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A FIRST STUDY ON HYBRID-MIXED STRESS FORMULATION ON TOPOLOGY OPTIMIZATION

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Abstract. Recently studies have proposed Topology Optimization based on the adoption of non-conventional formulations of the Finite Element Method in order to handle the presence of stress constraints, checkerboard pattern, incompressible media optimization and pressure load problems. In this sense, we present an alternative formulation for the topology optimization of continuum structures. The formulation adopted is the Hybrid-Mixed Stress Formulation, in which the stress as well the displacement on the domain and on the boundary, are the main variables. A minimum compliance with volume constraint problems fully described in terms of the stress are examined with the compliance computed through the calculation of the complementary energy. The well-know SIMP (Solid Isotropic Material with Penalization) was adopted to penalize the pseudo-density of intermediate volume fraction elements and an Optimality Criteria Method was adopted to solve the numerical optimization problem. This formulation has shown to achieve free checkerboard instabilities designs without the application of any filter techniques for this purpose and good agreement with available references results based on classic Finite Element Methods.

Keywords: Topology Optimization, Hybrid-Mixed Stress Formulation, Compliance Minimization

1. INTRODUCTION

The main concept behind the topology optimization is to determine the optimal distribution of material within a given domain, in most cases using a Finite Element Method approach to approximate and solve the continuous elastic problem (Bendsoe and Sigmund, 2004). Since the introduction of the method in Bendsoe and Kikuchi (1988), it has become very popular in industry and academia as a mechanical design tool for conceptual projects, and by its popularity many studies have been carried to improve its applicability as the methodology itself.

Recently as the problems analyzed through the topology optimization are involving more complex scenarios as pressure load problems, stress constraints and incompressible media, more robust schemes have been addressed to approximate and solve the continuous elastic problem, reference is made to Bruggi (2008), Bruggi and Venini (2007), Bruggi (2016), Sigmund and Clausen (2007). Some of the main problem in using a displacement-based formulation such as FEM are the arising of the check board problem when the adoption of low order elements is made and the required post-process to compute the stress field (Bruggi, 2008).

Within such scenario of adopting non-conventional formulations for solving the discrete problem, the aim of this paper is to introduce the Stress Hybrid-Mixed Formulation, called here as HMSF, to compose a Minimum Compliance with Weight Constraint (MCWC problem) topology optimization problems to examine its performance and limitation. Introduced by Teixeira de Freitas *et al.* (1999) the HMSF approximate independently the stress and the displacement in the domain of the element and the displacement on the boundary. The main feature of this formulation is the accuracy of the stress approximation in comparison with displacement-based formulation, which can be very useful for optimization problems with local or global stress constraints.

Coming back to the optimization problem, the SIMP approach was adopted and, although the HMSF has shown to achieve free checkerboard pattern solution, a filter was also implemented to ensure existence of solution and to produce mesh independent ones (Sigmund, 2001). For simplicity, the Standard Optimality Criteria method was implemented to solve the MCWC optimization problem.

2. COMPUTATIONAL PROCEDURE

The numerical methodology was implemented in FORTRAN and is mainly based on the algorithm described in Sigmund (2001). The domain was assumed to be rectangular and discretized by square Hybrid-Mixed Stress elements with the solver of the discrete problem based on Góis and Proença (2007) work which implemented the Babuška Method to solve the resulting linear system, as the solving system is not well conditioned.

In a sense to validate the HMSF, a plate with its third half subjected to a traction load is analyzed through a classical FEM and the in-case formulation, in order to check the accuracy of the stress field approximation and convergence pattern through the concept of energy norm, presented in Zienkiewicz and Zhu (1987).

To adopt a classical MCWC problem, objective function is computed through the complementary energy and the necessary sensitivities computations are derived analytically. The complementary energy U is written in terms of stress according the above equation, where f holds the flexibility matrix and σ the stress field:

$$U = \int \sigma f d\sigma \quad (1)$$

In what concerns the optimization algorithm, the Optimality Criteria was implemented and for simplicity the Lagrangian multiplier obtained from the optimal problem is obtained using a bi-sectioning algorithm. To overcome mesh-dependent solutions a filter was implemented modifying the sensitivities of the objective function through a convolution operator, which is mainly based on the distance between two elements and decays linearly with the distance from a given element.

3. RESULTS AND DISCUSSION

3.1 Hybrid-Mixed Stress Convergence Pattern

The benchmark problem used to validate the proposed formulation is described in Fig. 1, with Young Modulus $E = 1000 \text{ N/m}^2$, Poisson's ratio $\nu = 0.3$ and traction $p = 10 \text{ Nm}^2$. To evaluate the proposed formulation, the Superconvergent Patch Recovery - SPR, was implemented to recover the stress field (σ^*) of the structure and later to write the error estimator of the energy norm (Zienkiewicz and Zhu, 1987), which for elastic problems has the form:

$$\|e\| = \left[\int_{\Omega} (\sigma^* - \sigma)^T f (\sigma^* - \sigma) d\Omega \right]^{1/2} \quad (2)$$

A comparison in terms of the error estimator of the norm energy between a classical FEM and the HMSF in Fig. 2. shows that both formulations are convergent with almost the same rate of convergence. In meaning of accuracy of the solution the HMSF shows better results, which can be evaluated through the energy norm given by the same element size.

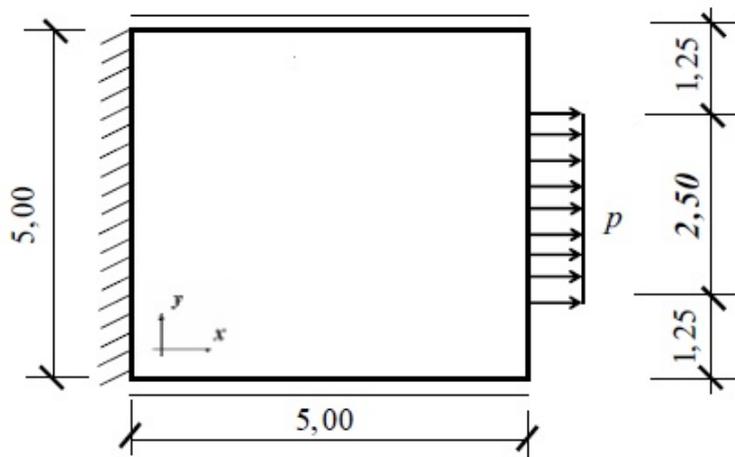


Figure 1. Benchmark Problem used to evaluate the HMSF. Units in SI.

To complement the comparison of the formulation with a displacement-based formulation, Fig. 3a represent the stress field on the load direction for both formulations, with a discretization of 16 elements. The main differences here lies on

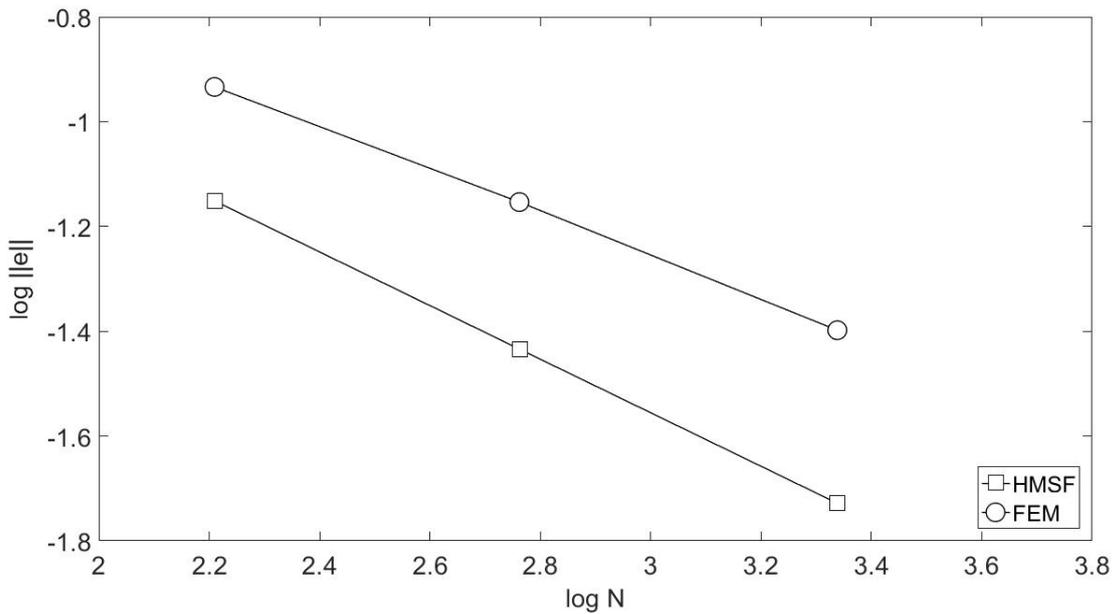


Figure 2. The convergence rates in h version of the HMSF and FEM, where N holds the number of degrees of freedom of the discretization. It should be noted that the rate slopes are approximately the same.

the smoothness of the stress field and the approximation of the stress peak. As it can be checked, although the HMSF demands more computational power due to the size of its linear system, a poor mesh can lead to an acceptable stress field comparing to the classical FEM.

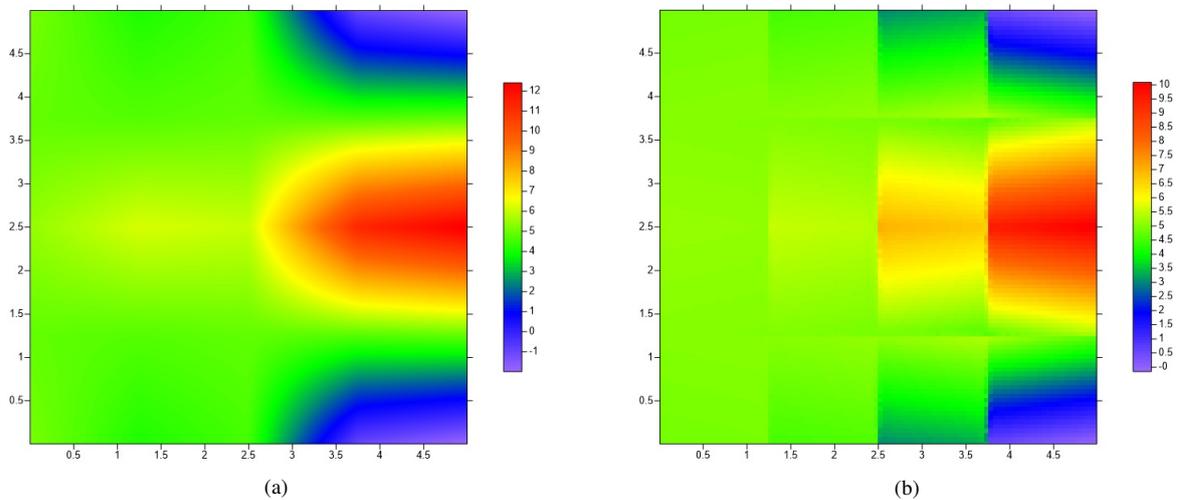


Figure 3. Stress Field on the load-direction for the HMSF at left (a) and a Displacement-based Formulation, at the right side (b).

3.2 Middle-Loaded Cantilever Beam

The test case considers a Middle-Loaded Beam with Young Modulus $E = 1 \text{ N/m}^2$ and Poisson's ratio $\nu = 0.3$. The discretization was made considering unitary elements, the load unitary at the middle of the left side of the structure and all the units in SI. Figure 4 shows the optimal layout found adopting a classical displacement-based formulation, on the top, and the non-conventional formulation, on the bottom, without the application of any filter. A comparison shows that for the HMSF, besides its design present gray members and it is not clean as the one found when applying filters in density or sensitivity, see Fig., 4, it does not present checkerboard pattern, in contrast to the designs found when applying a classical displacement-based formulation.

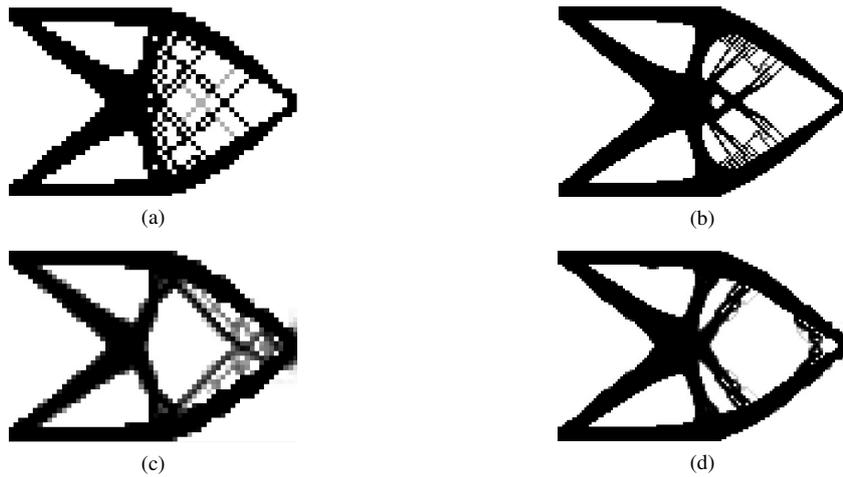


Figure 4. Optimal design for the rectangular cantilever middle loaded without the application of filters. FEM (top) and HMSF (bottom). The meshes used are (a)/(c) 60 x 40 and (b)/(d) 120 x 80 elements.

Although the formulation does not present checkerboard, it can be checked that the formulation proposed is mesh dependent. For this reason, the same test has been made with the application of filters, with the size of 0.03 the width of the domain. The optimal layouts found, Fig. 5 have no variation, independent of the mesh size or the type of filter applied, on the sensitivities, the top one, or on the densities, on the bottom. The main difference here lies on the presence of the shading effect, absent when applying density filter.

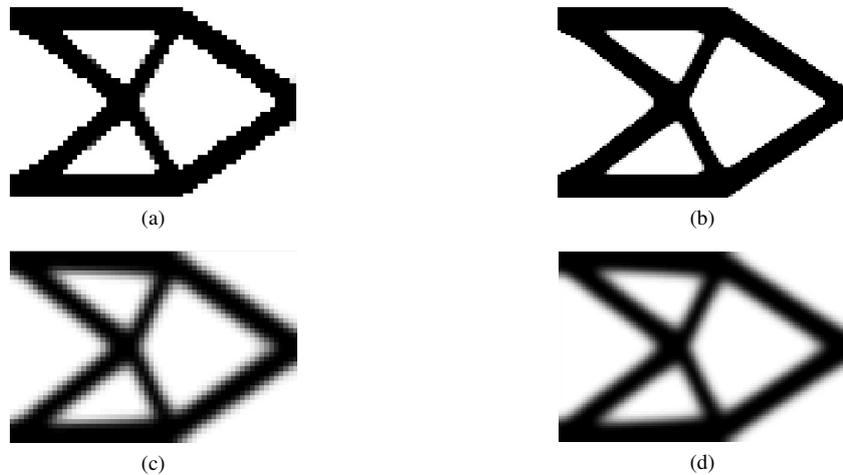


Figure 5. Optimal design for the rectangular cantilever middle loaded using sensitivity filter (top) and density filter (bottom). The meshes used are (a)/(c) 60 x 40 and (b)/(d) 120 x 80 elements.

4. CONCLUSION

The results obtained has shown to be in agreement with the classical topology optimization method. The chosen non-conventional formulation has proven to be checkerboard free, although it cannot achieve a clean and free mesh-dependency design, which is done when applying filters on density or sensitivity.

The next step on the study of Hybrid-Mixed Stress Formulation on Topology Optimization is the application of stress constraints, as the direct approximation of the stress field is the main advantage of the formulation over the classical Finite Element Method.

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