24th ABCM International Congress of Mechanical Engineering
December 3-8, 2017, Curitiba, PR, Brazil

COBEM-2017-0157
DAMAGE DETECTION IN A CANTILEVER BEAM BY A STOCHASTIC OPTIMIZATION ALGORITHM THROUGH THE VARIATION OF THE NATURAL FREQUENCIES

Gustavo Zeni
gugazeni@hotmail.com

Letícia Fleck Fadel Miguel
Mechanical Engineering Department – Federal University of Rio Grande do Sul
Rua Sarmento Leite, 425 – Porto Alegre,RS – Brasil – 90050-170
letffm@ufrgs.br

Leandro Fleck Fadel Miguel
Civil Engineering Department – Federal University of Santa Catarina
Leandro.miguel@ufsc.br

Abstract. *This paper presents the damage detection as an optimization problem using a stochastic optimization algorithm known as Backtracking Search Algorithm (BSA). This one uses the differences from the first five natural frequencies of a cantilever beam identified from the Stochastic Subspace Identification (SSI) as parameters for the objective function on the optimization process. The beam was modeled with ten Euler-Bernoulli elements through a FE model developed by the authors and the response for free vibration and ambient vibration was recorded using the Newmark method. In order to test the accuracy and efficiency of the method, four different damage scenarios were proposed and noise of 3% and 5% was introduced on the measurements. The proposed method for damage detection was accurate for free vibration test and low level of noise in the ambient vibration. As the noise level was increased, the identification of natural frequencies was less precise, thus the damage localization indicated some elements at the free end of the beam that in fact was not damaged.*

Keywords: *Structural Health Monitoring (SHM), Stochastic Subspace Identification (SSI), Stochastic optimization algorithm, Backtracking Search Algorithm (BSA), Ambient vibration*

1. INTRODUCTION

Practical methods of structural integrity control play an important role in the field of engineering. These methods use the control of structural parameters as an analysis tool. These parameters help to monitor the performance and reliability of the structure and can be used to locate and quantify structural damage. The damage is modeled as a variation on the mass matrices and rigidity of the system, so it can be used in models to compare it with a situation in which the structure under analysis is undamaged. This problem is in the field of Structural Health Monitoring (SHM).

The SHM aims to monitoring the dynamic response of the structure through experimental data. In recent developments SHM techniques, the focus has been placed on taking advantage of ambient vibration, such as traffic, the wind, or pedestrian-induced vibration to determine the spectral properties at any moment, without operational interference since there are some limitations in the use of classical analysis in problems involving bridges, for example. In these situations, in which only output measure data are available, techniques have been developed so that it is possible to identify the modal properties of the structure, such as Stochastic Subspace Identification (SSI) technique (Van Overshee and De Moor, 1993). The SSI method is capable of identifying models directly from time signals using the matrices of the state-space model. More details and applications of SSI in civil engineering can be found in Peeters (2000), for example.

Since SHM systems typically deal with differences between the theoretical and experimental responses, its basic idea is to modify de numerical model properties to achieve an optimum fit with the experimental data. Therefore, the whole process can be view as an optimization process, in which the objective functions must be formulated regarding the modal properties. Several studies with genetic algorithms have been done in the field of SHM (e.g., Au, Cheng, Tham, & Bai, 2003; Chou & Ghaboussi, 2001; Gomes & Silva, 2008; Hao & Xia, 2002; Meruane & Heylen, 2010).

Nowadays, the study of damage detection through the variation of the modal properties is still on debate, the focus is to compare optimization algorithms that can be robust enough to localize any level of damage. This work contributes to this study testing a new evolutionary algorithm, the BSA, on damage detection using a method for identifying natural frequencies through output data only, the SSI.

The process of damage detection through the variation of the natural frequencies proposed in this paper can be divided in three steps: (1) measurement of the vibration response of the structure; (2) modal identification using the stochastic subspace identification (SSI); (3) damage localization as an optimization process using the Backtracking Search Algorithm (BSA). The time-domain response is generated by a numerical model in MatLab using the Newmark method to represent the output data of the beam. The model is tested for free vibration and ambient vibration considering the noise effects

2. PROBLEM FORMULATION

This work deals with damage detection of a cantilever beam using the stochastic subspace identification (SSI) method. A stochastic optimization algorithm known as Backtracking Search Algorithm (BSA) is also used. Both the modal analysis and the optimization algorithm were implemented in a MatLab FE code developed by the authors. The beam is analyzed for free vibration and ambient vibration introducing noise on the measurements to evaluate the robustness of the procedure in situations similar to field conditions.

2.1 Damage detection as an optimization problem

The damage changes the stiffness and mass matrix of the structure. However, the changes in the mass matrix will be neglected once the changes in the stiffness are much higher than the loss of mass (Adams et al. (1978) and Hearn and Testa (1991)). The damage is introduced into the structure through the consideration of an elemental stiffness reduction factor (α_i). Thus, the global stiffness matrix [K] could be defined as a sum of damaged and undamaged element stiffness matrices $[\mathbf{k}]_i$.

$$[K]=\sum_{i=1}^{N}\alpha_i\,[k]_i \tag{1}$$

In which N is the total number of elements and α_i is the reduction factor. The reduction factor ranges from 0 to 1, where 1 signifies no damage in the element and zero means completely loss of its stiffness.

To solve this optimization problem, the objective function ($\prod$) must be formulated in terms of the differences between the experimental and analytical values. The choice of the objective function is of paramount importance in FE model updating because it can influence the performance of the optimization algorithm. In this work, the objective function is defined in terms of fractional changes in natural frequencies for the structure before and after damage.

$$\Pi(\alpha)=\sum_{i=1}^{NM}\left(\left(\frac{\delta\omega_i(\alpha)}{\omega_i}\right)^A-\left(\frac{\delta\omega_i}{\omega_i}\right)^E\right)^2 \tag{2}$$

In which the superscripts A and E represent analytical and experimental, respectively, ω_i are the natural frequencies for the *i*th mode of undamaged or healthy condition for both experimental and analytical conditions and $\delta\omega_i$ is a fractional change of experimental and analytical natural frequencies for the *i*th mode of the structure. A FE model should be used to represent the undamaged structure, or the reference target structure. Thus, the reduction factor of the FE model should be updated until the differences between the numerical frequencies in the undamaged and damaged condition converged to those observed in experimental frequencies in the pre- and post-damaged state.

2.2 Backtracking search algorithm (BSA)

This work deals with the problem of damage detection as an optimization problem using the Backtracking Search Algorithm (BSA). BSA is a new evolutionary algorithm (EA), proposed by Civicioglu (2013), for solving real-valued numerical optimization problems. EAs are popular stochastic search algorithms that are widely used to solve non-linear, non-differentiable and complex numerical optimization problems. EAs have been used for several applications, for example: mechanical design problems, communication applications, image processing applications, speech recognition and so on. These methods come up when the classical techniques fail to find a global optimum since the size of the search space increases with the dimension of the optimization problem. Civicioglu (2013) describes BSA and its efficiency. A gradient-free, or non-gradient algorithm, is preferred in this case because the objective function may have discontinuity, so that the solution may not converge.

The first step is determining the objective function. It states the relation between the system constraints and the system parameters. Usually the global optimum of the optimization problem is the global minimum of the objective function. The strategies for generating trial values to use on their objective function are the way EAs differ from one another. BSA uses three genetic operators to generate trial individuals: selection, mutation and crossover. In contrast with many other EAs, BSA has a random mutation strategy that uses only one direction individual for each target individual and uses a non-uniform crossover strategy that is more complex than the others algorithm.

BSA was designed to be a global minimizer and its algorithm can be explained as is done in another EAs: initialization, selection-I, mutation, crossover, and selection-II. These steps are described as follows [Civicioglu (2013)].

Initialization – The population of the algorithm is initialized according to (3) and (4), where U is the uniform distribution, lowj is the low boundary and upj is the up boundary of the variables.

$$P_{i,j} \sim U(lowj, upj) \tag{3}$$

$$oldP_{i,j} \sim U(lowj, upj) \tag{4}$$

In which i is the ith individual of the population, i=1, 2, …, S, S is the population size, j is the jth bit of the ith individual, j=1, 2, …, D, and D is the dimension size of variables.

Selection-I – oldP is the history population and in the beginning of each iteration it is introduced in BSA according to (5) and (6):

$$oldP = \begin{cases} P \text{ if } (a<b \mid a,b \sim U(0,1)) \\ oldP \text{ otherwise} \end{cases} \tag{5}$$

$$oldP := permuting(oldP) \tag{6}$$

Mutation – The process of mutation is defined according to (7):

$$M = P + F \cdot (oldP - P) \tag{7}$$

The experience from previous generation is used to generate a trial population and the historical information of the population penetrates the whole evolutionary process. F is the parameter that controls the amplitude of the search-direction matrix and its common value is equal to 3·randn, where randn ~ ND (0,1) (ND is the standard normal distribution).

Crossover – This process sets the final form of the trial population and the initial value of it comes from the mutation process. A binary integer-valued matrix (map) of size S·D guides the crossover directions of BSA algorithm. The crossover strategy of BSA is expressed in (8).

$$V_{i,j} = \begin{cases} P_{i,j} \text{ if } map_{i,j}=1 \\ M_{i,j} \text{ otherwise} \end{cases} \tag{8}$$

Selection-II – In this stage, the values of Vis that have better fitness than the corresponding Pis are used to update the Pis based on a greedy selection. The global minimum value obtained so far is compared to the best individual of P, and if P has a better fitness value, the global minimizer is updated to Pbest. This process is shown in (9).

$$P_i^{best} = \begin{cases} V_i \text{ if } f(V_i) \leq f(P_i) \\ P_i \text{ otherwise} \end{cases} \tag{9}$$

The structure of BSA is simpler than some EAs. Another aspect of this algorithm is that it uses de historical information in the updating process to improve its performance, that is, BSA is a dual-population algorithm that uses the current and the historical population, unlike some EAs [Civicioglu (2013)].

3. ANALYZED BEAM

The beam analyzed is 1000 mm long and it has square box cross section with external dimensions 25.4 mm and wall thickness equal to 1 mm. The beam was modeled with 10 Euler-Bernoulli beam elements (Fig. 1). The specific mass and elastic modulus are, respectively, 2800 kg/m³ and 68.6 GPa.

Numerically simulated dynamic tests are conducted through the consideration of two standard excitations: an impulsive and an ambient vibration. The ambient vibration is modeled by a Gaussian white noise signal generated with MatLab, with zero mean and standard deviation equal to one, applied at all the generalized coordinates of the structure (Brownjohn, Lee, & Cheong, 1999; Peeters and de Roeck, 2001). The impulsive loading is represented by the application of an impact at node 4 in the vertical direction.

There are four different damage scenarios to check the method's effectiveness on locate the damage. The elastic modulus is reduced 10% and 70% for element 8 and 50% for element 3. In the fourth damage scenario, the method is tested for damage in three elements simultaneously (elements 1, 2 and 3), with progressive reduction of the stiffness in 70%, 50% and 30%, respectively. Table 1 shows the first five theoretical frequencies for the first 3 damaged scenarios. This damage scenario was proposed in order to test the algorithm in some various levels of damage. The beam is numerically modeled using the FE MatLab code and the dynamic problem using the Newmark method. The time step of integration is equal to 10^{-5} s for the free vibration case and 10^{-6} s for ambient vibration (Fadel Miguel et al., 2012). The damping of the beam is assumed as proportional to the mass and the stiffness (Rayleigh damping) since the mass is distributed homogeneously in the beam.

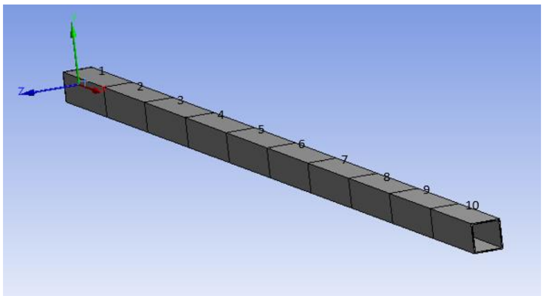

Figure 1 – Beam modeled with 10 elements

Table 1 – Five first theoretical frequencies for the 3 first proposed damage scenarios

Frequencies (Hz)				
Modes	Healhty	Scenario 1	Scenario 2	Scenario 3
1st	27.614	27.608	27.484	25.488
2nd	173.061	172.381	159.578	172.108
3rd	484.683	479.335	405.838	459.168
4th	950.449	941.619	854.353	903.321
5th	1,573.621	1,568.953	1,504.943	1,543.747

The data of the output-only system is carried out through the Stochastic Subspace Identification method (SSI). As earlier described, only the first five modes are available to represent situations that are close to field conditions. Afterwards, the procedure is repeated in order to assess the change in the stiffness caused by the damage in the element and the data is used in the BSA algorithm through the objective function specified in the Section 2. Tables 2 and 3 show the results for free vibration and ambient vibration, respectively, through the SSI method. Noise of 3% and 5% was introduced to the signal in order to test the robustness of the method to identify natural frequencies with the output signal only. Noise levels were simulated through the addition of white noise signals using the RMS amplitude of the mean measured response.

Afterwards, the optimization procedure is updated to assess the changes in the stiffness caused by the damage. The damage identification after the BSA updating for free vibration and ambient vibration and both noise levels are shown in Figures 2 to 5. Although some elements close of the damaged element show reduction in the stiffness it is observed that the damage localization for the case of 3% of noise is effective in the three proposed damage scenarios. For the case with 5% of noise, the damage scenario 1 with ambient vibration could not localize the damaged element.

Table 2 – Identified frequencies for free vibration

Frequencies (Hz)	Noise 3% (Hz)				Noise 5% (Hz)			
Modes	Healthy	Scenario 1	Scenario 2	Scenario 3	Healthy	Scenario 1	Scenario 2	Scenario 3
1st	27.619	27.607	27.481	25.483	27.619	27.601	27.483	25.486
2nd	173.033	172.382	159.555	172.097	173.115	172.371	159.583	172.081
3rd	484.486	479.099	405.918	459.461	483.407	479.549	405.481	458.043
4th	948.601	942.737	853.952	901.582	944.005	941.167	845.908	905.950
5th	1,573.695	1,567.038	1,502.629	1,536.799	1,565.048	1,564.048	1,506.058	1,530.854

Table 3 – Identified frequencies for ambient vibration

Frequencies (Hz)	Noise 3% (Hz)				Noise 5% (Hz)			
Modes	Healthy	Scenario 1	Scenario 2	Scenario 3	Healthy	Scenario 1	Scenario 2	Scenario 3
1st	27.595	27.667	27.628	25.557	27.787	27.576	27.491	25.412
2nd	172.871	172.362	159.637	172.221	173.567	171.876	159.624	171.943
3rd	484.450	479.424	406.086	459.501	482.786	478.196	406.559	459.411
4th	948.886	944.755	855.591	900.876	953.015	941.157	857.397	899.007
5th	1,582.151	1,567.714	1,514.681	1,588.652	1,578.884	1,560.386	1,533.757	1,555.203

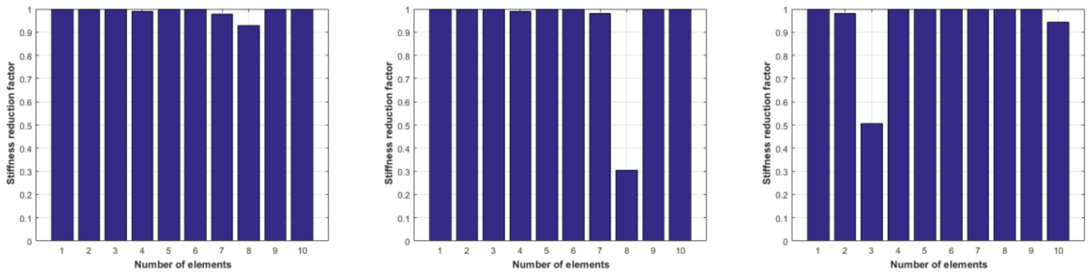

Figure 2 – Damage detection for free vibration with 3% noise on the measurement

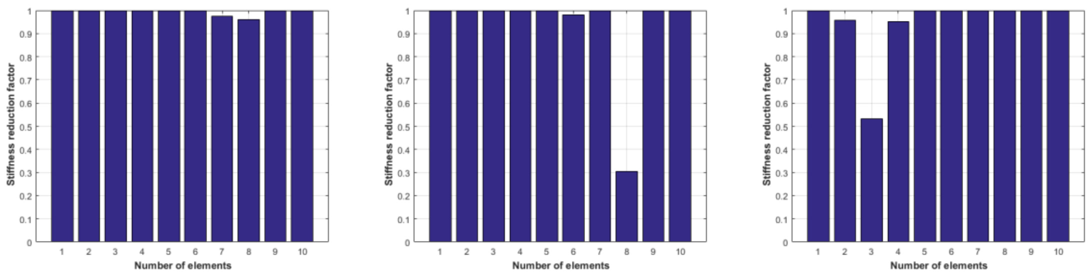

Figure 3 – Damage detection for free vibration with 5% noise on the measurement

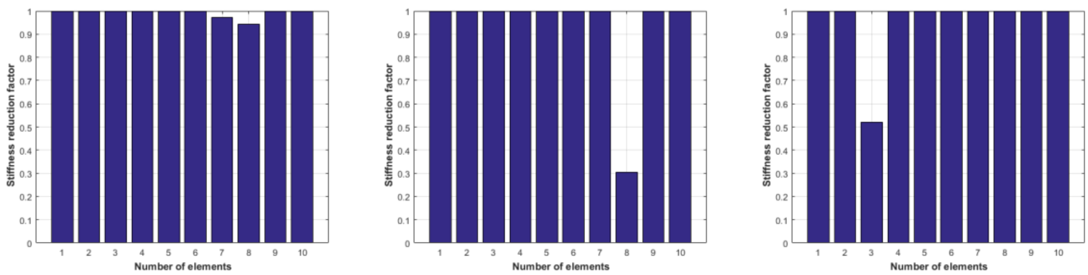

Figure 4 – Damage detection for ambient vibration with 3% noise on the measurement

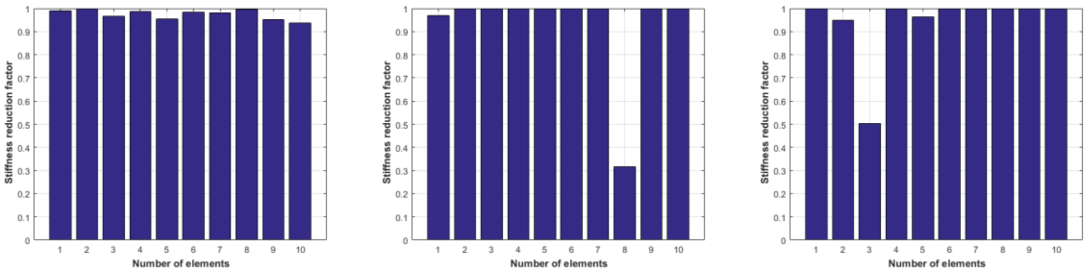

Figure 5 – Damage detection for ambient vibration with 5% noise on the measurement

A last damage scenario is proposed in order to test the method for a combined damage situation, as explained before. In this case, elements 1, 2 and 3 loses its stiffness 70%, 50% and 30%, respectively. This case deals with progressive damage from the fixed side of the beam. The identified frequencies for ambient vibration and free vibration with 3% and 5% of noise are shown in Table 4. Figures 6 and 7 show the damage detection for the last proposed damage scenario, for free and ambient vibration, respectively.

Table 4 – Identified frequencies for free vibration and ambient vibration for scenario 4

Frequencies (Hz)	3%	5%	3%	5%
Modes	Free Vibration		Ambient Vibration	
1st	18.850	18.885	18.926	18.911
2nd	148.798	148.792	148.663	148.766
3rd	422.352	422.864	420.879	422.582
4th	833.825	834.964	832.499	839.638
5th	1,395.933	1,418.520	1,402.352	1,436.694

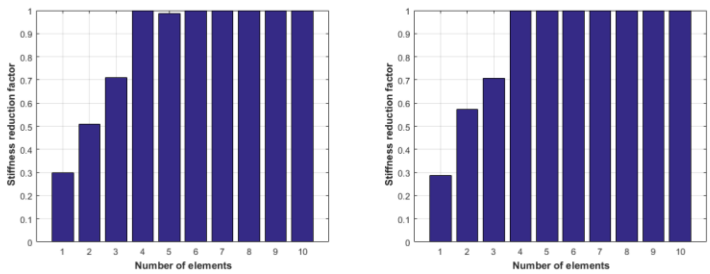

Figure 6 – Damage detection for free vibration and 3% and 5% of noise

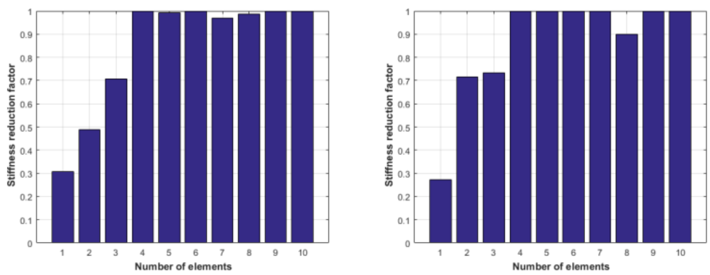

Figure 7 – Damage detection for ambient vibration and 3% and 5% of noise

Unlike for the ambient vibration case, it is observed that for free vibration the progressive damage localization is almost exact, and it does not depend on the both noise level proposed. There is an error in element 8 stiffness reduction for ambient vibration and 5% of noise.

4. CONCLUSIONS

This paper tested a method of damage localization and quantification as an optimization process for various levels of damage modeled as a reduction factor in the element stiffness. The method showed good results for free vibration even with noise on the measurement. It could localize the damaged element and quantify the reduction in the stiffness for all proposed scenarios with 3% noise. The SSI method proved to be effective even not using a large amount of data from the time-response of the beam, identifying the first five natural frequencies with small error.

The method has some limitations regarding the identification process using the SSI. The main limitation was the high processing capacity required since the output data was composed by thousands of numbers. When the natural frequencies identification step is not capable of identifying high frequencies accurately, the damage localization algorithm is not able to give acceptable results. Despite these method limitations, the Backtracking Search Algorithm showed good results converging fast for the solution.

The results for free vibration were not affected by the noise and the method could localize and quantify all four proposed scenarios, even in the cases for low damage level and moderate noise. For ambient vibration, the method could localize and quantify damage for 3% of noise. Thus, the authors believe that the proposed approach may be an interesting tool for structural health monitoring through real-time damage diagnosis.

5. REFERENCES

Adams, R., Cawley, P., Pye, C. J., Stone, B. (1978). A vibration technique for non-destructively assessing the integrity of structure. Journal Mechanical Engineering, 20, 93-100.

Au, F. T. K., Cheng, Y. S., Tham, L. G., & Bai, Z. (2003). Structural damage detection based on a micro genetic algorithm using incomplete and noisy modal test data. Journal of Sound and Vibration, 295, 1081-1094.

Brownjohn, J. M. W., Lee, J., & Cheong, B (1999). Dynamic Performance of a curved cable-stayed bridge. Engineering Structures, 21, 1015-1027.

Chou, J. H., & Ghaboussi, J. (2001). Genetic algorithm in structural damage detection. International Journal of Computers and Structures, 79, 1335-1353.

Civicioglu, P. (2013). Backtracking search optimization algorithm for numerical optimization problems. Applied Mathematics and Computation, 219, 8121-8144.

Gomes, H. M., & Silva, N. (2008). Some comparisons for damage detection on structures using genetic algorithm and modal sensitivity method. Applied Mathematical Modelling, 32, 2216-2232.

Hao, H., & Xia, Y. (2002). Vibration-based damage detection of structures by genetic algorithm. Journal of Computing in Civil Engineering, 16, 222-229.

Hearn, G., Testa, R. B. (1991). Modal analysis for damage detection in structures. Journal of structure engineering, 117, 3042-3063.

Meruane, V., & Heylen, W. (2010). An hybrid real genetic algorithm to detect structural damage using modal properties. Mechanical Systems and Signal Processing. doi: 10.1016/j.ymssp.2010.11.020.

Miguel, L. F. F., Fadel Miguel, L. F., Kaminski Jr, J., Riera, J. D. (2012). Damage detection under ambient vibration by harmony search algorithm. Expert Systems with Applications, 39, 9704-9714.

Peeters, B. (2000). System identification and damage detection in civil engineering, PhD Thesis, Department of Civil Engineering, K. U. Leuven, Belgium.

Peeters, B., & Roeck, G. (2001). Stochastic system identification for operational modal analysis: A review. Journal of Dynamic System, Measurement and Control, 123, 1-9.

Van Overschee, P., & de Moord, B. (1993). Subspace algorithm for the stochastic identification problem. Automatica, 29, 649-660.

6. ACKNOWLEDGEMENTS

The authors acknowledge the financial support of the Brazilian agency CAPES.

7. RESPONSIBILITY NOTICE

.

The authors are the only responsible for the printed material included in this paper.