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LARGEST LYAPUNOV EXPONENT AND ATTRACTORS APPLIED TO STABILITY STUDY OF NUCLEAR REACTORS

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Abstract. *Nuclear reactors are susceptible to instability, causing oscillations in reactor power in specific working regions characterized by determined values of power and coolant mass flow. During reactor startup, there is a high probability that these regions of instability will be present; another reason may be due to transient processes in some reactor parameters. The main aim of the instability phenomena studies in nuclear reactors is to try to identify points or regions of operation that can lead to power oscillations conditions. The problems of instability were analyzed using the classical concepts of non-linear systems, such as largest Lyapunov exponents, phase space and attractors. The Lyapunov exponents quantify the exponential divergence of the trajectories initially close to the phase space and estimate the amount of chaos in a system; the phase space and the attractors describe the dynamic behavior of the system. This method was applied to a set of signals coming from stability tests of the reactors Forsmark 1 and 2. In this work, there were calculated the largest Lyapunov exponents and also reconstructed the attractors for each signal considered. In the analyzed cases, all the largest Lyapunov exponents that were found are positive; this implies that system presents chaos.*

Keywords: *BWR Instability, Large Lyapunov Exponent, Attractor.*

1. INTRODUCTION

BWRs (Boiling Water Reactors) are subject to instabilities that may occur during specific operation points of power combined with coolant flow rate. Instability is a safety problem because large fluctuations in localized power could violate thermal margins and compromise fuel integrity. There is extensive research related to efforts to understand the reasons for the appearance of instability in terms of basic physics by identifying the causes and mechanical couplings between the distinct parameters of the system (Costa *et al.*, 2008).

A system that is disturbed may undergo an unstable behavior after such a disturbance. The initiating event of the disturbance may be internal or external to the system. Three types of system behavior can be found after a disturbance: stable-linear, stable-non-linear, and unstable-non-linear. If such behaviors are analyzed in space phase (Hetrick, 1971), the system trajectory converges to a fixed point, known as a focus, in the stable-linear case. The focus, then, represents a stable equilibrium point of the system. This type of behavior can be analyzed using linear methods of signal analysis. In the second case, the trajectory is a periodic orbit around the focus; this is the limit cycle of the system. The application of linear models of signal analysis has limited validity in this case. For the unstable-non-linear behavior, the limit cycle begins to bifurcate becoming aperiodic until it becomes a strange attractor.

It is well known that BWRs behave as linear systems under normal operating conditions (Hetrick, 1971; Pereira, 1993; Pereira *et al.*, 1992). For this reason, the dynamic system can be approximated by a linear model with a stochastic noise source included. However when operating at low flow levels and relatively high power values the dynamics is susceptible to the occurrence of power oscillations. The occurrence of these regimes is associated with increased feedback of reactivity due to changes in the vapor fraction of the refrigerant. This type of behavior is known as the BWR limit cycle. During a limiting cycle, power fluctuations can cause thermal fatigue to the fuel. Clearly, the limit cycles should be avoided by the above mentioned, since the BWRs are designed to normally operate in a stable-linear regime.

The boundary cycles that are generally associated with these oscillations are a typical phenomenon of the transition from a linear to a nonlinear regime, known as Hopf bifurcation. Under these conditions, the nonlinear behavior takes on a certain importance, so that the linear models used in the diagnostic systems are no longer valid.

One of the main problems that exist when studying the nonlinear dynamics is that the methods used are very sensitive to the noise contained in the signal, and this mainly affects the reconstruction of the behavior of the system in the phase space. This causes significant errors when the signal is characterized by the Lyapunov exponents (Moreno and Espinosa-Paredes, 2016).

In this work, we addressed the problem of instability using classical concepts of nonlinear systems such as phase space and Lyapunov exponents. The Lyapunov exponents quantify the exponential divergence of the trajectories initially close to the phase space and estimate the amount of chaos in a system.

Over the years, many algorithms have been developed to calculate the Lyapunov exponents from time-series experimental data (Liu *et al.*, 2005; Rosenstein, Collins and De Luca, 1993; Wolf *et al.*, 1985), and this fact of determining the Lyapunov exponents of a time series is a major challenge for any dynamic behavior analysis. In this work, the largest Lyapunov exponents were determined for the instabilities events of Forsmark reactor, for this purpose the Wolf algorithm was used; In addition, the attractors of each signal were rebuilt, using Takens method (Takens, 1981).

The study of instability phenomena in nuclear reactors investigates points or regions of operation that can lead to conditions of power oscillations. This approach to the problem of instability analysis was used for power signals of the Forsmark reactor (Verdú *et al.*, 2001).

2. METHODOLOGY

Forsmark 1 (F1) and Forsmark 2 (F2) are the same type of BWR built by ABB-Atom. F1 has been in operation since 1980 and F2 since 1981. Both reactors have been operated with a power of 2928 MWth and a maximum flow rate of 10400 kg/s. The core contains 676 fuels assemblies (Verdú *et al.*, 2001).

According to (Oguma, 1996) from 1989 to 1996, many stability tests were performed on the Forsmark 1 and 2 reactors. The data obtained are divided into six cases. In this work, only the data of Case 5 are used.

<u>Case 5</u>

This case is focused on the analysis of two aprm (average power range monitor) signals (aprm.1 and aprm.2); they were obtained during a small transient of the plant that resulted in an unstable behavior of the signals. This is the case most studied in the literature, because the oscillation signal presents drastic changes in amplitude (Verdú *et al.*, 2001). In Figures 1 and 2, the two power signals can be observed for case 5.

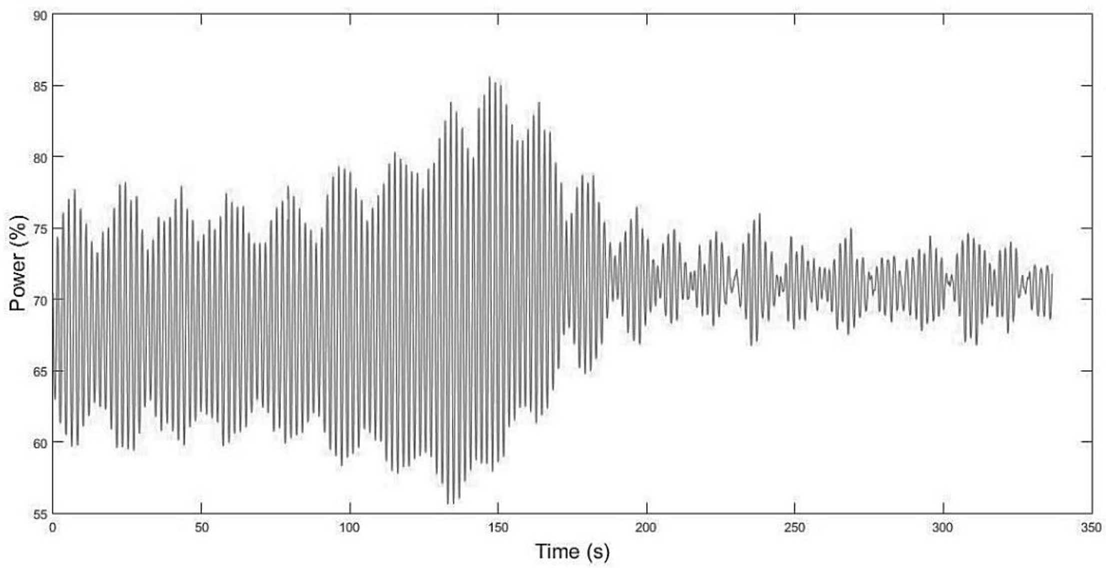

Figure 1. Power signal of case 5 aprm.1.

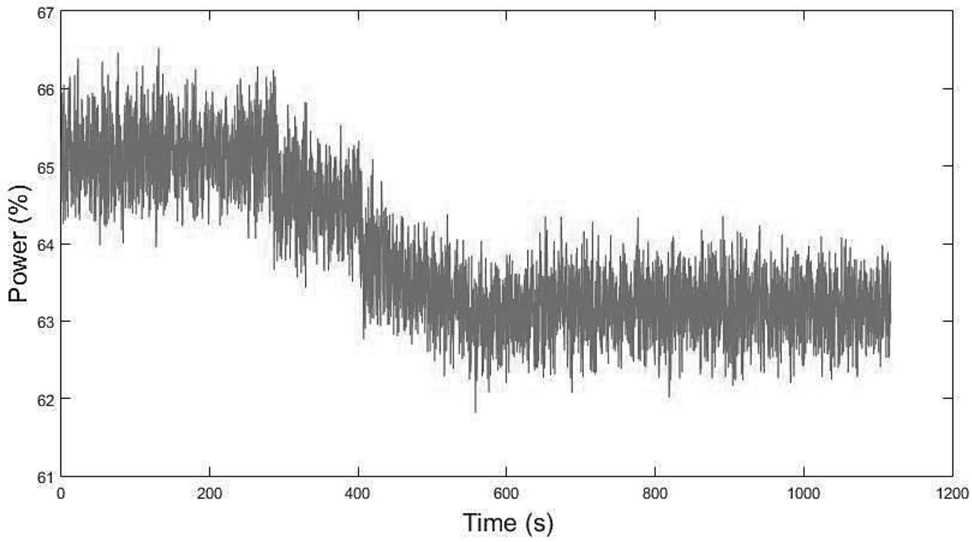

Figure 2. Power signal of case 5 aprm.2.

The purpose of this work is to obtain the attractors and the largest Lyapunov exponent for the Forsmark reactor power series. The Taken method was used to obtain the attractors, and the Wolf algorithm was used to obtain the largest Lyapunov exponent. Both methods are described in the next sections.

2.1 Attractor reconstruction

The most widely used method today was introduced by (Takens, 1981), and uses time delay coordinates. Demonstrations showed that the reconstruction using delays produces a topologically equivalent attractor, leaving the dynamic parameters invariant. Although the basic ideas are clear, the practical process of reconstruction has to overcome the difficulties presented by the use of finite system of points with noise, obtained from a system with an unknown number of degrees of freedom.

The method often used is the time-delay coordinate method, where each point in the phase space is an n-tuple of consecutive values of a series: $(x\,(t_i), x\,(t_i + \tau), \ldots, x\,(t_i + [m - 1]\,\tau))$, where $x(t_i)$ is the time series recorded, τ is the delay time and m the embedding dimension. The time τ is typically some multiple of the spacing Δ between the points in the time series. The attractors obtained in this way are called rebuilt attractors. In practice, attractors generated with small τ are closed and poorly defined, high values of τ generate dispersed attractors, whereas adequate values of τ generate open attractors with well-defined dynamics.

2.1.1 Obtaining embedding dimension

There were different methods proposed to determine the minimum incorporation size, but the most commonly used is the geometric approach, false nearest neighbors. This method of false nearest neighbors is based on the idea that when the trajectory is projected into the space of very small dimensions, the trajectory crosses and so-called neighboring false states occur. When the phase space reconstruction dimension increases, the number of trajectory auto-crosses and false neighbors decrease. When the dimension is large enough, both should disappear completely. To determine whether the neighbors are false or not, two criteria, mentioned in (Kennel, Brown, and Abarbanel, 1992), were used. It is calculated a fraction, $R\ (t_i)$, defined as

$$R(t_i) = \frac{|x(t_i + m\tau) - x^{VP}(t_i + m\tau)|}{\left||X_m(t_i) - X_m^{VP}(t_i)|\right|} \tag{1}$$

As the second criterion of falsity of the neighbours it is used fraction

$$\frac{|x(t_i + m\tau) - x^{NN}(t_i + m\tau)|}{R_A} \geq A_T \tag{2}$$

where R_A is radius of the attractor

$$R_A^2 = \frac{1}{N}\sum_{i=1}^{N}[x(t_i) - \bar{x}]^2 , \qquad \bar{x} = \frac{1}{N}\sum_{i=1}^{N} x(t_i) \tag{3}$$

2.1.2 Obtaining time delay

There is no suitable criterion for choosing the delay τ, however, it has been found that the mutual information of a time series becomes an excellent method for choosing that constant. Mutual information measures the general dependence of two variables. We are interested in measuring the dependence of the values $x\ (t\ +\ \tau)$ and $x(t)$.

$$I_\tau = \sum_{n=1}^{N} P(x_n, x_{n+\tau}) \log\left(\frac{P(x_n, x_{n+\tau})}{P(x_n)P(x_{n+\tau})}\right) \tag{4}$$

2.2 Algorithm Wolf

According to Wolf *et al.*, 1985, given a time series $x(t)$, an m-dimensional state space is reconstructed with delay coordinates; a point on the attractor is given by $\{x\ (t), x\ (t\ +\ \tau), \ldots, x\ (t\ +\ (m-1)\ \tau)\}$. A close neighbor is located at the initial point $\{x\ (t_0), \ldots, x\ (t_0\ +\ (m-\ 1)\ \tau)\}$ and denoted the distance between these two points $L\ (t_0)$.

In other words, considering a reference path (fiducial) $x\ (t_0), x\ (t_1), \ldots$ and $z_0\ (t_0)$ as the nearest neighbor in the reconstructed attractor of $x(t_0)$, $L(t_0)$ is defined as $L\ (t_0)\ =\ |\ x\ (t_0)\ - z_0\ (t_0)\ | < \varepsilon$. Therefore, $z_0\ (t_0)$ is within the hypersphere of radius ε centered at $x\ (t_0)$.

At an instant of time t_1, the distance between the two points exceeds ε, (L'_0); (t_1), so that it satisfies two criteria: its distance $L\ (t_1)$ of the evolved fiducial point is small (respecting ε) and the angular separation between the evolved and replaced points is also (see Fig. 3).

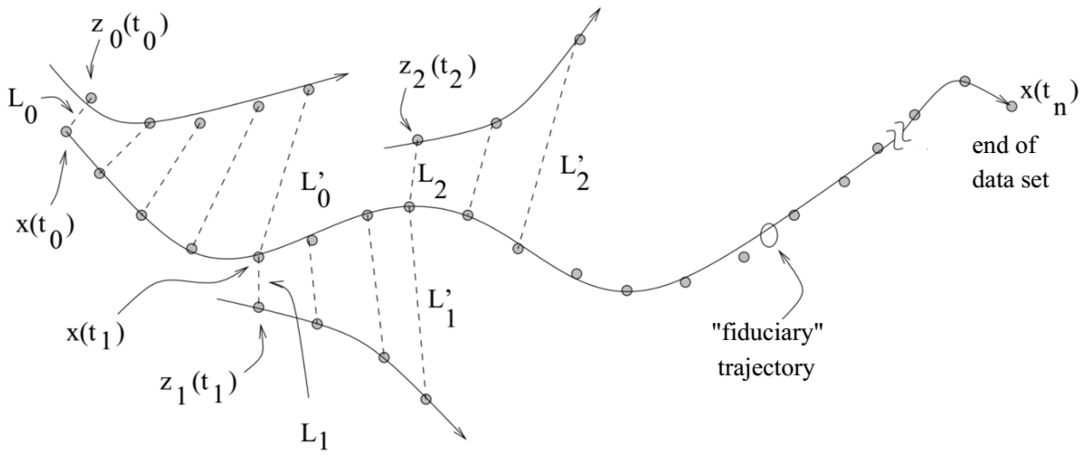

Figure 3. Representation of the Wolf method. The largest exponent of Lyapunov is calculated from the growth of the elements L_i. When the distance of the vectors between the two points exceeds ε, a new point is chosen next to the reference path, minimizing the distance L (Wolf *et al.*, 1985).

The time evolution of z_0 and y_0 is then traced until at an instant t_1 the distance between these points, L'_0, exceeds ε. At this point, we replace z_0 by a new neighbor, closer to $x\ (t_1)$, that is in the direction of the segment L'_0 and such that $L_1\ =\ |\ x\ (t_1)\ - z_1\ (t_1)\ |\ < \varepsilon$. The process proceeds until all points $x\ (t_i)$ have been traversed. The highest positive Lyapunov exponent is obtained as the mean of $ln\ \left(\frac{L'_i}{L_i}\right)$, along the fiducial trajectory, that is,

$$\lambda_1 = \frac{1}{N(t_n - t_0)}\sum_{i=0}^{M-1} \log_2 \frac{L'_i}{L_i} \tag{5}$$

where M is the number of times it has passed through the above loop, and N is the number of time steps in the fiduciary trajectory.

Through the method described above it is possible to obtain a numerical approximation for the largest Lyapunov exponent associated with a time series. The sequence of steps of the algorithm is:

1. Embed the data set.
2. Pick a point $x\ (t_0)$ somewhere in the middle of the trajectory.
3. Find the nearest neighbor from that point. Call that point $z_0\ (t_0)$.
4. Compute $||\ z_0(t_0)\ - x(t_0)\ ||\ =\ L_0$.
5. Follow the difference trajectory - the dotted line - advance in time, computing $||\ z_0\ (t_i)\ - x\ (t_i)\ ||\ =\ L_0\ (i)$ and increasing i to $L_0\ (i) >\ \varepsilon$. We call this value $L'_0$and time t_1.
6. Find $z_1\ (t_1)$, the "nearest neighbor" of x (t_1) and go to step 4. Repeat the procedure until the end of the fiduciary trajectory ($t\ =\ t_n$), keeping a record of L_i and L'.

The formula (5) is used to calculate λ_1 the largest (positive) Lyapunov exponent.

3. RESULTS

To reconstruct the attractors, the time delay method was used. The attractors for each signal are shown in Figures 3 and 4. The reconstructed limit cycle for the power oscillation event of the series aprm.1 is observed in Fig. 3, where a closed trajectory around an unstable focus is presented. In the Fig. 4 is shown the reconstructed attractor of the series aprm.2; apparently it is observed that the trajectory is around two instable foci. Using the Wolf algorithm the values of the largest Lyapunov exponents are obtained; these values are shown in Tab. 1.

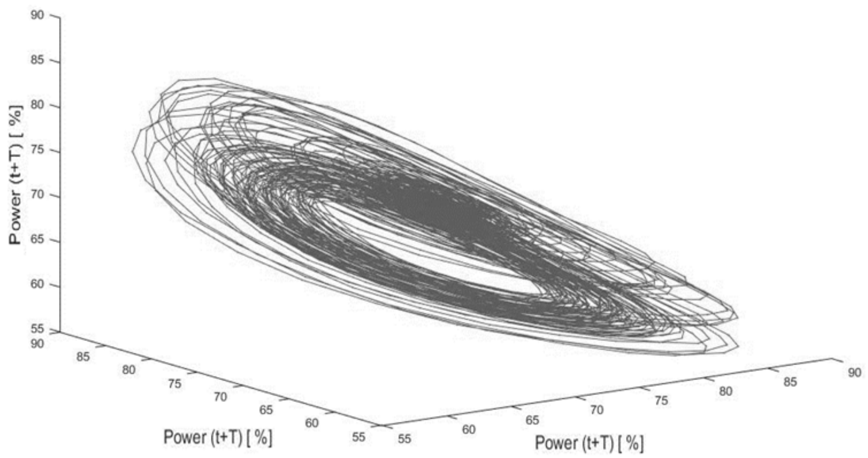

Figure 3. Attractor reconstruction for apmr.1 signal.

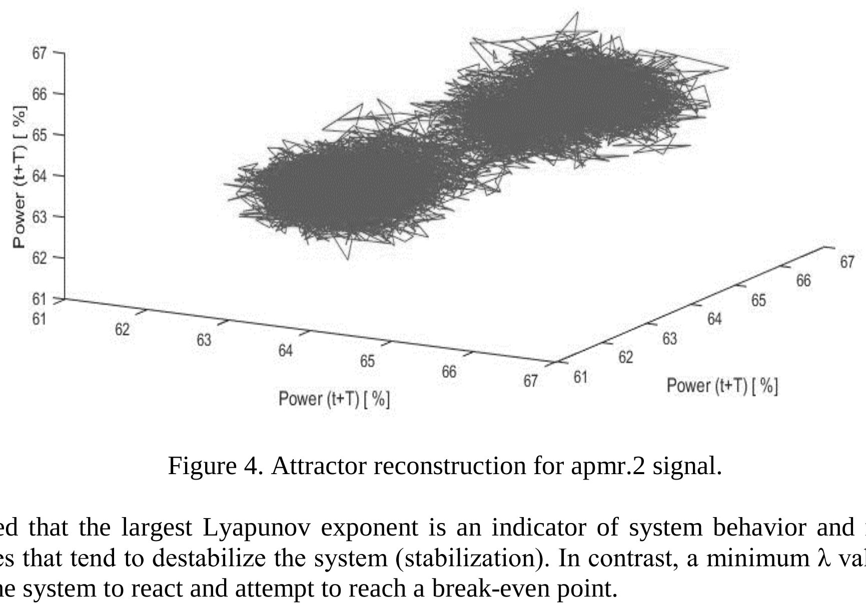

Figure 4. Attractor reconstruction for apmr.2 signal.

It can be deduced that the largest Lyapunov exponent is an indicator of system behavior and response, valid for monitoring the forces that tend to destabilize the system (stabilization). In contrast, a minimum λ value shown during a divergence causes the system to react and attempt to reach a break-even point.

Table 1: Values of the m, τ and largest Lyapunov exponents for the signals of the case 5.

Signal	m	τ	Largest Lyapunov exponent
aprm.1	3	4	0.9343
aprm.2	4	6	1.3432

4. CONCLUSIONS

In this work, the largest Lyapunov exponents were calculated with the Wolf algorithm and the attractors reconstructed for some of the signals used in this work. For both signals, the Lyapunov exponents are positive, indicating the presence of chaos in the signals and evidently instabilities.

The Takens method was used for the reconstruction of attractors. The trajectory of the attractors shows that both have instable foci, and present different behaviors, although it is a system with the same characteristics, that is, with the same neutronic processes, thermal hydraulic, heat transfer, etc. The applicability of the attractors in the monitoring of the nuclear reactors consists in aid to the operator; a boundary orbit is established with a radius that determines the transition between stable and unstable regions, this helps operators to determine whether the reactor is operating in a stable or unstable region. According to (Moreno, 2016), if the radius of attraction is almost 1.5 times longer than the radius established as stable, it should be considered that the reactor began to operate unsteadily.

5. ACKNOWLEDGEMENTS

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