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## NUMERICAL SIMULATION FOR FLOW ANALYSIS IN NON-ADIABATIC CAPILLARY TUBES

**Hugo Augusto Marinho Moreira**

Federal Institute of Pernambuco – Campus Caruaru  
Hugomarinho93@outlook.com

**Felipe Vilar da Silva**

Felipe.Vilar@caruaru.ifpe.edu.br

**Caio Vinícios Juvêncio da Silva**

Caiovinicios91@hotmail.com

**Maycon Ferreira Silva**

Mayconferreirasilva7@gmail.com

**Leonardo José Cavalcante**

Leonardo\_cavalcante2008@hotmail.com

**Abstract.** This article aimed to implement a code in the EES (Engineer Equation Solver) computational platform, using the finite difference method to simulate non-adiabatic capillary tubes and their behavior. The flow involves a biphasic process and due to the difficulty of solution is implemented a method of dividing the tube into small volumes of control. The effects of loss of pressure and heat are counted in each section assuming mean properties between input and output and the solution of conservation equations is found for each stretch using the simplifying hypotheses of constant properties, obtaining the thermodynamic state at the next point. The results show the temperature and pressure profile along the tube for different tube lengths and diameters, at the end of this work the code validation was performed comparing it with other literature.

*Keywords:* Tube capillary, Refrigeration, Finite differences.

### 1. INTRODUCTION

The capillary tube is an expansion device used to cause pressure reduction between the condenser and the evaporator. Its objective is to regulate the flow of refrigerant and the expansion of the fluid so that the evaporator reaches a determined thermodynamic state, being directly connected to the thermal load of the system.

Capillary tubes are not the most powerful expansion devices, the advantage is mainly in their cost and maintenance. Thus, any improvement provided by good scaling, or by optimization, is relevant. Overall, its performance is greatly reduced when it works outside the ranges of specificity.

Because of poor performance, coolant in the condition of saturated liquid may exit the evaporator and enter the compressor, which can result in serious mechanical problems, such as hydraulic chock. One way to solve this problem and even increase the efficiency of the system is to add a heat exchanger between the capillary tube and the suction line of the system. As a benefit, hot fluid from the condenser can be cooled and any remaining saturated liquid can be evaporated before being compressed in the compression stage.

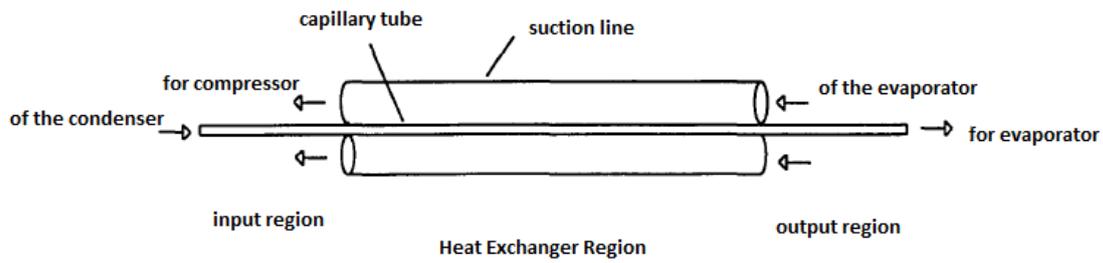


Figure 1. Model of the concentric capillary heat exchanger-suction line used in this work.  
 Source: Marangone (1995).

As a result, an increase in the system COP is noticed, since the high-pressure line fluid loses heat to the suction line. It is possible to observe in the graph below the condensation of part of the capillary tube fluid, because of the loss of heat.

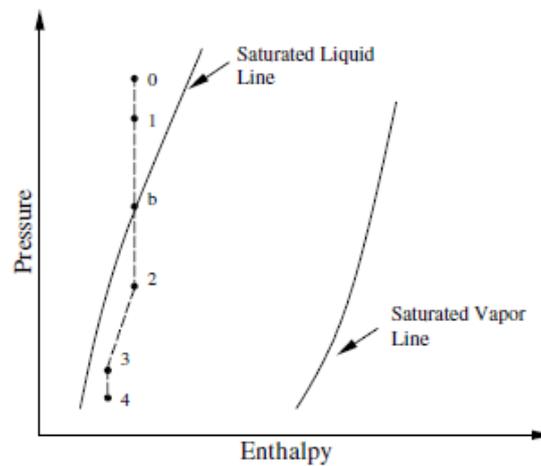


Figure 2. Representation of the thermodynamic change of the capillary tube fluid.  
 Source: Marangone (1995).

This arrangement has its performance greatly altered, due to the type of refrigerant used in the system. Domanski et al. (1994) performed this analysis, and it was showed that refrigerants such as CFC 12, HFC 134a and HC 600a show a significant increase in the performance coefficient under typical operating conditions. CFC 22, on the other hand, shows a reduction in the performance coefficient when used under the same conditions. Showing that an overall analysis in the system should be performed.

The numerical analysis used to analyze this problem consisted of discretization of the problem and its probable solution using the finite difference method. According to Chapra (2004) and Hoffman (1992), the specific objective of Numerical Analysis is the design and analysis of the techniques that provide approximate and precise solutions to problems whose analytical solutions are difficult or impossible to determine.

This work aims to analyze the flow of the fluid inside capillary tubes and the phenomena that occur in the biphasic mixture during the heat exchange with the suction line, as well as to evaluate the performance of capillary tubes.

## 2. METHODOLOGY

The modeling of the problem was carried out using the input and output balance of the control volume ( $\Delta L$  size element) with the application of the conservation equations of energy, momentum and mass, which are exposed in the following sections.

Due to the presence of phase change characteristics during the flow, the analytical solution of the problem becomes complex due to the effects involved in the biphasic flow. However, the pipe was divided into  $n$  control volumes with length  $\Delta x$ . Each control volume is evaluated by maintaining the mean thermodynamic properties between the input and output condition in the control volume. Thus, it is possible to identify the input condition of the control volume  $n + 1$  having computed the effect of loss of pressure and loss of heat in the previous control volume.

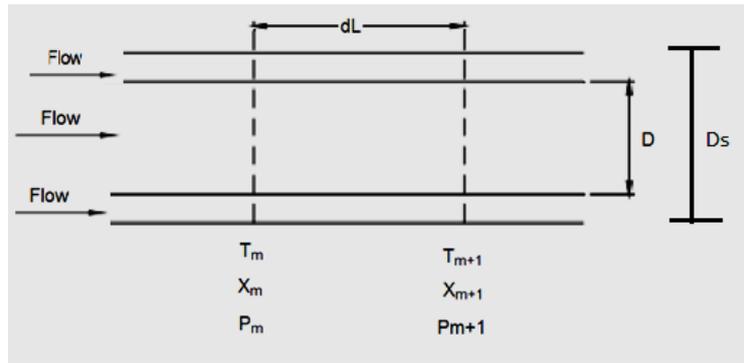


Figure 3. Discretization of the control volume of the capillary tube and the suction tube, with mass flows in the same direction.

## 2.1 First law of thermodynamics

For the modeling of the control volume for the conservation of energy, the 1st law of thermodynamics is applied to the capillary tube.

$$\frac{dE}{dt} = \dot{Q} - \dot{W} + \sum m e (h e + E c e + E p e) - \sum m s (h s + E c s + E p s) \quad (1)$$

Considering the non-adiabatic capillary tube, in permanent regime and neglecting the potential energy, we have:

$$-Q = \sum m e (h e + E c e) - \sum m s (h s + E c s) \quad (2)$$

$$h_1 - Q + \left( \frac{V_1^2}{2} - \frac{V_2^2}{2} \right) = h_2 \quad (3)$$

where it is given in [kJ/kg] and Q is given in [kW].

## 2.2 Momentum balance

For the modeling of the control volume for the momentum balance, the momentum conservation equation in the axial direction of the capillary tube is applied.

$$\rho \left( \frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} \right) = \rho g_z - \frac{\partial P}{\partial z} + \mu \left\{ \frac{1}{r} \cdot \frac{\partial}{\partial r} \left( \frac{r \partial v_z}{\partial r} \right) + \frac{1}{r} \cdot \frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2} \right\} \quad (4)$$

Disregarding the rotational flow, in steady state, without the effect of gravity, and with constant acceleration, we have that the pressure gradient is given by:

$$\frac{\partial P}{\partial z} = \mu \left\{ \frac{1}{r} \cdot \frac{\partial}{\partial r} \left( \frac{r \partial v_z}{\partial r} \right) \right\} \quad (5)$$

The flow in the capillary tube is given by:

$$Q = V \cdot A = u \cdot A = \frac{1}{4\mu} \cdot \frac{\partial P}{\partial z} (r^3 - rR^2) \cdot 2\pi r dr \quad (6)$$

From the energy conservation equation, we have:

$$h_e^T - h_s^T = h_i + E p_i + E c_i - h_{i+1} - E p_{i+1} - E c_{i+1} = 0 \quad (7)$$

Considering the negligible potential and kinetic energy, we have:

$$u_i - u_{i+1} + (P_i - P_{i+1})/\rho = 0 \quad (8)$$

$$\frac{\Delta P}{\rho} = h_l \quad (9)$$

Using Eq. (6) in Eq. (9), we have:

$$\frac{8\mu QL}{\pi R^4} = h_l \rho \quad (10)$$

Rearranging the term, we have the equation of loss of charge.

$$h_l = \frac{32\bar{V}\mu L}{\rho D^2} \quad (11)$$

where  $\frac{\mu}{\rho} = \frac{1}{\nu}$ .

### 2.3 Mass balance

The Mass Storage Eq., for the steady-state flow, it can be given by:

$$\frac{dN}{dt} = \frac{\partial}{\partial t} \int \eta \cdot dV + \int V \cdot dA \quad (12)$$

$$\frac{V_1 A}{v_1} = \frac{V_2 A}{v_2} = \dot{m} \quad (13)$$

$$\frac{V_1}{v_1} = \frac{V_2}{v_2} = \frac{\dot{m}}{A} = G \quad (14)$$

### 2.4 Constitutive relations

#### 2.4.1 Friction factor for single-phase region

Due to the small diameter of the capillary tube, the effects of the roughness of the walls should be considered. Thus, the capillary tube cannot be considered as a smooth tube, and the friction factor becomes a function of Reynolds number and relative roughness.

Churchill (1977) obtained the following empirical equation from experimental data of several authors. This equation is explicit and valid for laminar, transition and turbulent regimes.

$$f = 8 \left[ \left( \frac{8}{Re} \right)^{12} + \frac{1}{(A+B)^{\frac{3}{2}}} \right]^{\frac{1}{2}} \quad (15)$$

Where:

$$A = \left\{ 2,457 \left[ \frac{1}{\left( \frac{7}{Re} \right)^{0,9} + 0,27 \frac{\epsilon}{D}} \right] \right\}^{16} \quad (16)$$

$$B = \left( \frac{37530}{Re} \right)^{16} \quad (17)$$

The flow of the liquid into a capillary tube is, in most cases, turbulent. Experimental Evidence Boabaid Neto (1994) show that, in these cases, the Reynolds number is generally greater than  $7 \times 10^3$ .

#### 2.4.2 Friction factor for biphasic region

There are basically two ways of estimating the friction factor in the biphasic region. In the first way, the Reynolds number is calculated using one of the many correlations available for biphasic viscosity, and the friction factor is obtained through an equation for single-phase flow. The other way is to use empirical correlations, which are equations obtained through the analysis of experimental data.

Boabaid Neto (1994) conducted an extensive literature review on this subject and compared the results of his model using a series of correlations available in the literature with experimental results for adiabatic capillary tubes. His

conclusion was that the correlation proposed by Erth (1980) is the most appropriate for the refrigerants HFC 134a and CFC 12. The correlation is given below:

$$f = \frac{3,1}{\sqrt{Re}} \cdot \exp\left(\frac{1-x^{0,25}}{2,4}\right) \quad (18)$$

This equation provides an average value of friction factor along the entire biphasic stretch as a function of the Reynolds number and the quality, both at the capillary entrance. Thus, if the sub cooled liquid condition occurs at the input, the quality is zero and the equation becomes a function only of the Reynolds number.

### 2.4.3 Coefficient of heat transfer for single phase flow

For single-phase flows that have already been extensively studied, there is a wide range of correlations for the Nusselt number available in the literature.

According to Kakaç et al. (1987), such correlations should be used with caution in the case of tubes with diameters smaller than 2 [mm]. This is because mechanisms with turbulent vorticity are reduced by reducing the cross-section of the tube, resulting in lower heat transfer coefficients, regardless of the level of flow turbulence, as is commonly seen.

In a capillary heat exchanger-suction line, there is liquid flow in part of the capillary and superheated steam in the suction line. The following equation is the correlation of Gnielinski (1976), which is recommended by Kakaç et al. (1987):

$$Nu = \left(\frac{f}{8}\right) \cdot (Re - 1000) \cdot \frac{Pr}{1 + 12,7 \cdot \left(\frac{f}{8}\right)^{0,5} \cdot \left(\frac{2}{3} \cdot \frac{Pr - 1}{Pr}\right)} \quad (19)$$

This expression is valid for Reynolds numbers between  $2.3 \times 10^3$  and  $5 \times 10^4$  and for Prandtl numbers greater than 0.5.

### 2.4.4 Coefficient of heat transfer for biphasic flow

The determination of a correlation for the coefficient of heat transfer in the region of phase change seems, in principle, of great importance for the modeling of the problem.

In the heat exchanger capillary-line suction line, an evaporation process occurs with cooling, making it difficult to model empirical correlations for the Nusselt number. In some cases, condensation may also occur along the flow, depending on the rate of heat transfer.

Many authors realized that adopting a mathematical or other correlation for the calculation of the convection coefficient, with phase change, provided similar results, thus its impact is not so great. They considered the convective coefficient infinite. The reason why they did it is that when the heat transfer involves phase changes, the heat transfer coefficient becomes extremely high, not implying significant changes in the thermal resistance calculation.

The model used is described by Pate (1982) based on the so-called net speed method. According to this method of analysis, only the properties of the liquid affect the heat transfer and the only effect of the vapor is to increase the velocity of the liquid.

$$Nu = 0,023 \cdot Re^{0,8} \cdot Pr^{0,3} \cdot \left(\frac{1-x}{1-\alpha}\right)^{0,8} \quad (20)$$

where the fraction of void is given by:

$$\alpha = x \cdot \frac{v_g}{v_l} \quad (21)$$

## 3. RESULTS

This section deals with the results of the analysis performed on non-adiabatic capillary devices. The fluid addressed in these results was ammonia, a fluid commonly used in absorption refrigeration systems, which operate below the freezing temperature of the water and in vapor compression systems, which is more common today.

### 3.1 Validation of the code in the EES platform

For the validation of this work, data from the simulation performed by Marongane (1995) were used. The validation consisted essentially of performing a simulation with the same input data as Marongane (1995), and comparing the pressure gradient along the capillary tube. Subsequently, the final error was calculated.

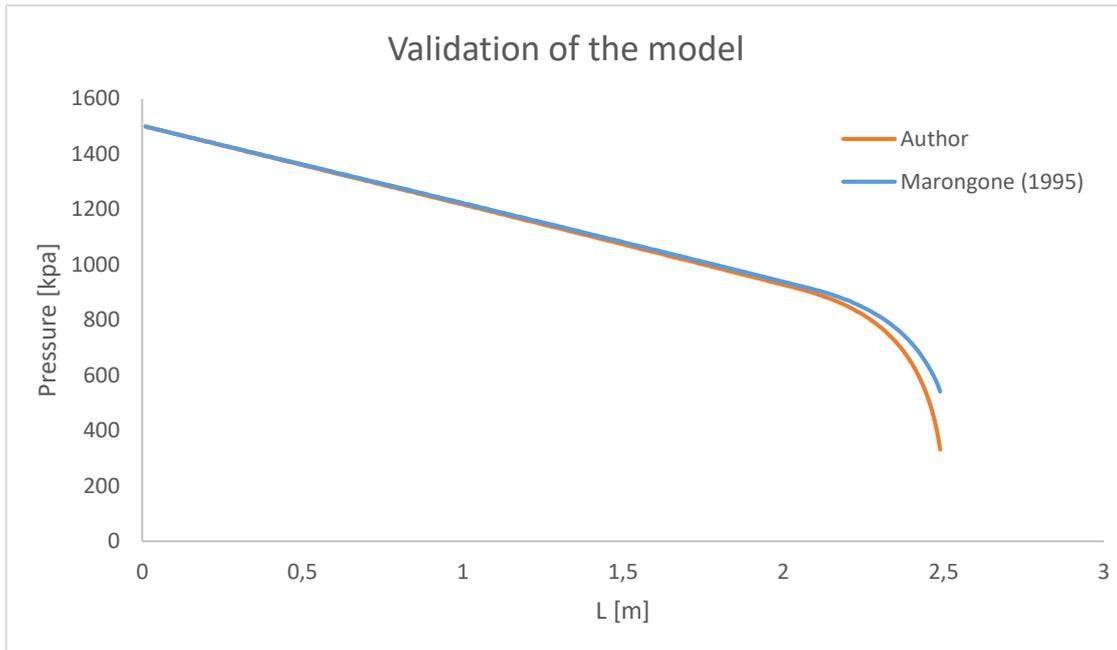


Figure 4. Validation of the computational code for the ammonia flow in non-adiabatic capillary tubes.

The final error observed in the simulation was 7,71%, when compared to the Marongane (1995) simulation. It was a high error, considering that correlations were used for the friction factor and Nusselt number for the non-adiabatic flow specified for the case.

The error is attributed mainly to the poor refinement of the differential element in the region where the highest concentrations of phase change of the fluid occurs. Complications involving phase shift flow of the fluid increased the final error, thus further refinement of the element of that region would be required for significant reduction of error. However, due to the limitations found in the EES, it was not possible to perform the refinement necessary to reduce the final error.

### 3.2 Analysis of the influence of differential element size

In order to verify the influence of the refinement (size of the differential element) in the simulation results, three (3) simulations were performed for comparison and final calculation of the error.

It can be observed that with an element of  $\Delta L = 0.05$  [mm], the resulting error is extremely high. The error becomes smaller when the element size is reduced, decreasing significantly for elements smaller than  $\Delta L = 0.01$  [mm]. The result converges to a satisfactory result when the differential element is less than 0.01 [mm], as shown in table 1.

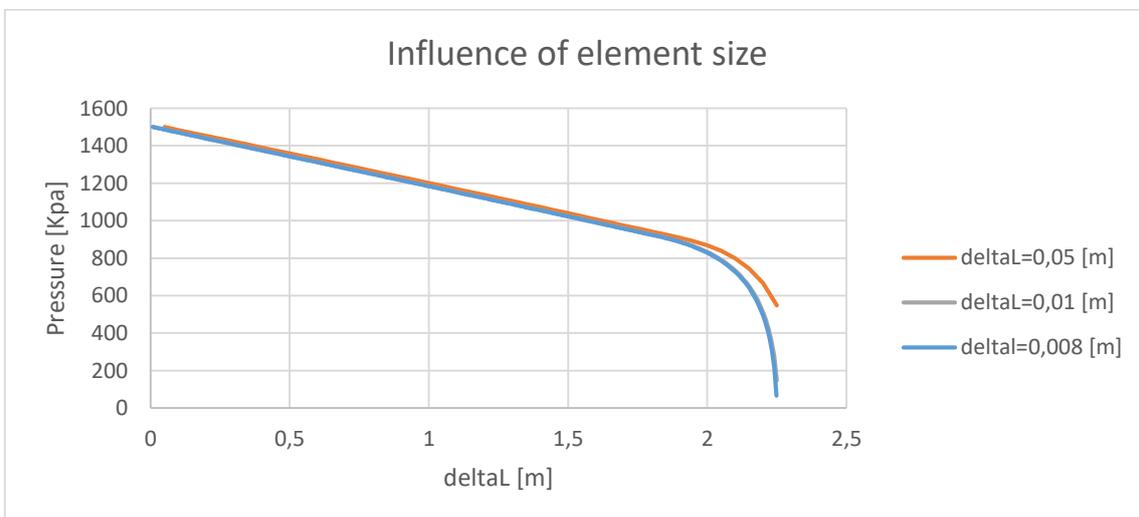


Figure 5: Graph to compare the curves after the refinement of the element size.

Table 1: Final error for element refinement.

DeltaL [mm]	0,05	0,01	0,008
Error (%)	32,164	5,553	Ref.

### 3.3 Influence of mass variation

As the capillary tube must be designed for a specific operating condition basically operating in a steady state, the influence of the mass flow variation on the thermodynamic state of the capillary outlet was studied.

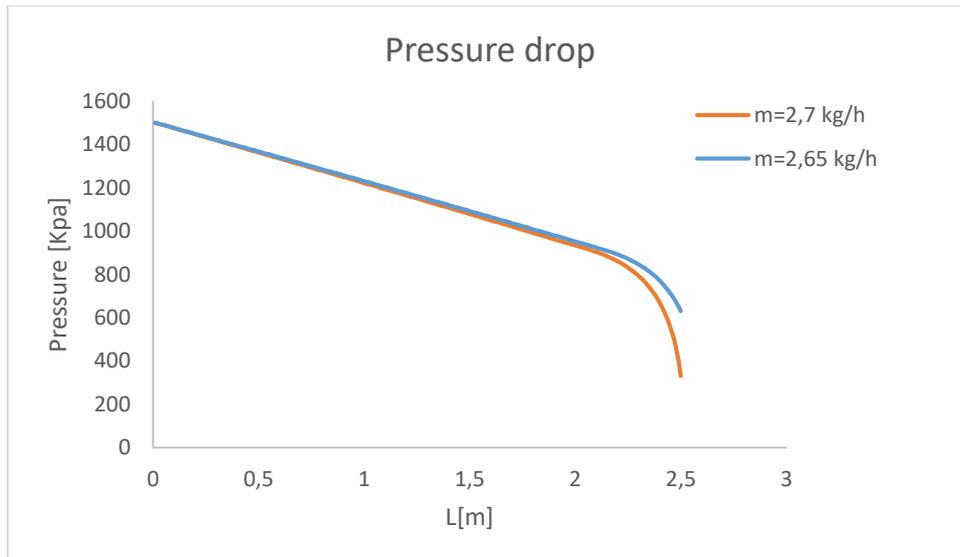


Figure 6. Influence of the mass variation on the pressure gradient along the capillary tube.

It is possible to observe that, with a variation of only 50 grams/h in the mass flow in the capillary, the output pressure is reduced by up to 26.4%, imposing a non-ideal system operating condition.

It can also be seen a change in the capillary output enthalpy, where the enthalpy is directly associated with the COP of the refrigeration system. The change in this specific case represented a reduction of 75 [W] of thermal load on the evaporator, or approximately 8.1%.

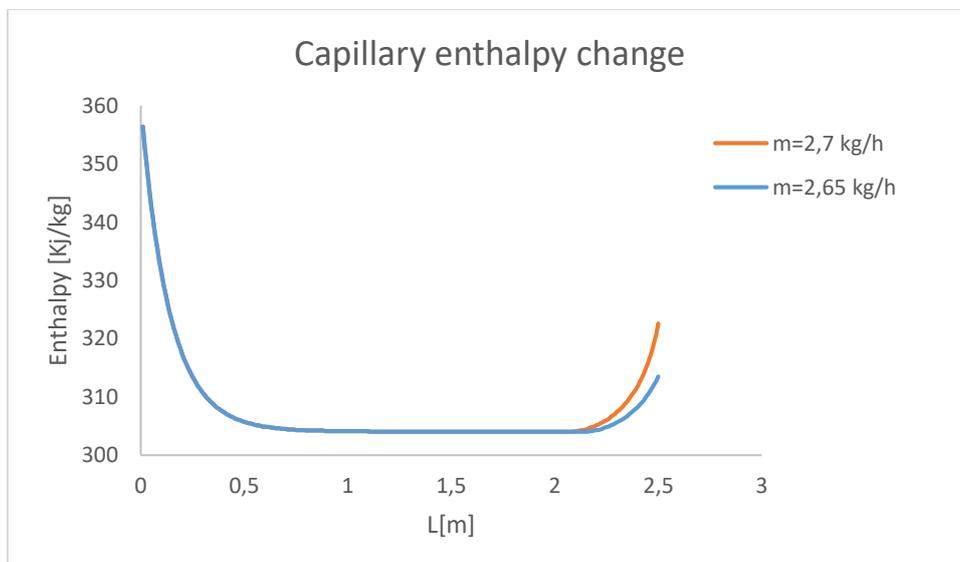


Figure 7. Variation of the enthalpy of the exit of the capillary tube.

### 3.4 Specific cases and analysis of results

Two specific cases were studied for the analysis of capillary tube output. The input data were taken from experimental practical cases of MARONGANE (1995). However, the results were not compared. The validation of the model is present in the first section of this chapter.

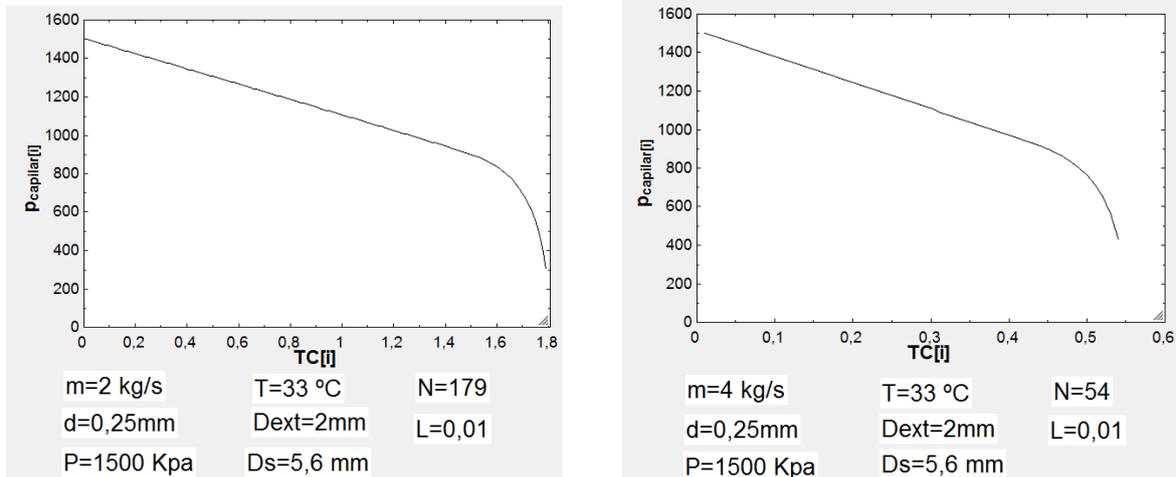


Figure 8. Pressure gradient [Kpa] as a function of capillary tube length [m].

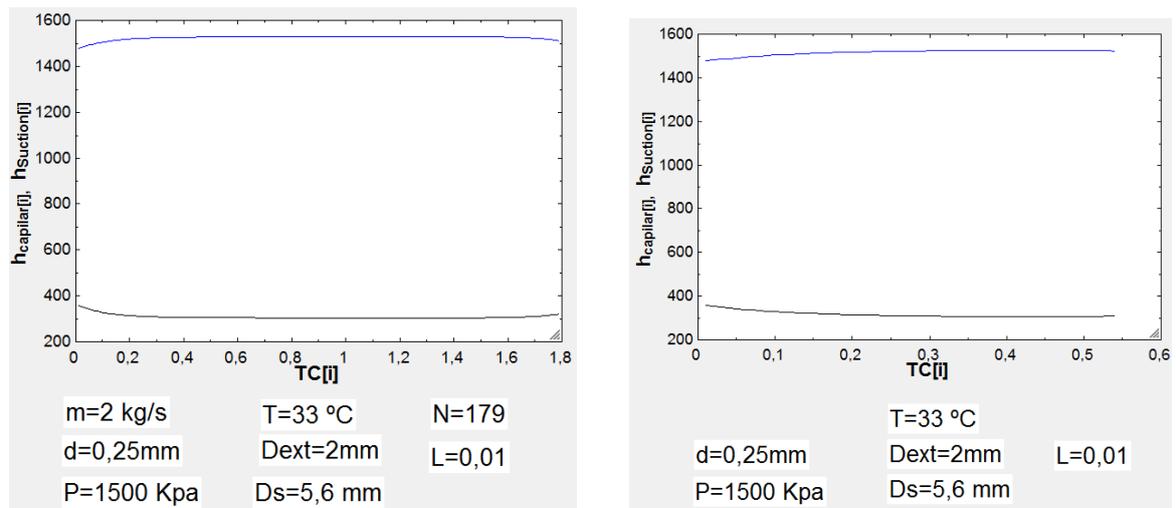


Figure 9. Variation of the enthalpy [kJ/kg] of the suction tube (blue) and the capillary tube (black) along the length [m].

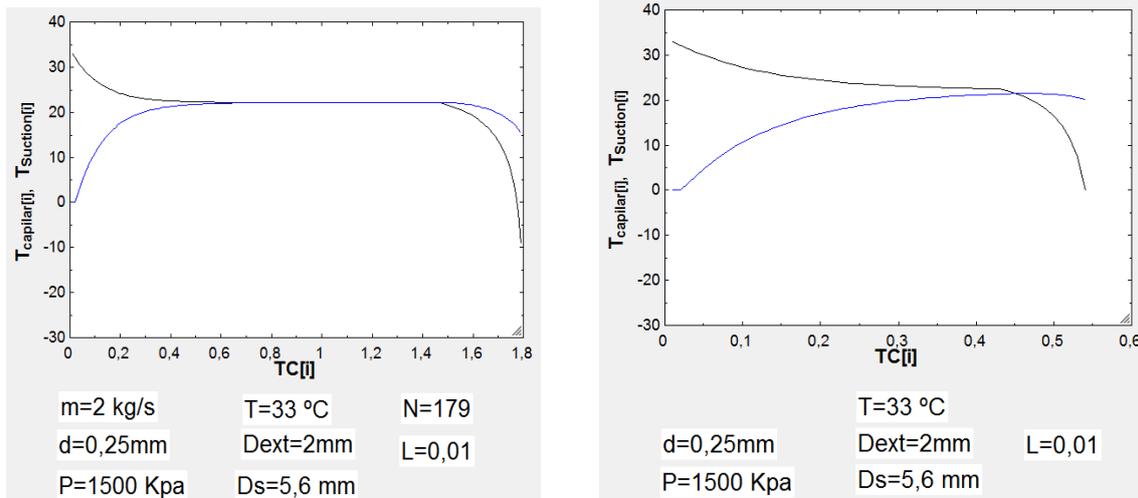


Figure 10. Temperature profile of suction tube (blue) and capillary tube (black) along length [m].

It can be seen in figure 10 that, for the first case, the temperature of the capillary tube and the suction tube equals and remains constant for a certain length, whereas such phenomenon does not occur in the second case. This phenomenon can be explained due to the condensation of not only the capillary fluid, but also due to condensation of the suction tube fluid. It is not the purpose of this arrangement to promote the condensation of the suction tube fluid, since some damage to the compressor can be caused by the inflow of refrigerant in the saturated liquid condition. In this way, the heat exchanger used must have an optimized length for the best performance of the arrangement.

In figure 11, it is possible to observe the condensation of part of the capillary tube fluid, due to heat loss to the suction tube, the purpose of this arrangement.

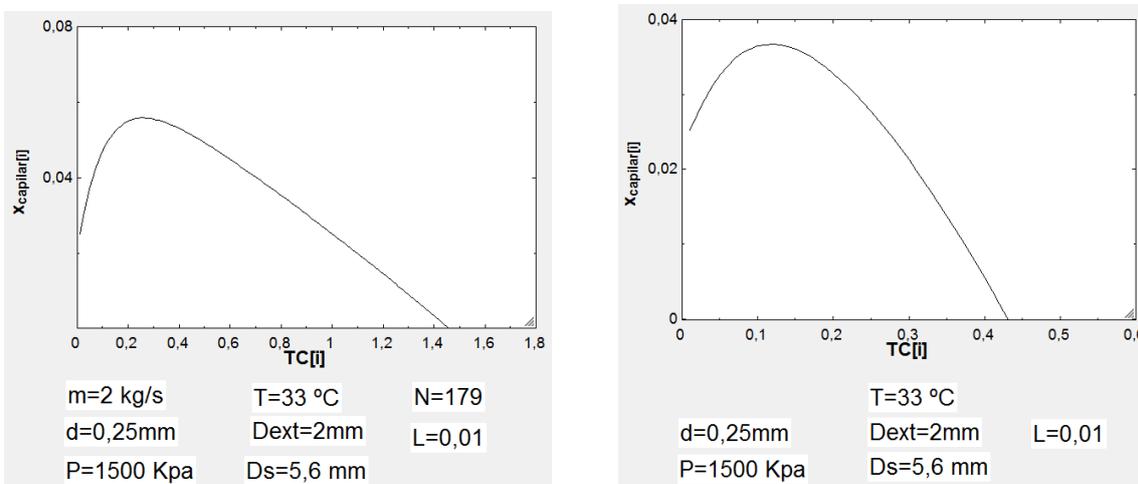


Figure 11. Re-condensation of the fluid along the capillary, due to loss of heat to the suction tube.

#### 4. CONCLUSIONS

The present work presented an alternative method for the analysis of non-adiabatic capillary tubes for the ammonia flow, using the equations of conservation of energy and moment, solving the set of equations by a progressive method of finite differences from a code implemented in the EES platform, where processing time and accuracy were the determining factors for their choice. With a relatively small number of interactions, it was possible to observe agreement with the compared models.

The difficulties encountered in the analysis of biphasic heat transfer were solved through specific correlations found in the literature, and the effect of pressure loss was computed numerically through the sum of the pressure loss in each differential element. However, there are still difficulties in establishing specific correlations for the phase change of the fluid, which can result in a greater final error.

Further, it was possible to observe that the arrangement can still be optimized by reducing the length of heat exchange, which will be the focus of future work. A methodology can be created relating inlet and outlet data for calculating the optimum length of the capillary heat exchanger.

For future work a generalized correlation can be developed, based on the combination of operating parameters for analysis of the COP.

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