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## AN INVERSE HEAT CONDUCTION STUDY USING TIME TRAVELLING REGULARIZATION

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**Abstract.** *The Time Travelling Regularization (TTR) is an inverse methodology that is used together with an optimization technique, in this case with the Golden Section method. One of its great advantages is that it allows reducing the noise on the output data in inverse models. In order to compare its versatility, this work presents an alternative way to determine the heat imposed in a stainless steel plate. The inverse problem technique was based on the use of the Golden Section method which was used together with the Time Travelling Regularization (TTR). The direct model is based on the one-dimensional heat diffusion equation solved through the Finite Difference method on a MATLAB code. In order to validate the methodology, a set of lab experiments were conducted on Stainless Steel AISI 304 samples. The heat flux was estimated through the TTR and Function Specification method (FSM) presenting similar results. As the number of future time-steps is increased, the noise on the output data is reduced. The similarity between the results of the TTR and Function Specification techniques and the small difference between the experimental and numerical temperatures validated the proposed methodology.*

**Keywords:** *Time Travelling Regularization, inverse problems, optimization, heat transfer, Function Specification method*

### 1. INTRODUCTION

The field of inverse problems has grown in the last decades due to the necessity of industrial application and science in general. These problems can be applied in computerized tomography, nondestructive experiments, shape optimization and geophysics problems for example. Also, the inverse heat conduction problem (IHCP) can be used to study complex manufacturing problems as welding and machining processes.

In a simple way, two problems can be called inverse to each other if the formulation of one problem involves the solution of the other; the results of the direct problem are the effects (experimental temperatures) generated by a particular cause (heat flux). So the effects are obtained by monitoring these causes. Inverse heat transfer problems are those which experimental temperatures are used to estimate some unknown parameter, for example, the heat flux in boundary regions, heat exchange coefficients or thermal properties of materials. Thus, inverse problems in heat transfer are indispensable in industrial situations, where parameter measurement is difficult or even impossible.

In IHCP, certain parameters are difficult to obtain, for example, in the determination of the inside lining of a furnace wall from temperature measurements. As the 1500 °C internal temperature prevents the direct analysis of the problem, Radmonser and Wincor (1998) modeled an inverse problem, in which they considered the furnace wall composed of different materials and calculated its thickness numerically.

The pioneer of IHCP studies was Stolz (1960) who proposed a method to determine the heat flux on the surface of spheres during a quenching process, using experimental temperatures in its interior. The Stolz Method is of easy implementation; however it is very sensitive to noise in the measured temperature data. This sensitivity causes large errors in the estimated heat flux; therefore it is not a good approach to the real problem.

In order to minimize the noise problem of temperature data, Beck *et al.* (1985) improved the Stolz Method by using data from future temperatures to estimate the heat flux at present time (Function Specification Method). Furthermore, the authors improved the method by using multiples temperature sensors which increases the accuracy of the solution with more stable results.

Since Stolz (1960), several techniques have been developed capable to solve linear and nonlinear inverse problems: Conjugated Gradient method with adjoint equation (Alifanov, 1974); BFGS (Broyden-Fletcher-Goldfarb-Shanno) (Avriel 1976); Tikhonov Regularization (Tikhonov and Arsenin, 1977); Golden Section (Vanderplaats, 2005).

Gaikawad and Deshmukh (2004) studied the thermoelastic inverse problem in a circular plate submitted to unknown heat imposed on one of the surfaces. To solve the problem, the authors proposed an analytical solution using Bessel functions to calculate the heat flux, and thus, calculate the body deformation.

Huiping *et al.* (2006) using the Golden Section Optimization method, estimated the heat transfer coefficient on the surface of a metal during quenching process according to the temperature curve obtained experimentally.

Yang *et al.* (2011) used the Conjugate Gradient method to estimate the unknown heat generation at the interface of cylindrical bars subjected to friction welding process. The authors obtained excellent results for the cases considered in the study.

Magalhães *et al.* (2016) estimated the heat flux during a welding process in 6061 T5 aluminum plates using the BFGS (Broyden-Fletcher-Goldfarb-Shanno) optimization method. From the results obtained, the author correlated the microstructure found in HAZ (Heat Affected Zone) with the cooling rate of the process.

To estimate the boundary heat transfer coefficient of a rod with one endpoint contacted with some liquid media, Wang *et al.* (2016) used the measured nonlinear temperature field and applied a regularizing scheme by minimizing a cost functional. They proposed a steepest iterative scheme to solve the non-quadratic optimization problem.

This work proposes a new methodology to solve inverse heat conduction problems. The Time Travelling Regularization (TTR) is an optimization method based on the Golden Section Method. It corrects the noise problem caused due to errors in the temperature measurements. Different from other approaches, the TTR analyses multiples temperature data ahead the current time to estimate the heat flux at that time instant. To validate this methodology, experiments in laboratory were performed using a stainless steel AISI 304 sample heated by a Kapton resistive heater. The results obtained from the TTR method was compared with the measured heat flux and the Function Specification method.

## 2. METHODOLOGY

### 2.1 Direct Model

The direct heat conduction problem studied in this paper is shown in Fig. 1. A stainless steel AISI 304 flat plate is subjected to a heat flux,  $q(t)$ , generated by a resistive heater that heats the upper surface uniformly. Furthermore, all the other surfaces are insulated, ensuring one-dimensional the heat flux in direction  $z$ . By assuming the thermal properties of the material constants and the initial temperature  $T_0$ , the problem is modeled by Eqs. (1):

$$\frac{\partial^2 T(z,t)}{\partial z^2} = \frac{1}{\alpha} \frac{\partial T(z,t)}{\partial t} \quad (1a)$$

$$-k \frac{\partial T(z,t)}{\partial z} \Big|_{z=0} = q''(t) \quad (1b)$$

$$\frac{\partial T(z,t)}{\partial z} \Big|_{z=L} = 0 \quad (1c)$$

$$T(z,0) = T_0 \quad (1d)$$

where  $\alpha$  is the material thermal diffusivity. The temperature signals,  $T_e$ , are measured in the center of the surface opposite the heated surface by a thermocouple.

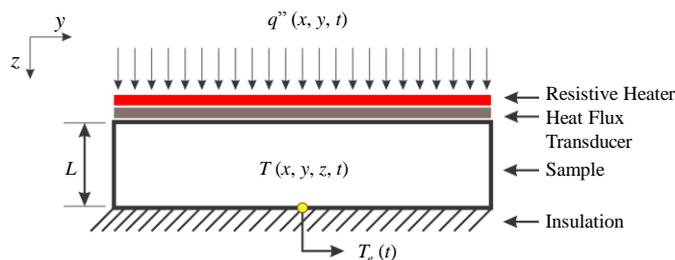


Figure 1. One-dimensional model.

## 2.2 Function Specification Method

The Function Specification method is used in this paper to estimate the heat flux in the inverse heat conduction problem. Through this method, it is possible to estimate the heat flux on the boundary of the sample by using the measured temperatures.

To solve the inverse problem, the Objective Function defined by the difference between the measured temperature ( $Y_i$ ) and the calculated temperature ( $T_i$ ), at a time instant  $i$  is used, as seen in Eq. (2):

$$S = \sum_{i=1}^r (Y_{M+i-1} - T_{M+i-1})^2 \quad (2)$$

where  $r$  is the total number of future time steps.

To determine the heat flux, the sequential method proposed by Beck *et al.* (1985) is used. It admits a constant function form of the heat flux. Thus, the heat flux values,  $q$ , are constants from time  $M$  to  $r$  future time steps:

$$q_M = q_{M+1} = q_{M+2} = \dots = q_{M+r-1} \quad (3)$$

By using a numerical approximation of Duhamel's theorem and the heat fluxes of Eq. (3), the temperatures,  $T_i$ , are given by Eq. (4):

$$T_{M+i-1} = T_M + q_M \sum_{i=1}^r X_{M+i-1} \quad (4)$$

where  $X$  is defined as the sensitivity coefficient of the material at the position analyzed. The sensitivity coefficient at time step  $i$  is calculated by Eq. (5):

$$X_i = \frac{\partial T_i}{\partial q_i} \quad (5)$$

The combination of Eqs. (4) and (1), and the optimization of the heat flux results in:

$$q_M = \frac{\sum_{i=1}^r (Y_{M+i-1} - T_{M+i-1}) X_i}{\sum_{i=1}^r X_i^2} \quad (6)$$

Equation (6) provides a direct expression to estimate the heat flux at each time step. It is observed that the greater the future time steps ( $r$ ), the better the algorithm stability is.

## 2.3 TIME TRAVELLING REGULARIZATION

The heat flux estimation technique proposed in this work is based on the Time Traveling concept. Therefore, it consists of an initial guess which analyses a hypothetical timeline. Then, it compares the parameter to be estimated in a future time step. If the analyzed timeline does not suit the optimum condition, the method goes back to the present time and changes the guess. In all kinds of experimental measurements, there is a response time between excitation and measure. This methodology reduces time response in order to maximize the estimation. Therefore, this configuration reduces the number of required future time steps in the conventional regularization techniques. Figure 2 represents the Time Traveling methodology, where  $r$  represents the number of future time steps,  $i$  is the counter for the Time Traveling methodology and  $j$  is the counter for the model analyzed.

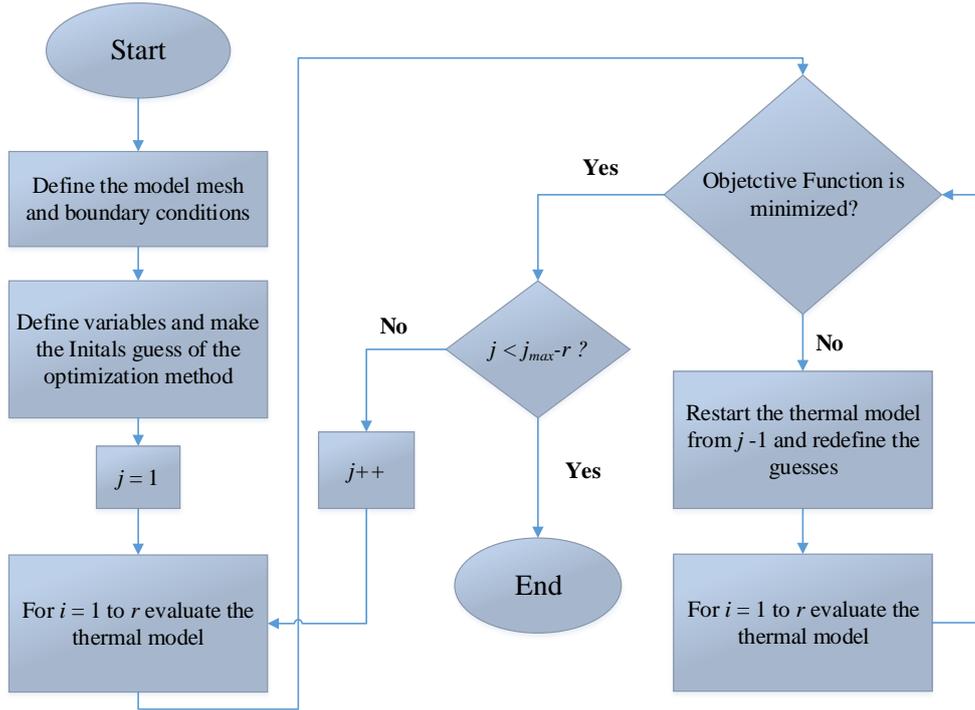


Figure 2. Flowchart for the Time Traveling methodology.

## 2.4 Objective Function

In order to estimate the heat flux in this work, the Golden Section Technique was applied. This estimation technique requires the Objective Function. The objective function based on the square difference between experimental and numerical temperature was used due to the characteristic high thermal gradient of the process. The Objective Function adopted in this work is defined by:

$$F_{obj}^j = \sum_{i=1}^r (Y^{j+i} - T^{j+i})^2 \quad (7)$$

where  $F_{obj}^j$  is the Objective Function, and  $Y$  and  $T$  are the experimental and numerical temperatures respectively at time step  $j + i$ .

## 3. EXPERIMENTAL AND COMPUTATIONAL METHODOLOGY

In order to present the methodology versatility, experiments were performed at the Heat Transfer Laboratory (LabTC) of the Federal University of Itajubá. The experiments have only the feature of proving the methodology capability. Therefore, only three experiments were conducted in order to assure the repeatability.

Figure 3 shows the equipment used to perform the temperatures measurements during the experiment. The stainless-steel plate has dimensions of 50 x 50 x 10 mm<sup>3</sup>. The kapton resistive heater has 15 ohms resistance and dimensions of 50 x 50 x 0.2 mm<sup>3</sup>. It is connected to a MCE 1051 digital power supply to provide the necessary heat flux. An Arctic Silver 5 thermal paste was used to minimize the contact resistance between the heater and the sample. This paste minimizes air gaps in the assembly and has excellent thermal conductivity. To improve the contact between components, the assembly was made under pressure with the use of weights. The assembly was isolated with 50 mm expanded polystyrene (EPS) plates to minimize the heat loss to the environment and ensure the one dimensionality of the problem. The temperatures were measured by using K-type thermocouples (30AWG) welded by capacitive discharge. The thermocouples were calibrated by using a Marconi MA184 calibration bath with a resolution of ±0.01 °C. The thermocouples were connected to a computer controlled Agilent 34980A data acquisition system. To obtain more accurate results, the experiments were carried out at controlled room temperature.

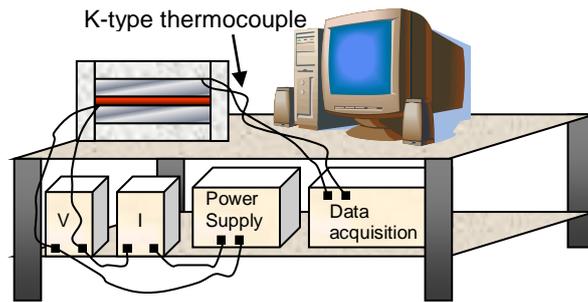


Figure 3. Experimental setup.

The samples were assembled symmetrically so that the heat flux was equally divided between the two samples, as shown in Fig. 4. Thus, the heat flux measurement errors are minimized. Figure 5 shows the position of the thermocouple on the sample.

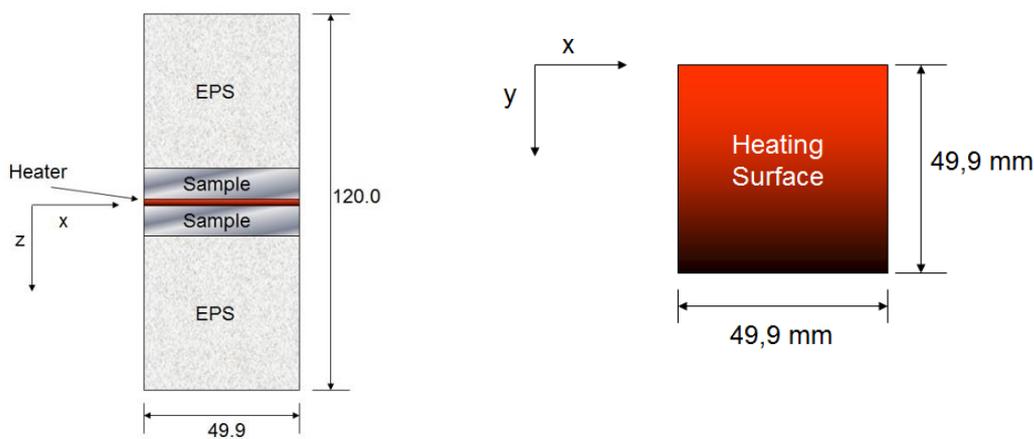


Figure 4. Symmetrical assembly and heating surface.

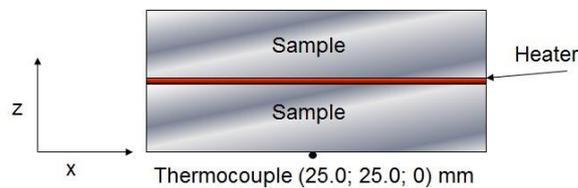


Figure 5. Thermocouple location on the sample.

A Finite Difference model based on the one-dimensional heat diffusion equation was developed in MATLAB code. Two inverse models, one based on the TTR and the other based the Function Specification, were also constructed in the code. The thermal properties were considered constant. The values of the thermal conductivity  $\lambda = 14.7 \text{ W/mK}$  and thermal diffusivity  $\alpha = 3.77 \times 10^{-6} \text{ m}^2/\text{s}$  were adopted from Carollo *et al.* (2012).

#### 4. RESULTS

From the experiment 1596 measurements were obtained with a time interval of 0.2 seconds, consequently the experiment lasted 319.2 seconds. The samples were heated from the experimental time of 60 seconds. The heating was divided in two steps. The first one, a uniform heat flux of 1150 ( $\text{W/m}^2$ ) was applied. The heating process remained for 100s. In the second step, the heat flux was reduced to about 350 ( $\text{W/m}^2$ ) until the end of the experiment.

In Figure 6 is presented the estimated heat flux by the FSM and the Golden Section with TTR. Both methods presented the same behavior in the heat flux estimation. The results show good stability until the end of the experiment. Although it is not possible to notice the difference between the FSM and the Golden Section with TTR estimation, there are small differences between the estimated heat flux results. Figure 7 presents the heat flux residuals defined as the difference between the estimated heat flux by the FSM and the Golden Section with TTR.

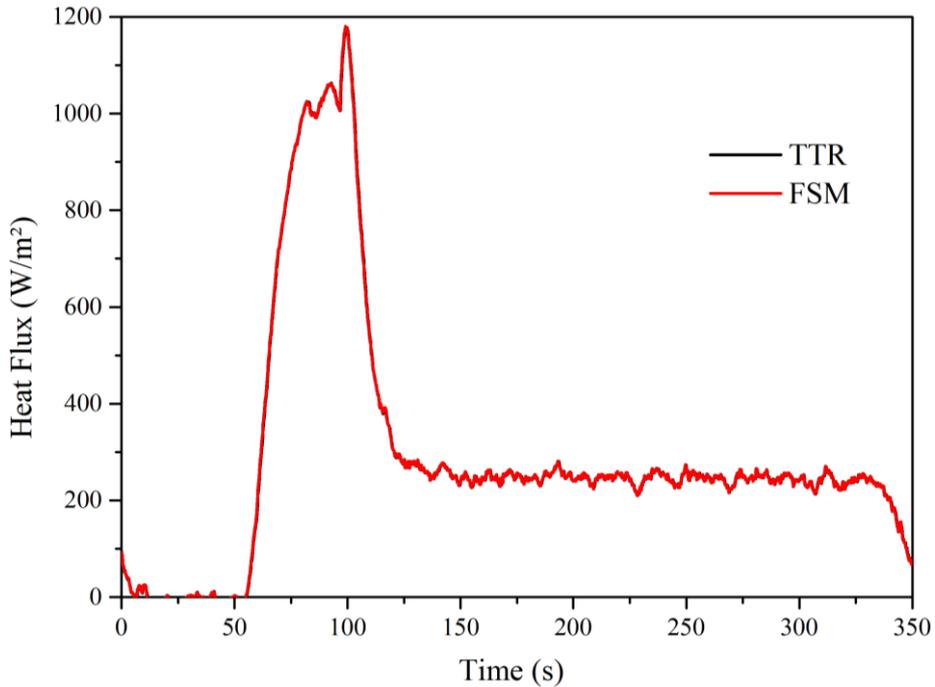


Figure 6. Heat flux estimated by FSM and Golden Section technique with TTR, for  $r = 50$ .

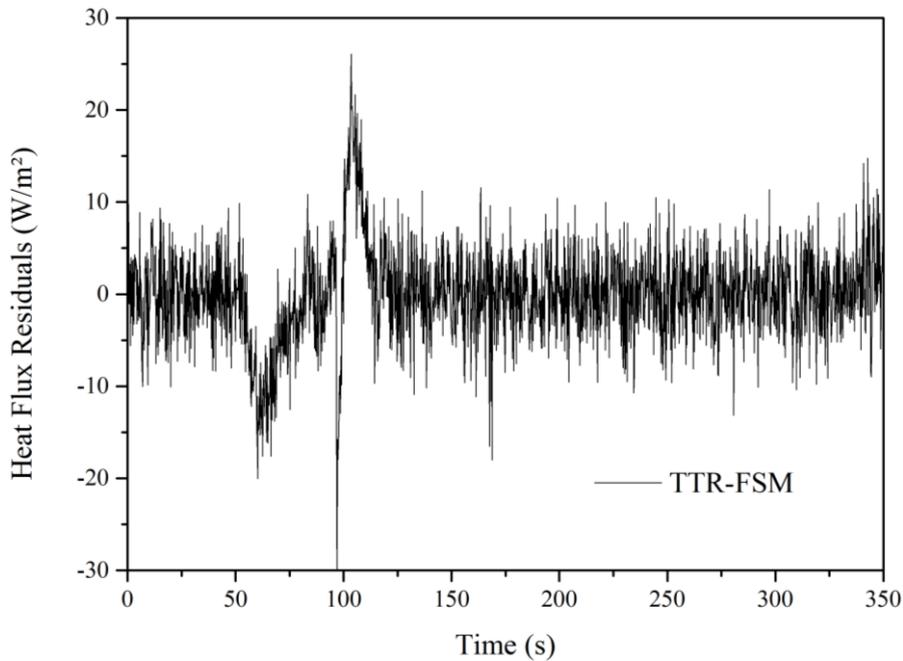


Figure 7. Heat flux difference between FSM and Golden Section technique with TTR

Figures 8 present a comparison between the temperatures estimated by the methods of FSM and Golden Section technique with TTR and the measured temperatures. The three techniques presented a good agreement in relation to the experimental temperatures. Figure 9 presents the temperature residuals defined as the difference between the FSM and experimental temperatures, and the difference between the Golden Section technique with TTR and experimental temperatures. It could be noticed, that the heat flux residuals for the TTR method presented a lower noise than the residuals for the temperatures that represented the FSM. This proves that both techniques prioritize different points. The FSM eliminates the noise from the heat flux estimation, while the TTR aims to find a solution that better approximate the experimental values.

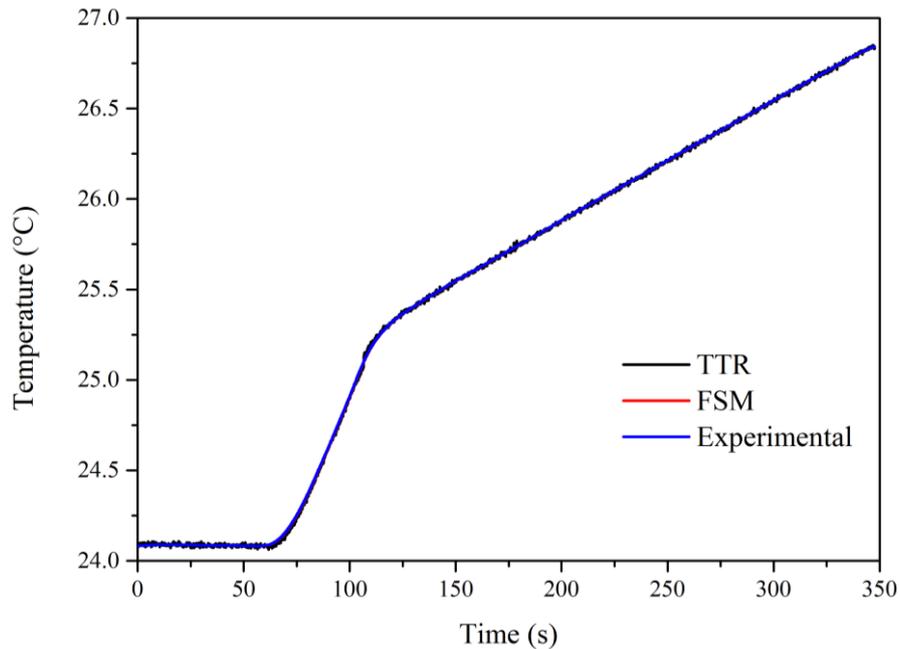


Figure 8. Comparison between the temperature estimated by using the heat flux obtained with the Function Specification, Golden Section technique with TTR and the measured temperature.

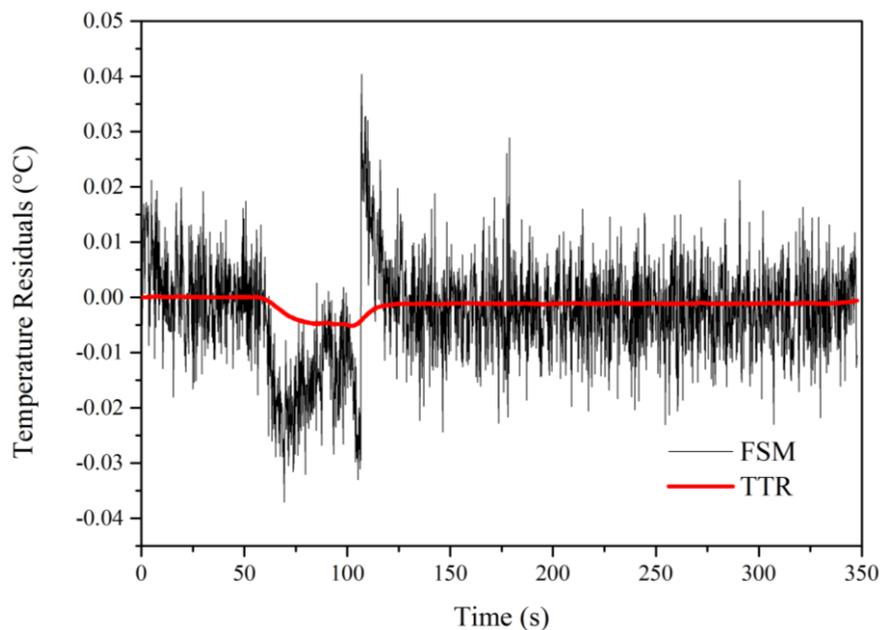


Figure 9. Temperature Residuals for both estimation methods.

## 5. CONCLUSIONS

This work presented a comparison between two inverse techniques. The Golden Section with TTR and the Function Specification method. In order to obtain a good comparison, lab-controlled experiments were conducted. In those experiments, a Stainless Steel AISI 304 sample was heated in two steps. From the experimental data, an objective function was minimized for the two tested inverse techniques. Both methods presented a good agreement between the experimental and numerical obtained data. The FSM and Golden Section with TTR presented almost the same result. However, from the temperature residuals analyses, the Golden Section with TTR presented a lower residual than the FSM.

## 6. ACKNOWLEDGEMENTS

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## 7. REFERENCES

- Alifanov, O.M. (1974), "Solution of an Inverse Problem of Heat Conduction by Iteration Methods, *Journal of Engineering Physics*, Vol. 26, pp. 471-476.
- Avriel, M. (1976), *Nonlinear Programming. Analysis and Methods*, Prentice-Hall, Englewood Cliffs, NJ.
- Beck, J. V., Blackwell, B. and Clair, C. St. (1985), *Inverse Heat Conduction: Ill-posed Problems*, Wiley-Interscience Publication, New York, NY.
- Gaikwad, M.N. and Deshmukh, K.C. (2004), Thermal deflection of an inverse thermoelastic problem in a thin isotropic circular plate, *Applied Mathematical Modelling* 29, p.797-804.
- Huiping, L., Guoqun, Z., Shanting, N. and Yiguo, L. (2006), Inverse heat conduction analysis of quenching process using finite-element and optimization method, *Finite Elements in Analysis and Design Journal*, Vol. 42, pp. 1807-1096.
- Radmoser, E. and Wincor, R. (1998), Determining the inner contour of a furnace from temperature measurements, *Industrial Mathematics Institute, Johannes Kepler Universität Linz*, Technical Report 12.
- Stolz Jr., G. (1960) "Numerical solution to an inverse problem of heat conduction for simple shapes", *Journal of Heat Transfer*, Vol. 82 No. 1, pp. 20-26
- Tikhonov, A. N. and Arsenin, V. Y. (1977), *Solutions of Ill-Posed Problems*, V. H. Winstons and Sons, Washington, DC.
- Vanderplaats, G.N. (2005), *Numerical Optimization Techniques for Engineering Design*, Vanderplaats Research and Development Inc, USA.
- Wang, L. et al. (2016), Identification of the boundary heat transfer coefficient from interior measurement of temperature field, *Applied Mathematics Letters*, Vol. 63, pp. 6-13.
- Yang, Y. C., Chen, W. L. and Lee, H. L. (2011), A Nonlinear Inverse Problem in Estimating the Heat Generation in Rotary Friction Welding. *Numerical Heat Transfer - Part A*, Vol. 59, pp. 130-149.

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