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RANDOM WALK IN PETROPHYSICS

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Abstract: *The transport properties (Surface to Volume ratio, Porosity, Permeability and Tortuosity) with the current simulations methods can be quick, but requires high computational cost. The article: "Estimate of transport properties of porous media by microfocos x-ray computed tomography and random walk simulation" (Nakashima & Watanabe, 2012) shows how to get these properties through the random walk. The technique requires little computational cost and gets good results. In this work is implemented the algorithm described by Nakashima & Watanabe (2012). The program is applied to a problem equivalent to those studied by the authors as a way of validating the implementation. After this,is done the study of digital rocks obtained by x-ray microtomography from reservoir rocks. The digital rocks have 1000^3 voxels and are described in Raeini, et al., 2017.*

Keywords: *Monte Carlo, Porous Media, Random Walk.*

1. INTRODUCTION

The Young-Laplace Method-YLM (Hazlett, 1995; Magnani, et al., 2000; Hilpert, 2001) is a fast simulation method because it simplifies the physics of the flow problem in porous media considering only the geometry of the invasion front. In addition, it has the advantage of simulating any type of geometry, however complex it may be. Other numerical methods may even simulate any kind of geometry, such as the Lattice Boltzmann method (LBM) (Qian, et al., 1992; Chen, et al., 1992), but the simulations are more time-consuming. Of course, the accuracy of the description of the invasion front of the LBM is much more accurate than that of the YLM (Wolf, et al., 2013). In addition, the LBM has the advantage of simulating the flow dynamics, an impossible task for YLM. It would be very interesting to have a method able to reconcile the advantages of the LBM with those of the YLM, that is, the ability to describe the dynamics of the invasion with low computational cost. If, in addition, the method is more accurate in describing the dynamics and the invasion front than YLM, this method would acquire a relevant role in the area. There is an attempt in the literature to develop such a method through the so-called Random Walk Simulations (RWS) (Nakashima & Watanabe, 2012). In RWS virtual particles are generated that move through the porous medium following a law of displacement and thus "map" the entire medium. The great difficulties are (i) to identify the law of displacement based on a minimalist physics of the porous medium and (ii) to be able to extract the relevant information from the statistics of this Random Walk. The work of Nakashima & Watanabe (2012) is quite simple in terms of numerical implementation and has great potential for development to obtain significant results.

2. COMPUTATIONAL PROCEDURES

To obtain the transport properties (Permeability, Surface to Volume ratio and Tortuosity) of a porous medium, the RWS was implemented, as described by Nakashima & Watanabe (2012). The rocks used were obtained from Raeini, et al., 2017, and are described in Table 1.

Table 1. All the rocks used in simulations[1] have 1000^3 voxels.

Name	Type	Porosity	Resolution (μm)
Bentheimer	Sandstone	0.216	3.0035
Doddington		0.194	2.6929
Estaillades	Carbonate	0.109	3.3114
Ketton		0.132	3.0001

Each walker starts from a random starting point in the rock so that it is guaranteed that all connected porous voxels from the Digital Rock are accessed. Each walker walks a random path inside the rock and can move only to the first neighbors, that is, a voxel poro at a time. At each steps or time a new direction it will be generated. When a solid part of the rock appears during the course, the walker should stay in the same place and wait for the next step to sortition new direction. Figure 1 shows the path evolution of the walk through time:

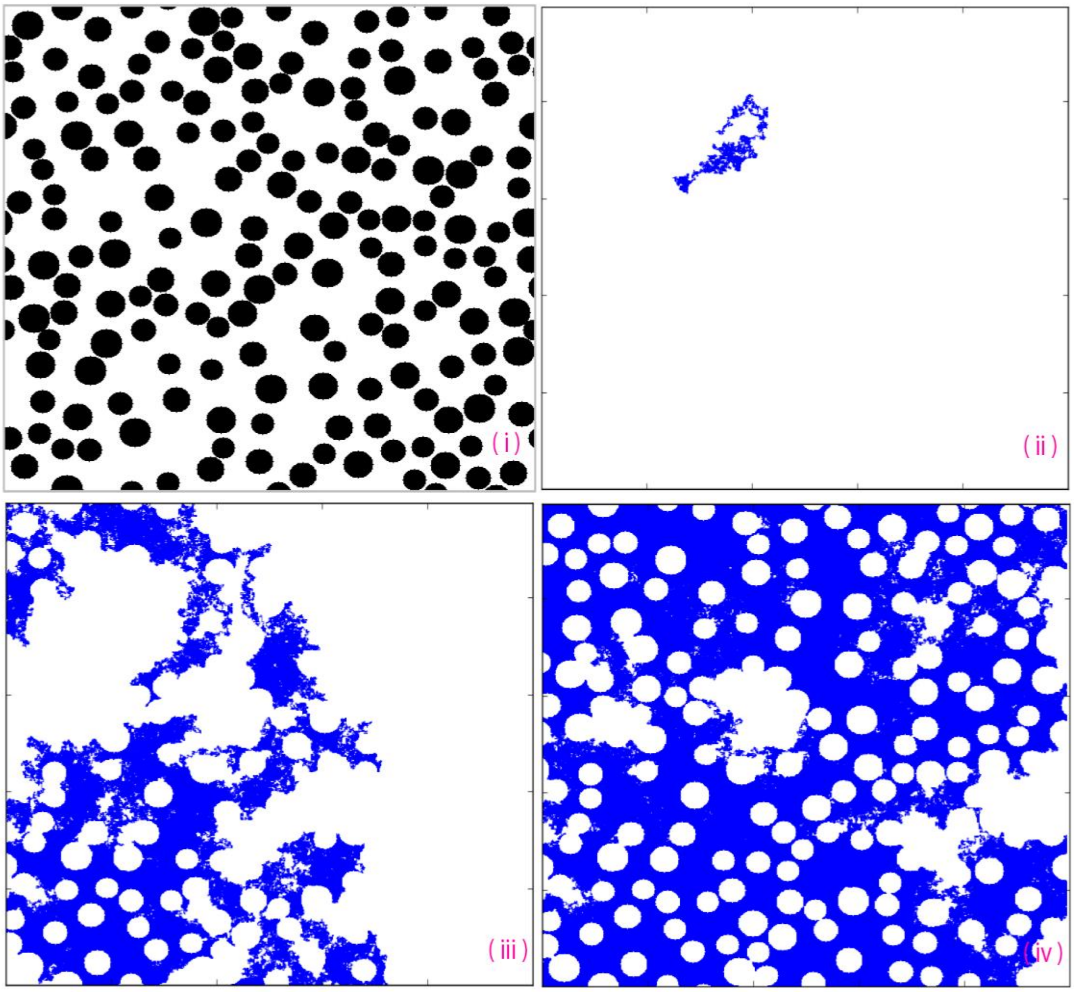

Figure 1. Example 2D of Random Walk. (i) artificial 2D geometry where the black represents the solid parts and white pore. (ii) to (iv) path evolution of a walker in relation to the number of steps.

[1] Available at:<*http://www.imperial.ac.uk/earth-science/research/research-groups/perm/research/pore-scale-modelling/micro-ct-images-and-networks/* >

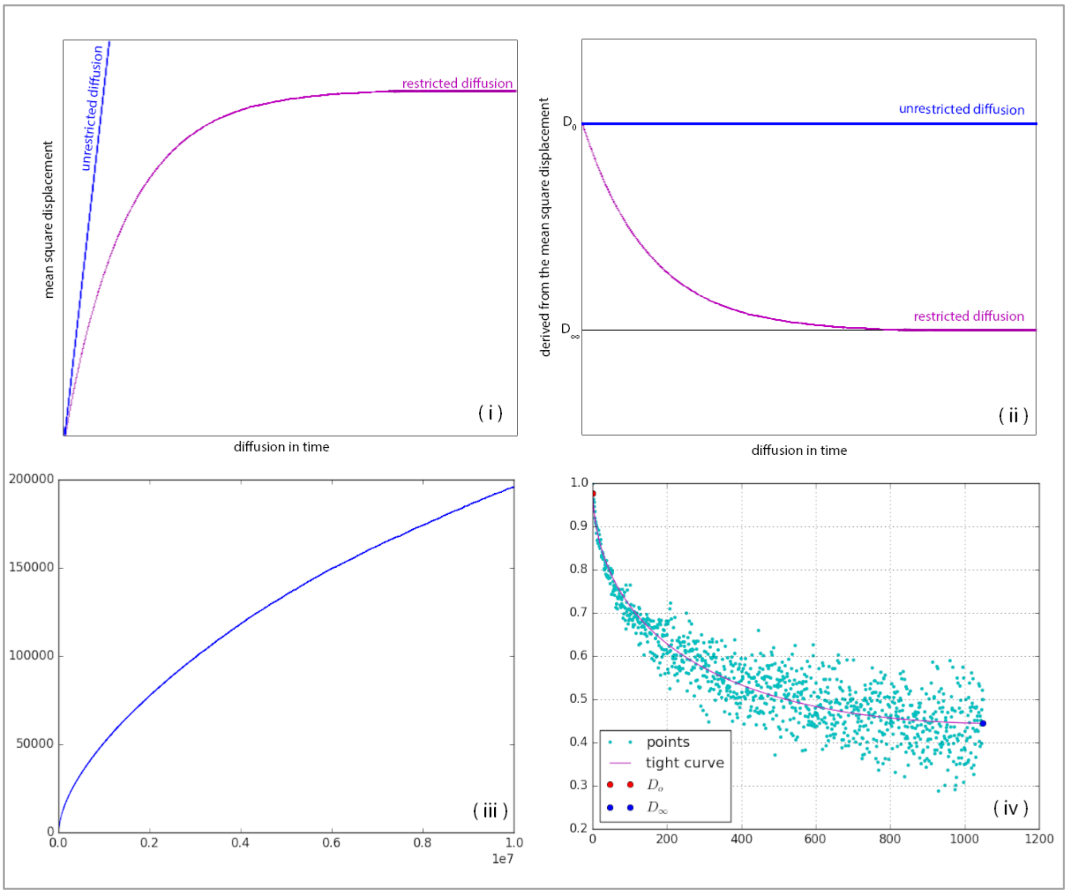

Figure 2. (i) Mean square displacement as a function of each step or time τ for a walk in a fully porous region (unrestricted diffusion) and a region with solid voxels and voxels porous (real rock, restricted diffusion). Figure adapted from fig. 1 (b) from Nakashima & Watanabe (2012). (ii) Derivative of the mean squared displacement. Factor of applied correction of 1/6 and values divided by D_0. Figure adapted from fig. 1 (c) from Nakashima & Watanabe (2012). (iii) Graph of mean square displacement as a function of time τ of the Estaillades carbonate rock for $2x10^5$ particles with 10^7 steps. (iv) Fitting of eq. 4 to the first 1050 points (Fig. 4). The permeability is obtained using the results of the fit in the equation (6).

It must be taken care must be taken with the random number generator so that the walking directions do not have statistical vices, this is why an OpenMP[2] library is used with the L'Ecuyer (1999) "RngStreams" generator (multiple independent streams of pseudo-random numbers), as it is suitable for parallelized workstations. From its origin in the rock is calculated the displacement traveled at each step. Then is calculated the average square of the distances traveled in step τ to all walkers:

$$\langle \|\vec{r}(\tau)\|^2 \rangle = \frac{1}{n}\sum_{i=1}^{n} \|\vec{r_i}(\tau)\|^2 \tag{1}$$

By throwing the walker in a fully porous medium its mean square displacement as a function of time τ will have a linear behavior. In the real case of a rock with solid and porous voxels its mean square displacement as a function of τ will have a curvature as shown in Fig. 2i. The number of particles defines the precision, the higher their number, the better their curve will be generated. A long time (or many steps) is important to check if walkers have traveled all over the rock. With the mean square displacements obtained as a function of each time or step τ the next step is to apply a numerical derivation in the data. Using the numerical derivative as only consecutive differences (Nakashima & Watanabe, 2012) we will have:

$$\frac{d\langle \|\vec{r}(\tau)\|^2 \rangle}{d\tau} = \langle \|\vec{r}(\tau+1)\|^2 \rangle - \langle \|\vec{r}(\tau)\|^2 \rangle \tag{2}$$

[2] Matthew Bognar; Department of Statistics and Actuarial Science, University of Iowa, <*http://www.stat.uiowa.edu/~mbognar/omprng*>.

A correction factor is applied, the values obtained are multiplied by 1/6. The procedures are described in Nakashima & Watanabe (2012). The data it is divided by D_0 (value of the first derivative) after a curve adjustment is fitted to, the data is done with at equation:

$$\frac{D}{D_0} = 1 - \frac{4}{9\sqrt{\pi}}\left(\frac{S}{V}\right)_{pore}\sqrt{D_0 t} + C_1 t \;\; as\; t \to 0 \tag{3}$$

where: C_1 is a constant, is the surface ratio by volume $\left(\frac{S}{V}\right)_{pore}$ of the pore space and t → 0 represents that only the initial points are adjusted. Therefore eq.3 is valid only for the first steps. For a given rock, the "first steps" are those in which the walker is inside the original pore or have not gone too far away. For adjustment the equation is reduced to:

$$y = y_0\left(1 - a\sqrt{t} + bt\right) \;\; as\; t \to 0 \tag{4}$$

where: $a = \frac{4}{9\sqrt{\pi}}\left(\frac{S}{V}\right)_{pore}\sqrt{D_0}$; b = c_1 e $y_0 = D_0$

therefore: $$\left(\frac{S}{V}\right)_{pore} = \frac{9\sqrt{\pi}\, a}{4\sqrt{D_0}} \tag{5}$$

3. RESULTS AND DISCUSSIONS

3.1 Surface to Volume ratio

In order to obtain the Surface to Volume ratio of the rock, equation (5) is used with parameters of the curve fitting of eq. 4. For the simulation 2×10^5 particles with 10^7 steps are used in all rocks. Table 2 presents the data for Surface to Volume ratio of the porous medium by means of RWS fitting and with direct measurement.

Table 2. Surface to Volume ratio obtained by the RWS and by direct measurement of the digital rock. Units in 10^4 m^{-1}.

Name	RWS fitting	Measured
Bentheimer	9.21	11.06
Doddington	8.36	8.09
Estaillades	10.15	11.05
Ketton	6.57	7.33

3.2 Absolute Permeability

The absolute permeability can be estimated with using the Kozeny-Carman equation (Tiab and Donaldson, 1996), using the volume surfaces obtained from adjust the curve fitting:

$$k \approx \frac{\phi}{\left(\frac{D_0}{D_\infty}\right)\left(\frac{S}{V}\right)^2_{pore}} \tag{6}$$

where: k is the permeability, ϕ is the porosity and D_∞ is the asymptotic (Fig. 2ii).

For the purpose of comparison, absolute permeabilities were obtained through the LBM (Lattice Boltzmann Method). Permeabilities from Raeini et al, 2017, are also tabled for reference. The values are presented in table 3:

Table 3. Permeability obtained through the RWS, LBM and the values of Raeini et al, 2017. Units in Darcy (D).

Name	Raeini	LBM	RWS
Bentheimer	3.60	3.37	298.12
Doddington	3.81	3.07	32.84
Estaillades	0.22	0.24	0.36
Ketton	5.99	3.80	20.13

3.3 Tortuosity

The tortuosity by RWS is obtained by dividing D_0 by D_∞ of the adjusted curve. For comparison purposes the tortuosities where also obtained by means of the LBM. The results are shown in Table 4:

Table 4. Tortuosities obtained through RWS and LBM.

Name	LBM	RWS
Bentheimer	1.60	1.93
Doddington	1.64	1.80
Estaillades	2.24	2.20
Ketton	1.66	1.68

4. CONCLUSIONS

It is necessary to apply equation (6) to obtain the permeabilities. It depends on the D_0 and D_∞ values, which are fundamental for obtaining the data of interest. It is important to note that the longer the walker's movement, the closer it will be to a reasonable value for D_∞. Consequently, it is a significant data for the actual permeability of the rock. The study rocks have 1000^3 voxels and therefore require many steps for is walkers traverse the entire porous medium and to obtain a good enough value for D_∞, the number of necessary steps should be enough to get to asymptote (Fig. 2i). We used 10^7 steps with $2x10^5$ walkers, but as can be seen in Fig. 2 iii, walkers did not they the entire rock, that is why the curve did not reached an asymptotic behavior. The method applied to obtain the permeability was promising, however, subjecting the particles to longer walks is essential to obtain data closer to the results obtained with the LBM. Such conclusions and tests will be shown in future articles.

Obtaining the Surface to Volume ratio and Tortuosity in the porous medium by the RWS, unlike permeability, is not dependent on a long walk, is limited only to curve fit. However, one must be aware of the number of points that will be adjusted in the equation because they interfere with the result. An ideal number of points may vary for the type of rock and its size. A rock with a typical pore size (average pore size) allows the walker to traverse the porous medium and to reach the asymptote (Fig. 2i) faster than a rock with pores of atypical sizes. It can happen that the walker starts in a closed porous region, this causes an erroneous data because it does not represent the region of interest, to make this walk data negligible, it suffices to have an expressive number of walkers. The results of Table 2 are shown expressive compared to the data measured directly in the porous medium. As a result of this, the method is promising, since with a reasonable particle walk (with 10^7 steps, they were able to traverse ≈ 1/6 of the porous medium) thus obtaining relevant data. Is fits to subject the particles to longer walks and identify means to minimize the arbitrariness of the number of points in the curve fit.

5. REFERENCES

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