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CONTROL OF ROTATING MACHINES BASES RESONANCE BY MEANS OF AXIAL FORCE

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Abstract. *In this work, we study the effect of the alteration of the geometric stiffness, by means of axial forces application, in the natural frequency of bases of rotating machines. Two models are studied: a beam and a planar portal frame, all metallic, to keep them out of the resonance range. This region, near resonance, is considered an unsafe region, and comprises the range starting at 20% below this frequency and to 25% above it. Two tools were used to carry out the study: Rayleigh's Method and the Finite Element Method (FEM). For the beam, a rectangular tube (60x60 profile) in structural steel is studied, using Rayleigh's Method and the FEM. For the planar portal frame we used only the FEM. In this case, more profiles were analyzed. In the case of the beam, without any loading, we forced it to be inside the dangerous zone, by both Rayleigh and FEM. An axial force intervention was applied to change its frequencies. In the planar portal frame, the 60x60 profile was not inside the dangerous zone, while the others, 80x80 and I profile, were within the region {0.8Ω, 1.25Ω}. The study showed that the geometric stiffness has a very important influence on the natural frequencies of these machine bases.*

Keywords: *Geometric stiffness, Resonance, Fundamental natural frequency, natural frequencies, Finite Element Method (MEF), Rayleigh Method, Rotating Machine.*

1 INTRODUCTION

Rotating machine bases are mechanical systems subjected to mechanical vibrations. These bases can be modeled as showed in the Fig.1 below.

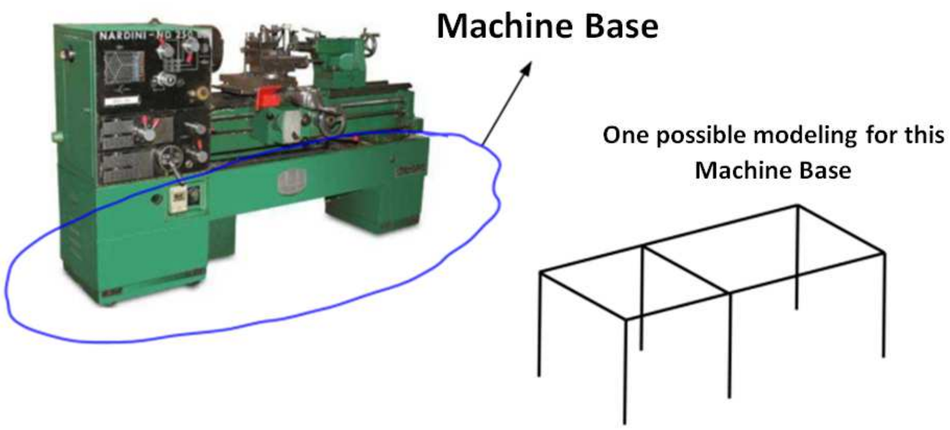

Figure 1 A possible modeling of a manufacturing machine base.

One manner of control undesirable frequency ranges, especially resonance conditions, is the application of axial forces. That it is going to change the value of the stiffness and natural frequencies.

In this paper, we present a study on the effect of axial loads upon the geometric stiffness of metal beams supporting unbalanced rotating machinery. Figure 2 shows a physical model of the problem. It is well known that the natural undamped free vibrations frequencies of mechanical systems depend on their mass and stiffness. Here we consider the effect of the geometric stiffness.

Geometric stiffness is the portion of the system's stiffness that depends on the external forces, especially axial forces. The dynamics characteristics of a structure depend of the mass, stiffness and damping. The two first influence in the natural frequency and shape modes. However, the initial stiffness of a structure, in the unloaded condition, is affected by the presence of the axial loading that changes its geometric stiffness and thus the natural frequency.

Compression loads tend to decrease the stiffness and the natural frequencies. Compression loads may even lead to the instability of the system (buckling). In the other hand, traction forces tend the increase the stiffness and the natural frequencies.

Previous work on the subject are to be found in references (Brasil, 2014), (Brasil and Silva, 2013) and (Clough and Pieze, 1995).

2 MODELING

As mentioned, two modeling tools were used, the Rayleigh Method and the Finite Elements Method (FEM). They are both based on the concept of shape functions. The FEM has been widely used in the engineering community.

Our physical model of the beam is shown in Fig.2, and that of the planar portal frame in Fig.3.

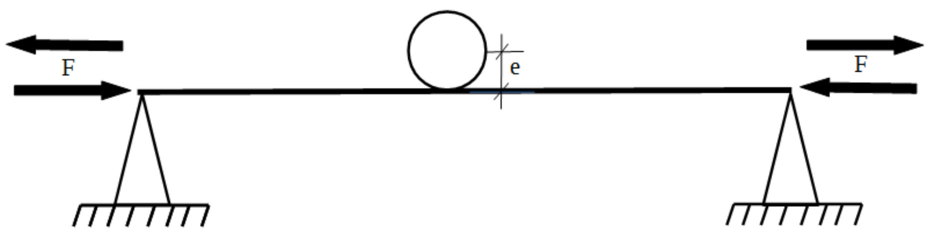

Figure 2 Physical model of the beam.

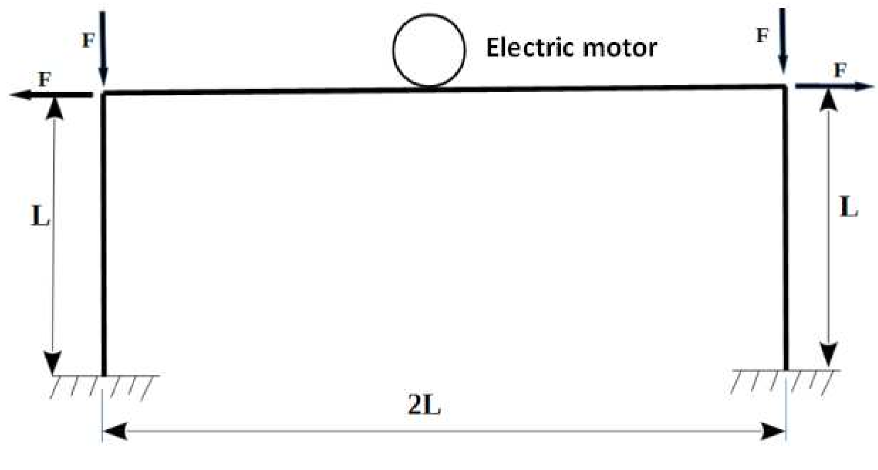

Figure 3 Physical Model of the planar portal frame.

2.1 Modeling by Rayleigh's Method

Our system has an infinite number of degree of freedom, but we are going to use Rayleigh's Method to transform it into a Generalized One-Degree of Freedom model with the concept of shape function. Here we disregard damping.

The equation of motion of a One-Degree of Freedom (ODF) system is (Brasil and Silva, 2015; Rao, 2004; Clough and Pezien, 1995):

$$m\ddot{x} + \mathrm{k}\mathrm{x} = 0 \quad (1)$$

For multiple degrees of freedom models, we have a matrix equation of motion

$$[m]\{\ddot{x}\} + [\mathrm{k}]\{\mathrm{x}\} = \{0\} \tag{2}$$

The beam of Fig.2 is a continuous mechanical system. The vibration analysis of continue systems require the solution of a partial differential equation (PDE), which may be difficult or even impossible (Rao, 2004). One alternative manner of work with continuous systems is the use of shape functions and the Rayleigh Method.

Lord Rayleigh (1842-1919) proposed a manner to derive the natural frequency of continue systems in a simplified manner (Clough and Pezien, 1995). This method utilizes the idea of conservation of mechanical energy with the concepts of shape function, generalized mass and stiffness.

Frequency is given by Eq.3.

$$\omega = \sqrt{\frac{\bar{k}}{\bar{m}}} \tag{3}$$

where $\bar{m}$ e $\bar{k}$ are called generalized mass and stiffness, respectively.

The generalized stiffness has two parts (Clough and Piezen, 1995), the generalized elastic stiffness $\overline{k_0}$ and the generalized geometric stiffness $\overline{k_g}$, as in Eq. 4.

$$\bar{k} = \overline{k_0} + \overline{k_g} \tag{4}$$

To compute the values of $\bar{k}$ e $\bar{m}$ we will use formulas (5-7) (Clough and Pezien, 1995), which may be derived from the Virtual Work Theorem, among other possibilities:

$$\bar{m} = \int_0^L m_{\mathrm{esp}}(x)\varphi(x)^2\,\mathrm{dx} \tag{5}$$

$$\overline{k_0} = \int_0^L \mathrm{EI}(x)\varphi''(x)^2\,\mathrm{dx} \tag{6}$$

$$\overline{k_g} = F\int_0^L \varphi'(x)^2\,\mathrm{dx} \tag{7}$$

where: $m_{\mathrm{esp}}(x)$ → unit density / φ(x)→ shape function / E → elasticity / I → inertial cross section / F → axial force and L is length of beam, or elements of beam.

The motion of the beam axis is $u(x,t)$, expressed by Eq.8.

$$u(x,t) = \varphi(x)q(t) \tag{8}$$

To describe a function as product of others two functions, we are using the separated-variables-concept, which follows some rules: i) $u(x,t) = 1$ in some point of domain, where our generalized coordinate is chosen; ii) $u(x,t)$ must satisfy the geometric boundary conditions. Figure 4 illustrates such points.

The shape function describes the bending displacements of the beam axis. In other words, we may say that the function tries to imitate the form that the beam assumes during its motion.

As we can to see in Eq.7, the generalized geometric stiffness depends on the applied axial force.

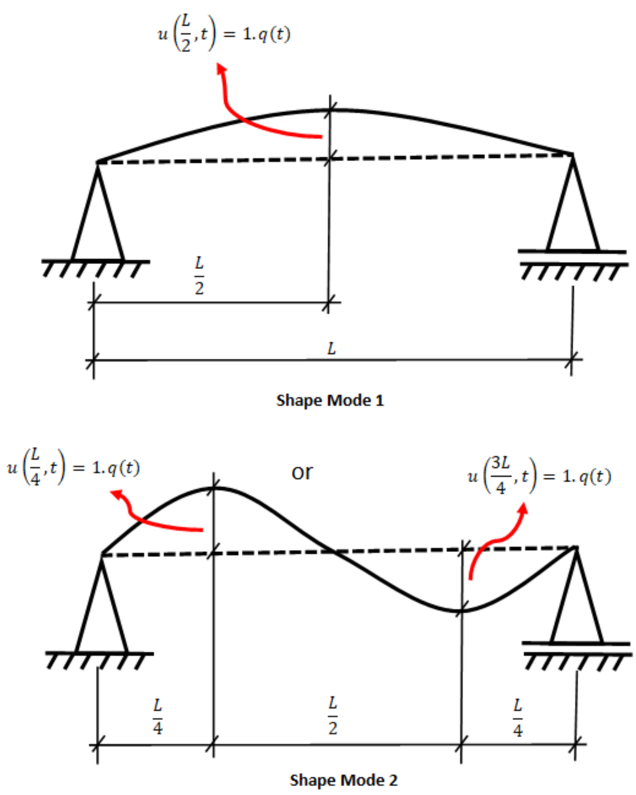

Figure 4 Possible shape functions.

2.1.1 The shape function as a cubic polynomial

As said, the shape function describes the bending motion of the beam axis.

One possible shape function will be a cubic polynomial function. We use this cubic function because the shape functions usually used in a beam Finite Element are also cubic.

$$\varphi(x)=a_0+a_1x+a_2x^2+a_3x^3 \tag{9}$$

Using the boundary conditions, we obtain the coefficients of the Eq.9.

$$\varphi(0)=0 \Rightarrow a_0=0$$

$$\varphi\left(\frac{L}{2}\right)=1 \Rightarrow a_1\frac{L}{2}+a_3\frac{L^3}{8}=1$$

$$\varphi'\left(\frac{L}{2}\right)=0 \Rightarrow a_1+a_3\frac{3L^3}{4}=0$$

$$\varphi''(0) = 0 \Rightarrow a_2 = 0$$

Thus, we adopt:

$$\varphi(x) = \frac{3}{L}x - \frac{4}{L^3}x^3 \tag{10}$$

2.1.2 Calculation of the values of the $\overline{m}$, $\overline{k_0}$ e $\overline{k_g}$

Given the adopted shape function, we proceed to determine the system dynamic parameters $\overline{m}$, $\overline{k_0}$ e $\overline{k_g}$.

Integrals Eq. 5, 6 and 7, will be carried out from 0 to L/2, because the cubic function, that is not symmetrical, is defined only for half the beam. The results will be multiplied by 2.

Applying Eq. 5, with the consideration that $m_{esp}(x) = m$ (constant unit density), we will get the generalized mass:

$$\overline{m} = \frac{17}{35}\text{mL} \quad (11)$$

To this generalized mass we must add the motor lumped mass M.

$$\overline{m} = \frac{17}{35}\text{mL+M} \quad (12)$$

The generalized elastic stiffness is given by Eq.6. The beam of the Fig.1 is considered to have EI(x) = EI (prismatic and uniform). Thus:

$$\overline{k_0} = \frac{48\text{EI}}{L^3} \quad (13)$$

The generalized geometric stiffness is given by Eq.7 and depends on the axial force, as can be seen in Eq.14.

$$\overline{k_g} = \frac{24F}{5L} \quad (14)$$

With Eqs. 11, 12 and 13, we have the general expression to find the first, or fundamental, natural frequency using Rayleigh's Method.

$$\omega = \sqrt{\frac{8400\text{EI}+840\text{FL}^2}{85\text{mL}^4+175\text{ML}^3}} \quad (15)$$

2.2 Modeling by FEM

The Finite Elements Method is a discretization technique of continuous systems, a numerical approach to the solution of its differentials equations (Wahrhaft, 2008).

Here we use a six degrees of freedom plane beam finite element, shown in Fig. 5.

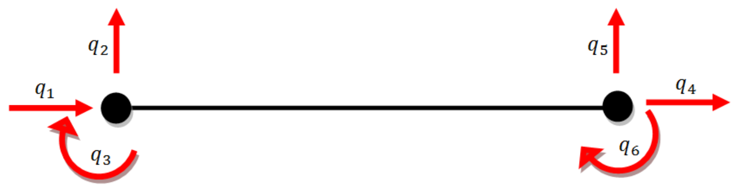

Figure 5 – A 6 degrees of freedom planar beam Finite Element.

Utilizing this model of finite element we have matrices (16-17):

$$[k] = \begin{bmatrix} \frac{\text{EA}}{L} & 0 & 0 & \frac{-\text{EA}}{L} & 0 & 0 \\ 0 & \frac{12\text{EI}}{L} + \frac{6P}{5L} & \frac{6\text{EI}}{L^2} + \frac{P}{10} & 0 & \frac{-12\text{EI}}{L^3} - \frac{6P}{5L} & \frac{6\text{EI}}{L^2} + \frac{P}{10} \\ 0 & \frac{6\text{EI}}{L^2} + \frac{P}{10} & \frac{4\text{EI}}{L} + \frac{2\text{PL}}{15} & 0 & \frac{-6\text{EI}}{L^2} - \frac{P}{10} & \frac{2\text{EI}}{L} - \frac{\text{PL}}{30} \\ \frac{-\text{EA}}{L} & 0 & 0 & \frac{\text{EA}}{L} & 0 & 0 \\ 0 & \frac{-12\text{EI}}{L^3} - \frac{6P}{5L} & \frac{-6\text{EI}}{L^2} - \frac{P}{10} & 0 & \frac{12\text{EI}}{L^3} + \frac{P6}{5L} & \frac{-6\text{EI}}{L} - \frac{P}{10} \\ 0 & \frac{6\text{EI}}{L^2} + \frac{P}{10} & \frac{2\text{EI}}{L} - \frac{\text{PL}}{30} & 0 & \frac{-6\text{EI}}{L^2} - \frac{P}{10} & \frac{4\text{EI}}{L} + \frac{2\text{PL}}{15} \end{bmatrix} \quad (16)$$

$$[m]=\rho AL\begin{bmatrix} \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{13}{35}+\frac{6I}{5AL^2} & \frac{11L}{210}+\frac{I}{10AL} & 0 & \frac{9}{70}-\frac{6I}{5AL^2} & \frac{-13L}{420}+\frac{I}{10AL} \\ 0 & \frac{11L}{210}+\frac{I}{10AL} & \frac{L^2}{105}+\frac{2I}{15A} & 0 & \frac{13L}{420}-\frac{I}{10AL} & \frac{-L^2}{140}-\frac{I}{30A} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{9}{70}-\frac{6I}{5AL^2} & \frac{13L}{420}-\frac{I}{10AL} & 0 & \frac{13}{35}+\frac{6I}{5AL^2} & \frac{-11L}{210}-\frac{I}{10AL} \\ 0 & \frac{-13L}{420}+\frac{I}{10AL} & \frac{-L^2}{140}-\frac{I}{30A} & 0 & \frac{-11L}{210}-\frac{I}{10AL} & \frac{L^2}{105}+\frac{2I}{15A} \end{bmatrix} \quad (17)$$

[k] is the generalized stiffness matrix of the element, including the geometric stiffness k_g and the elastic stiffness k_0.

$$[k] = [k_0] + [k_g] \quad (18)$$

[m] is the mass matrix considering the moment of inertial I of the section, to consider rotation inertia, disregarded in the Rayleigh Method model.

These matrices will be found in the literature (Venancio, 1975).

2.3 Results and Discussion

Our FEM models are shown in Fig. 6 and Fig.7.

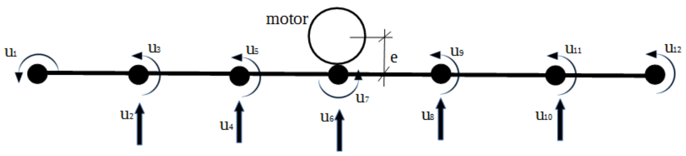

Figure 6 Beam's FEM mesh.

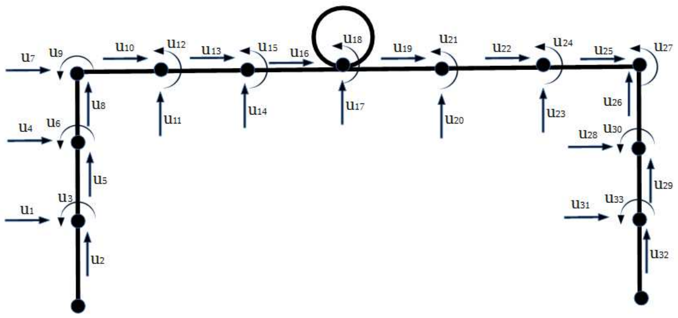

Figure 7 Planar portal frame FEM mesh.

Our electric motor works at 1715 RPM or 29 Hz in steady state operation mode, the only one we analyzed. Resonance occurs when the excitation source, in this case the electric motor, has its frequency equal or near some natural frequency of the structure. When this happens, possible undesired effects may occur (Brasil and Silva, 2013). We will assume a resonance zone of 0.8Ω to 1.25Ω, where Ω is the motor frequency. Thus, the structure should stay out of the range {23 to 36}.

We will derivate the frequencies for axial forces F = {-280, -210, -140, -70, 0, 70, 140, 210, 280}, as displayed in Table1. The frequencies, for the Rayleigh's Method, are in the Tab.2 and for FEM are Tab.3.

Table 1 Data of several used metallic profiles.

Greatness	Values	Description
A (60X60)	$1.0598\times10^{-3}m^2$	Cross Section Area
I (60X60)	$5.4230\times10^{-7}m^4$	Area Inertial Moment
$\bar{m}$ (60x60)	$8.27\,kg/m$	Linear Densit
A (80x80)	$1.4719\times10^{-3}m^2$	Cross Section Area
I (80x80)	$1.3895\times10^{-6}m^4$	Area Inertial Moment
$\bar{m}$ (80x80)	$11.48\,kg/m$	Linear Densit
A (profile **I**)	$1.05\times10^{-3}m^2$	Cross Section Area
I (profile**I**)	$1.04\times10^{-6}m^4$	Area Inertial Moment
(profile **I**)	$8.19\,kg/m$	Linear Densit
ρ	$7800\,kg/m^3$	Specifc Mass – Structural Steel 1020 NBR 8261:1983
L	2 m	Length of Beam and Portico
L	1 m	Height of Portico
E	210 GPa	Young Modulus Strutural Steel NBR 8261:1983
σy	270 MPa	Yield Tension in this estractural steel NBR 8261:1983

Table 2 Values of beam frequencies for several axial forces using Rayleigh Method.

LOAD (kN)	FREQUENCIES (Hz)
-280	3.9
-210	15.4
-140	21.4
-70	26.1
0	30.0
70	33.5
140	36.6
210	39.9
280	42.3

Observing Tab.2, we should apply compression or traction axial forces larger than 100 kN to avoid the resonance range.

In Fig.8 we have the curve of Rayleigh Method. We can see that as we increase the axial force the natural frequency also increases.

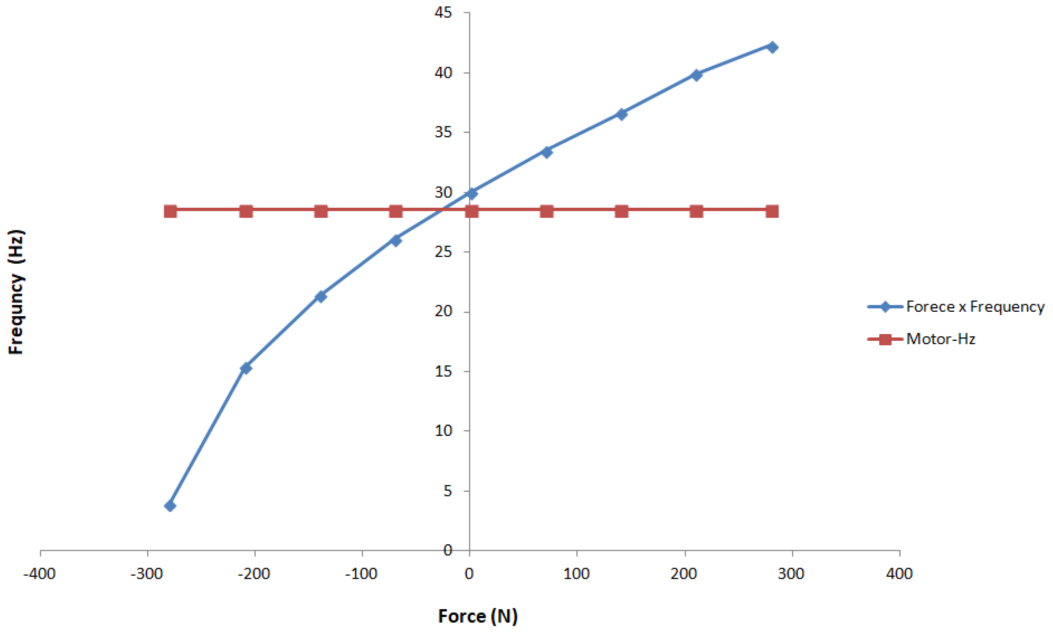

Figure 8 Axial force vs frequencies for beam model, Rayleigh Method.

In Fig.9 we have the FEM model, whose behavior is equal to that of the Rayleigh Method, except at the point of resonance. One should observe that without axial force the beam is in the resonance range.

For the portal frame, we analyze several axial forces combinations, as shown Tab.4. The best combination is FMHED.

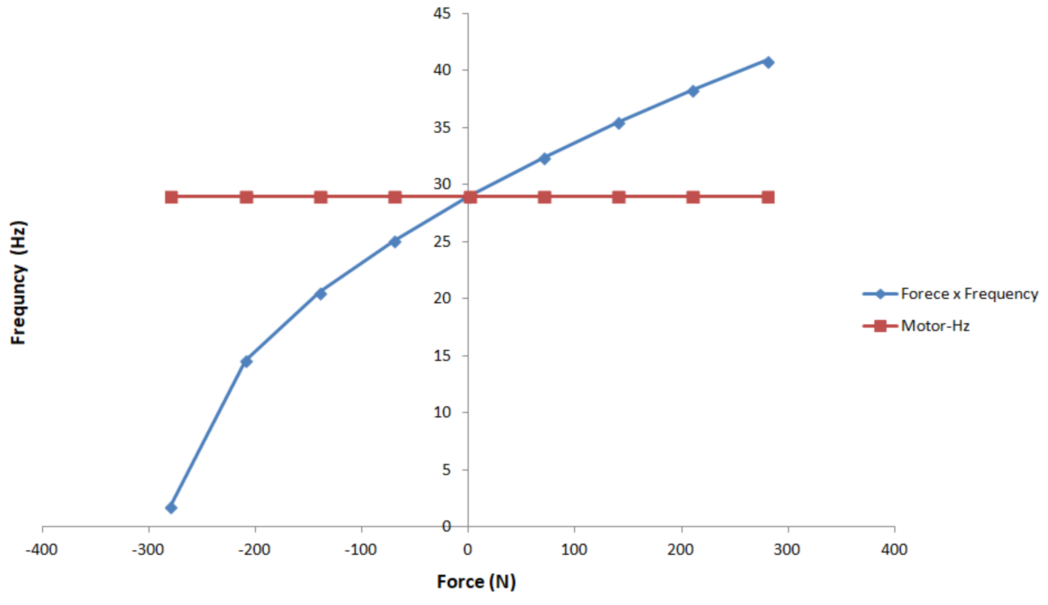

Figure 9 Axial force vs frequencies for the beam FEM model.

Table 3 Data for the planar portal frame, rectangular profile 60x60mm.

Greatness	Values	Description
A (60X60)	$1.0598\times10^{-3} m^2$	Cross Section Area
I (60X60)	$5.4230\times10^{-7} m^4$	Area Inertial Moment
$\bar{m}$ (60x60)	$8.27\, kg/m$	Linear Densit
A (80x80)	$1.4719\times10^{-3} m^2$	Cross Section Area
I (80x80)	$1.3895\times10^{-6} m^4$	Area Inertial Moment
$\bar{m}$ (80x80)	$11.48\, kg/m$	Linear Densit
A (profile **I**)	$1.05\times10^{-3} m^2$	Cross Section Area
I (profile**I**)	$1.04\times10^{-6} m^4$	Area Inertial Moment
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σy	270 MPa	Yield Tension in this estractural steel NBR 8261:1983

In Tab.4: FMVE/D – axial force applied upon left or right vertical members / FMV- axial force applied on both vertical member / FMH- axial force applied to the horizontal member / FMHE/D- axial force applied to the horizontal member and to one vertical member, left or right, and, finally, FEMHED- axial force applied to all members.

Table 4 Natural frequencies for the planar portal frame, for the several force combinations

Loads(kN)	FMVE/D	FMV	FMH	FMHE/D	FMHED
-280	16.2	12.5	13.2	9.1	1.2
-210	17	14.5	15.1	12.7	9.7
-140	17.8	16.2	16.4	15.3	13.6
-70	18.5	17.7	17.9	17.3	16.6
0	19.1	19.1	19.1	19.1	19.1
70	19.6	20.3	20.1	20.6	21.2
140	20.1	21.4	20.9	21.9	23.2
210	20.5	22.4	21.7	23.1	25
280	20.9	23.4	22.4	24.2	26.7

Joining results of both Rayleigh Method and FEM for the beam, we have Tab.5

Table 5 Comparison between the beam frequencies by Rayleigh's Method and FEM.

LOAD (kN)	FEM - FREQUENCIES (Hz)	RAYLEIGH - FREQUENCIES (Hz)	DIFFERENCE %
-280	1.8	3.9	117%
-210	14.6	15.4	5%
-140	20.6	21.4	4%
-70	25.1	26.1	4%
0	29.0	30.0	3%
70	32.4	33.5	3%
140	35.5	36.6	3%
210	38.3	39.9	4%
280	40.9	42.3	3%

As it can be seen in Tab.5, the beam without axial force has its natural frequency on the resonance range, that is, 23 to 36 Hz, in both Rayleigh Method and FEM. Therefore, immediate intervention is necessary.

Changing the geometric stiffness by means of the axial force, we change immediately its natural frequency avoiding dangerous vibrations. The axial force must be larger than 150 kN, both in the traction and compression, for the FEM case, and up of 130kN, for the Rayleigh case.

In conclusion, both analysis methods indicate that changes in the geometric stiffness (axial forces applied) interferes immediately in the natural frequency of the beam and planar portal frame.

Thus, application of axial force in a rotating machine base is a possible strategy to avoid undesired resonance conditions.

3 CONCLUSION

In this paper, we have performed the dynamic analysis of a metal beam supporting an unbalanced machine, under axial forces that modify its geometric stiffness and, therefore, its natural frequencies, to avoid possible resonance conditions. We computed the natural frequencies of the beam for several axial traction and compression forces values. First, we utilized the Rayleigh's Method, using a cubic polynomial shape function, because these are the same functions used in the usual FEM beam element. Comparing these two methods, the results differ, in the average, 3.5%.

Next, we analyzed a simple portal frame with several combinations of applied forces. The most favorable combination is when we apply axial forces in all members and the same directions.

The study showed that the geometric stiffness has very important influence on the natural frequencies of these machine bases. Thus, we conclude that changing the geometric stiffness can be a possible strategy of control or avoid resonance conditions, a practical concern of civil and mechanical engineering.

4 ACKNOWLEDGEMENTS

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