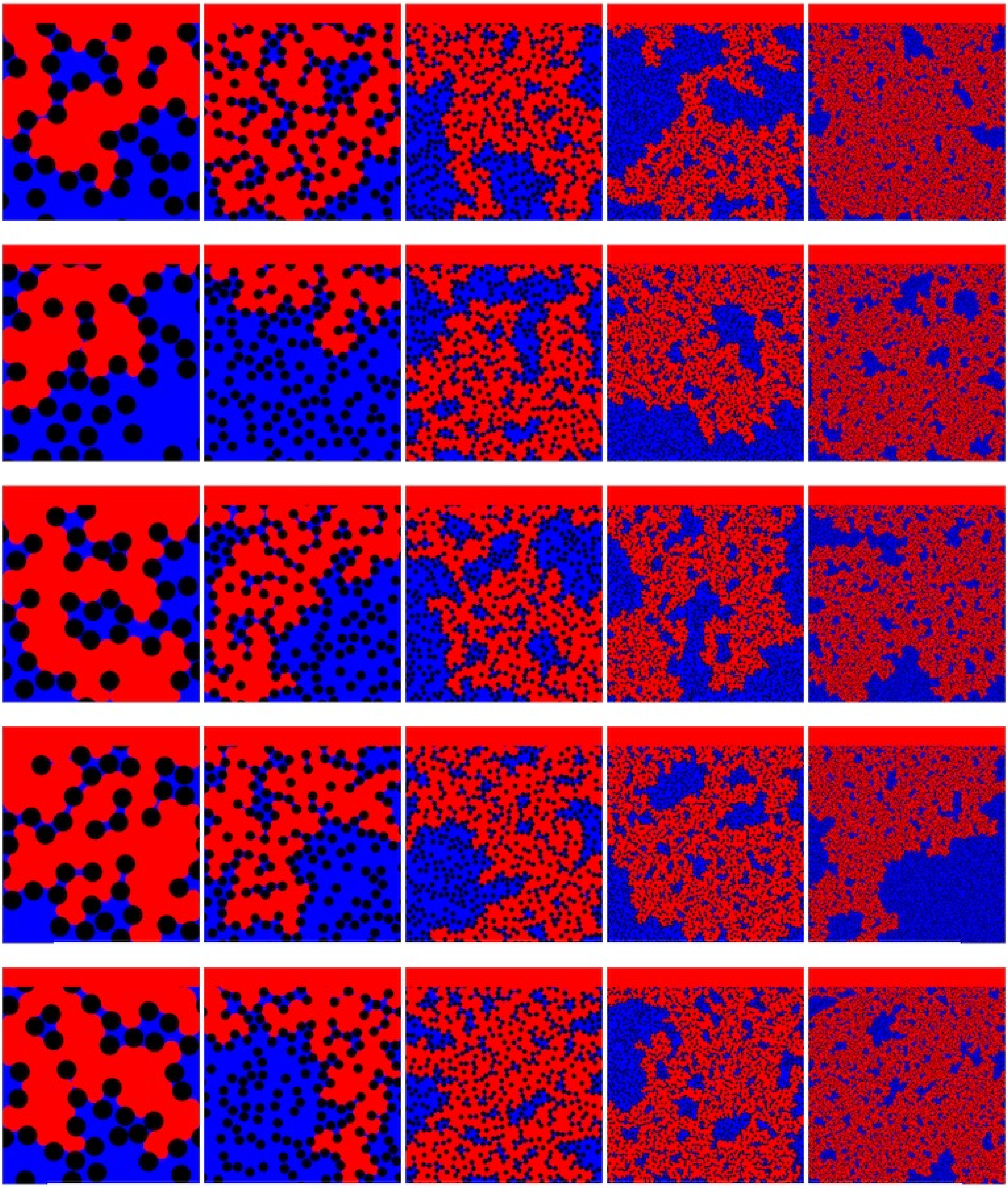

Figure 10. Invasion simulation with YLM for $\phi = 0.47$ and $\sigma/\sigma_{max} = 1$. From left to right the geometry sizes increases (Tab. 1). From top to bottom are shown different realizations of the simulation, for error analysis.

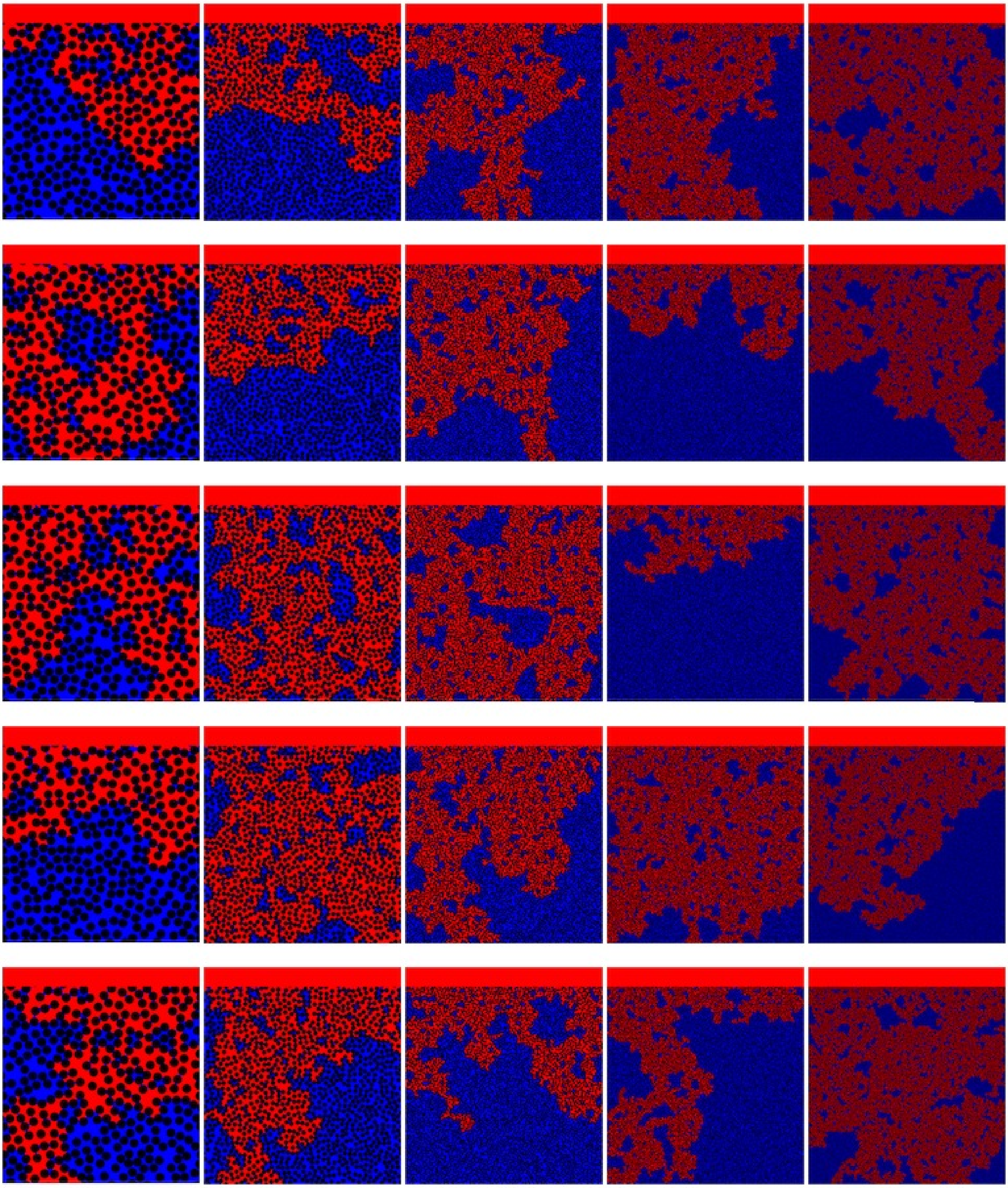

Figure 11. Invasion simulation with YLM for $\phi = 0.72$ and $\sigma/\sigma_{max} = 1$. From left to right the geometry sizes increases (Tab. 1). From top to bottom are shown different realizations of the simulation, for error analysis.

4. CONCLUSION

I showed that the Young-Laplace Method is not affected by the disorder of the geometry. The saturation curves were essentially the same for different disorders. The conclusion holds for different porosities. It was also shown that all the geometries were above the minimum Representative Elementary Volume, although it has not been determined.

All the evidences suggests that the pore size distribution is the key parameter for the success or not of the YLM to give good saturation curves. Further investigation are needed to corroborate this hypothesis.

5. ACKNOWLEDGEMENTS

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7. RESPONSIBILITY NOTICE

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