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APPLICATION OF TAGUCHI'S LOSS FUNCTION AND SIGNAL-TO-NOISE RATIO

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Abstract. *Genichi Taguchi revolutionized the Japanese economy through his ideas in quality engineering, showing the quadratic loss function and the signal-to-noise ratio. The Loss Functions was a great advanced in quality engineering as it managed to improve and translate the monetary loss in production, improving the concept of conforming and nonconforming products. The signal-to-noise ratio enables to analyses the expected response with the noises present. Bootstrapping is an intensive computational technique, nonparametric, that allows estimate some parameters, variances and confidence intervals. This paper presents some of Taguchi's concepts and provide an evaluation of his statistical methods with resampling Bootstrap technique in food industry. We compared the quality of five different products. Comparisons were made between similar products and between work shifts in the same manufacturing process. An application "Experimento Taguchi" was developed to execute and show all the steps of this process, facilitating its posterior statistical analysis.*

Keywords: *Bootstrap, Loss Function, Signal-to-Noise Ratio, Taguchi, Quality.*

1. INTRODUCTION

In the last decades, the importance of the quality has grown. It is possible to see quality as a list of proprieties which one product should have, like Garvin (1987), or Feigenbaum (1991) that understands that the quality is based only on determination and experience of consumers. However, what is interesting is the concept that makes the quality more operational like Taguchi (1924) describes. He presents the idea of quality as the loss which a product cause to the society after it be produced.

In the same line of thinking of Deming, who give emphasis in statistical methods, Taguchi invented new ways of measure, compare and improve the quality. Taguchi separates the quality control processes in two areas: "on-line", that is on production line where is applied the statistical control process techniques (SPC) and "off-line", out of production line, where there are the phases project design and parameter optimization. It is in off-line area who Genishi Taguchi proposed the quadratic loss function and the signal-to-noise ratio.

Nowadays, there are plenty companies who provide the same product in the marketplace, that differ only by price and quality service. Thus, the competitive between companies is very high. For that companies who desire stand out and gain advantage among similar companies is necessary a huge investment in quality of its products.

At this scenario, the development of an app for a specific objective turns to be advantageous. A good app makes your aims achieved in the most precise and fast way, which is, turning your company more competitive in the most efficient way.

In this pretext, the app "*Experimento Taguchi*" was developed to provide a Taguchi's analysis. The app has the function to provide an easy and fast way to apply the methodology, aiming its uses for students, professionals and researchers.

Apply the Taguchi's techniques and allying with technology of the app developed, it may analyses the characteristics of the products to supply a consistent analysis of the current quality in process of production. With these data, it is possible that managers of production can take decisions for improvement of quality desire.

2. THEORETICAL ASPECTS

2.1 Quadratic Loss Function of Quality

In the classic conception, we seek produce products that are inside the specifications. For Taguchi's sight, with the loss function, the search for improvement only ends when achieved the perfection, that is, a centered process and variability equals zero. The loss function implies a philosophy of continuous quality improvement.

The Taguchi's quality loss function, Fig. 1, represents the loss generated when a quality characteristic has a deviation from the target. This loss is occasioned for production's cost and damage suffered by the consumers (repairs, loss of business and others) (Phadke, 1989).

The loss function is given by:

$$L(y) = k(y - T)^2, \tag{1}$$

where L is the loss in monetary units when the observation it's at y, T is the target and k is a constant to be defined in the future for different cases.

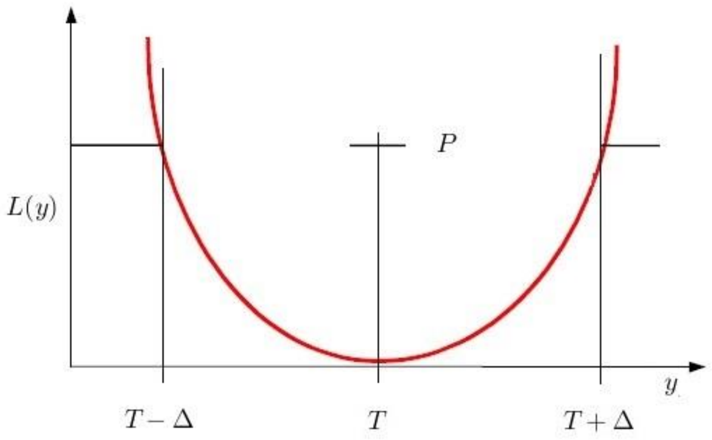

Figure 1. Taguchi's Loss Function

With this formulation, the loss function is calculated for each observation, that is, for each product individually. In the most of cases, it have many observations, so the loss function goes by the average quality loss function and it is calculated by mean square deviation (MSD) from the sample.

$$\bar{L}(y) = k(MSD) \tag{2}$$

There are three types of loss functions, according with the type of functional characteristic analyzed. They are:

i) Nominal-the-Better (NTB)

$$\bar{L}(\mathrm{y}) = \frac{P}{\Delta^2}[S^2 + (\bar{y} - T)^2]; \tag{3}$$

ii) Smaller-the-Better (STB)

$$\bar{L}(y) = \frac{P}{\Delta^2}(S^2 + \bar{y}^2); \tag{4}$$

iii) Larger-the-Better (LBT)

$$\bar{L}(y) = P\Delta^2\left(\frac{1}{y^2}\right). \tag{5}$$

Where P represents the maximum loss value, Δ the tolerance of targets' deviation.

The function have some interesting aspects, it exposes the quality and cost at the same time, allowing a better comprehension of the importance of process enhancement, such as it deals the economic aspects and engineering (Creveling and Fowlkes, 1995).

2.2 Signal-to-Noise Ratio

According Taguchi:
"The signal-to-noise ratio (S/N) is a measurement scale that has been used in the communication industry for nearly a century. A radio measures the signal or the wave of voice transmitted from a broadcasting station and converts the wave into sound. The larger the voice sent, the larger the voice received. […] Actually, the input is mixed with the audible noise in the space. […] Taguchi has generalized the concept of S/N ratio as used in the communication industry and applied it for the evaluation of measurement systems as well as for the function of products and processes." (Taguchi, 1924).

The S/N is a proceeding utilized to enhance the robustness of a product or process. It reflex the response variability of the system made by the noises, doesn't depends of adjustments on the mean. It is utilized to compare the proposes and doesn't accounts the interactions between the analyzed factors. The S/N have the functional characteristics:

i) Nominal-the-Better Type I (NTB-I)

$$S/N_{NTB-I} = 10\,log\left(\frac{\bar{y}^2}{S^2}\right); \tag{6}$$

ii) Nominal-the-Better Type II (NTB-II)

$$S/N_{NTB-II} = -10\,log(S^2\,); \tag{7}$$

iii) Smaller-the-Better (STB)

$$S/N_{STB} = \;-10\,log(S^2 + \bar{y}^2); \tag{8}$$

iv) Larger-the-Better (LTB)

$$S/N_{LTB} = -10\,log\left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{y_i^2}\right)\right]. \tag{9}$$

2.3 Bootstrap

The bootstrap is a computationally intensive non-parametrical statistical technique that permits the evaluation of statistics variabilities based on data from one single existing sample. Efron introduced this technique in 1979 and since then it has been part of theoretical and applied statistical studies. The bootstrap surged when Efron studied the estimative of sample distribution of a statistic $S(x)$ based on sample data of n size, from a unknown distribution (Chaves Neto, 1991).

The bootstrap is indicated for problems that standard statistical methods doesn't exist or when is difficult its application. The purpose of this method is to obtain the standard deviation, confidence interval, estimative of probability distribution and statistic calculus through the computational use.

Operationally, the bootstrap consists of NBS resampling with replacement of the original sample $x = [x_1, x_2, \dots, x_n]$ of size n. The resamples (x_i^*) are obtained with the same size n. Then, we calculate the statistics of interest $S(x_i^*)$ for each bootstrap resample, and the set of bootstrap values of the statistic corresponds to a true sample distribution's estimative of the statistic in question. The Figure 2 represents the bootstrap procedure to obtain NBS statistic of interest.

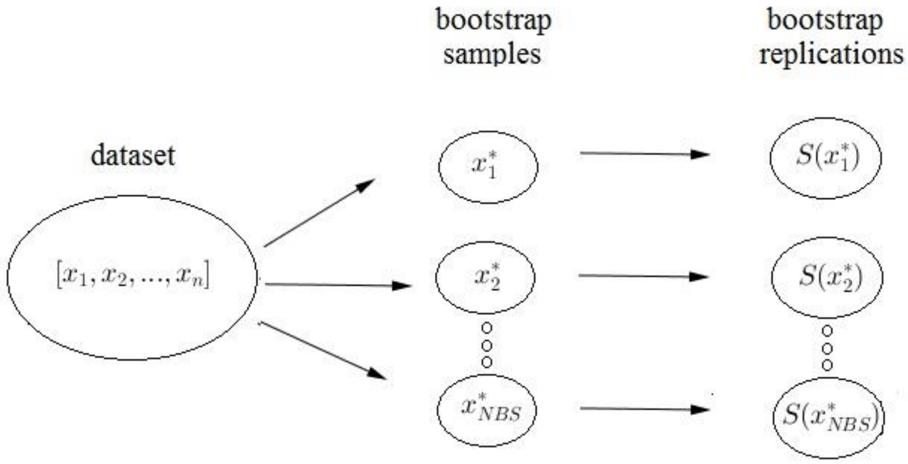

Figure 2. Methodology resample Bootstrap, adapted (Efron and Tibshirani, 1993)

The bootstrap confidence intervals present on this work are (Marques, 2015):

i) Standard Confidence Interval (Z)

$$IC_{boot\ Z} = [S(x) \pm Z_\alpha SE_{boot}], \tag{10}$$

where Z_α is the o $\alpha - th$ value from the score of standard normal distribution and $SE_{boot}$the standard deviation bootstrap.

ii) Confidence Interval t

The efficacy of this method to confidence interval calculus is reliable since the statistical distribution of interest being approximately normal and small addictions (Efron and Tibshirani, 1993).

$$IC_{boot\ t} = [S(x) \pm t_{df} SE_{boot}], \tag{11}$$

being n the master sample size, $df = (n-1)$ degrees of freedom.

iii) Percentile Confidence Interval (p)

$$IC_{boot\ percentil} = \left[S(x^*) - P_{97,5\%} diferenças;\ S(x^*) - P_{2,5\%} diferenças\right], \tag{12}$$

where $diferenças$ is the difference of the average bootstrap estimatives with each bootstrap estimative and $S(x_i^*)$ is the bootstrap estimative itself.

iv) Bias Corrected Confidence Interval (BC)

The estimative method for the BC confidence interval utilizes as interval the extremes the percentiles of adjusted bootstrap distribution, to correct the addiction and the asymmetry of the distribution in question.

$$IC_{BC} = \left[P_{P_i}\big(S(x^*)\big), P_{P_s}\big(S(x^*)\big)\right], \tag{13}$$

where P_i and P_s are the selected percentiles calculated by the correlation parameters from the addiction and $Z_{\alpha/2}$.

v) Bias Corrected and Accelerated Confidence Interval (BCa)

Another method to estimate the confidence interval is denominated BCa, being convenient its use when the asymmetry is present in a very strong way.

$$IC_{BCa} = [P_{P_i}(S(x^*)), P_{P_s}(S(x^*))], \tag{14}$$

the interval is similar to the BC method, the difference is an adjustment on the acceleration constant a on the calculus of the percentiles P_i and P_s (Andrews and Buchinsky, 2002).

3. APP

The development of the app "*Experimento Taguchi*" was realized on the period from January to July of 2017, in Windows environment, using the programming language MATLAB, as it permits the creation of graphic interfaces and is currently one of most powerful computational systems of math and engineering.

The app utilizes the classic concepts of statistics and the new concepts presented by Taguchi. The aim was the construction of a simple interface to the user, without unnecessary information noticeable. All the information is concentrated in just one place in a not confusing way for data analysis, making all the process as simple as possible.

The main window of the app, Fig. 3, have sites to select the number of factors, the levels, the degree of interactions between the level of factors, the entry or importation of data (.xls archives), the number of bootstrap resamples, nominal value, as well as buttons for calculus of analysis of variance (ANOVA), graphic options, characteristics selection of the process ("Opções" button) and exportation of the results. The main window also have individual tab to exhibit data and result of each step made. They are: "Matriz Experimento" (experimental design), "Amostra" (sample), "ANOM" (analysis of mean), "ANOM-SN" (analysis of mean for signal-to-noise ratio), "ANOVA", "ANOVA-SN" (analysis of variance for signal-to-noise ratio) and "Geral" (general).

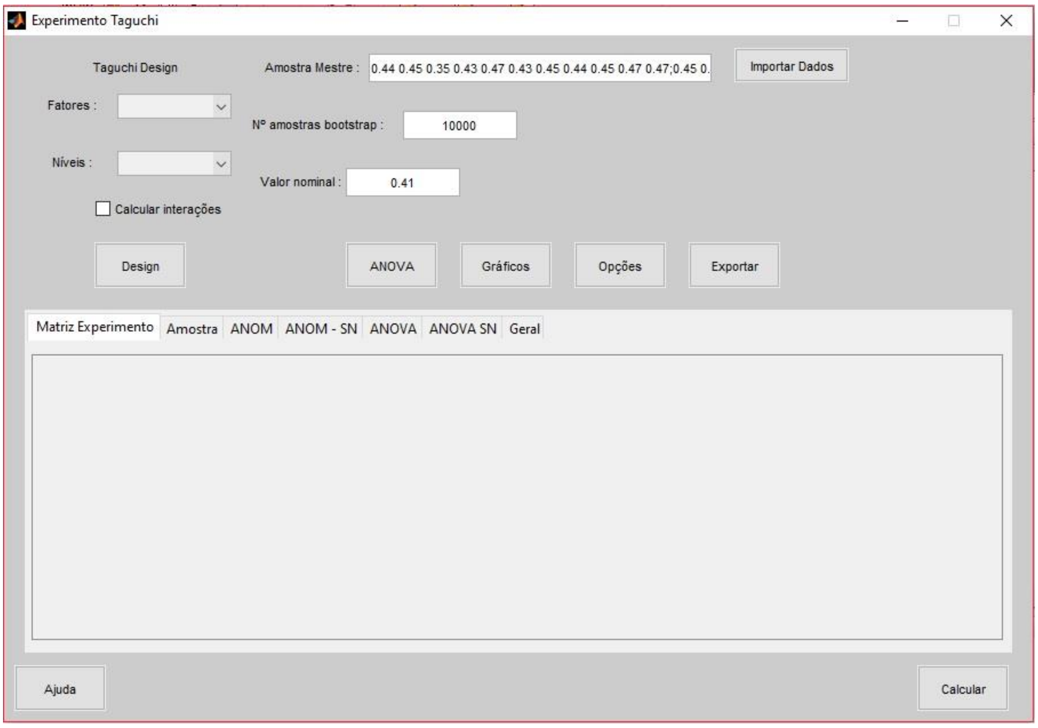

Figure 3. Main window

In the option selection window, Fig. 4, is presented the choices of process characteristics (NTB-I, NTB-II, STB, LTB), that will be applied to all analysis. As optional, there is the average loss function (that is mandatory the entry of proportionality constant k, or loss parameters P and target deviation Δ), process capability index Cp, Cpk, Cpm (that need the entry of upper specification limit USL and lower specification limit LSL). The confidence intervals of parameters are calculated from bootstrap samples generated, with the confidence levels 95% and 99%, with the choices: confidence interval Z, confidence interval t, percentile confidence interval, BC confidence interval, BCa confidence interval and automatic mode, which the program choose the best option for each situation. The confidence intervals will be calculated only for those parameters that were selected.

The result exhibition is shown in the last tab of the main window, denominated "Geral", Fig. 5.

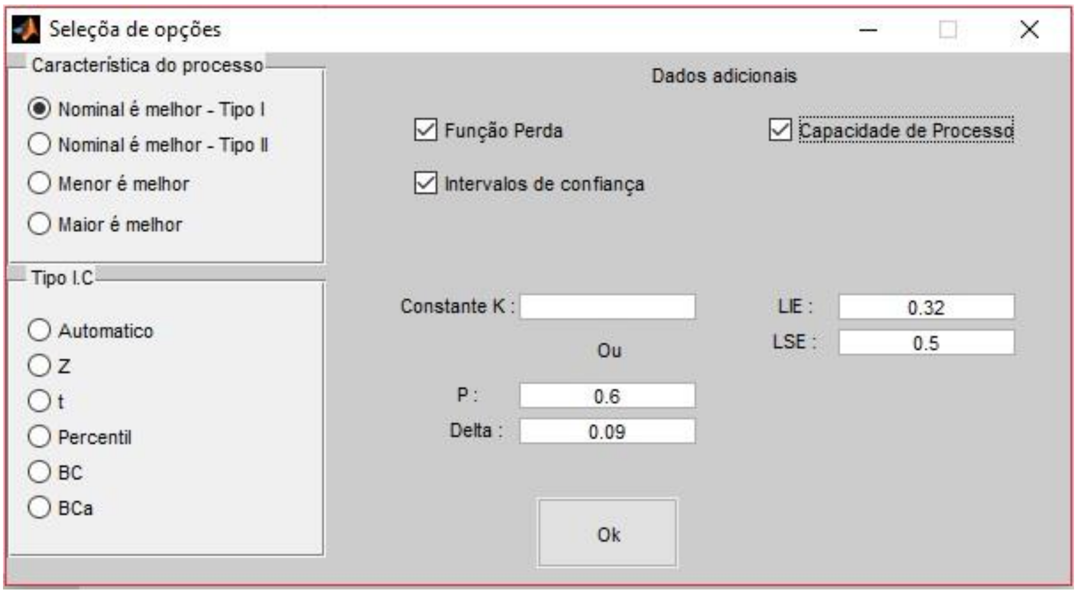

Figure 4. Option window for analysis

The "Geral" tab presents: the orthogonal array selected together with the attribution of factors and interactions, mean of the sample, variance of the sample, S/N ratio, confidence intervals level 95% of S/N ratio lower bound (SN-LI95%) and upper bound (SN-LS95%), confidence interval level 99% of S/N ratio lower bound (SN-LI99%) and upper bound (SN-LS99%). Also exhibited average loss function, confidence intervals level 95% of average loss function lower bound (FP-LI95%) and upper bound (FP-LS95%), confidence intervals level 99% of average loss function lower bound (FP-LI99%) and upper bound (FP-LS99%). As well as Cp, confidence intervals level 95% of Cp index lower bound (Cp-LI95%) and upper bound (Cp-LS95%), confidence intervals level 99% of Cp index lower bound (Cp-LI99%) and upper bound (Cp-LS99%), Cpk, confidence intervals level 95% of Cpk index lower bound (Cpk-LI95%) and upper bound (Cpk-LS95%), confidence intervals level 99% of Cpk index lower bound (Cpk-LI99%) and upper bound (Cpk-LS99%), Cpm, confidence intervals level 95% of Cpm index lower bound (Cpm-LI95%) and upper bound (Cpm-LS95%) and confidence intervals level 99% of Cpm index lower bound (Cpm-LI99%) and upper bound (Cpm-LS99%).

	Fator 1	Fator 2	e	Média	Variância	S/N	SN-LI95%	SN-LS95%	SN-LI99%	SN-LS99%
1	1	1	1	115	3.1667	36.2079	33.3241	44.3631	31.5303	44.3631
2	1	2	2	113	1	41.0616	36.2465	44.3242	36.2465	45.6278
	Função Perda	FP-LI95%	FP-LS95%	FP-LI99%	FP-LS99%	Cp	Cp-LI95%	Cp-LS95%	Cp-LI99%	Cp-LS99%
1	0.7917	0.4808	1.3809	0.3846	1.5385	3.7463	2.9532	5.2874	2.7931	7.0293
2	1.2500	0.7885	1.7308	0.6538	1.9158	6.6667	5.2662	10.2494	5.1247	14.5896
	Cpk	Cpk-LI95%	Cpk-LS95%	Cpk-LI99%	Cpk-LS99%	Cpm	Cpm -LI95%	Cpm -LS95%	Cpm -LI99%	Cpm -LS99%
1	3.7463	2.7180	4.8050	2.7115	5.1640	3.7463	2.9814	5.5635	2.8886	7.6285
2	6	4.5233	7.5580	4.5233	8.1680	2.9814	2.8165	3.1699	2.7912	3.2374

Figure 5. Example of results display

4. MATERIAL AND METHODS

For this work, it was analyzed data from food industry. They were five same type of products (product A, product B, product C, product D and product E) that differ only in their flavors. The observations refer to acid levels, in which every flavor have its specification. Each product is produced in three different work shifts: work shift 1 (6 A.M. to 12 P.M.), work shift 2 (1 to 9 P.M.) and work shift 3 (11 P.M. to 5 A.M.). The measurements were made each 2 hours from production line.

In that particular industry, 64% of the times the product was retained for not fit to the specifications, after reanalysis was totally released. The other 36% have some type of discard of the produced material, being it partial or complete.

As the cost for production doesn't evolve only the raw material and packing costs, the analyze and improve the production can reduce another expenses such as: time spend on reanalysis, consumer complaints and others. Clearly, the improvement of production bring economic gains not only for the better utilization of the raw material, consequently a better product, but it also brings the competitively for the brand.

The analysis was realized with the app "*Experimento Taguchi*" described in the previous section, with the S/N nominal-the-better type I, average loss function and 10.000 bootstrap resamples to generate the confidence intervals for the analyzed parameters.

5. RESULT AND CONCLUSION

The Table 1 shows the results of S/N for each work shift and product. The product A on work shift 1 has the best performance (higher S/N) in comparison to all the products at every work shift. The work shift 1 has the majority of best process executions (for products A, C and D), soon after the second work shift (products B and E). The third work shift does not showed a better execution for any product.

Table 1. Signal-to-noise ratio for the products.

S/N	Product A	Product B	Product C	Product D	Product E
1 Shift	32.116	18.870	26.676	26.752	22.359
2 Shift	27.974	28.213	22.723	24.670	30.112
3 Shift	28.609	23.816	23.727	23.349	26.688

Consequently, to the performance of their processes, the Tab. 2 show the results of the average loss function. Work shift 1 has the higher expenses for products B and E, work shift 2 for the product C and work shift 3 for the products A and D.

Table 2. Average loss function for the products.

Loss Function	Product A	Product B	Product C	Product D	Product E
1 Shift	0.2931	8.164	0.657	0.546	1.544
2 Shift	2.0806	0.435	1.072	0.864	1.214
3 Shift	2.9586	1.799	0.885	1.162	0.781

It was also made the calculus of confidence intervals for all the parameters before mentioned, that are presented on the Tab. 3 and Tab. 4.

Table 3. Confidence intervals for the S/N.

S/N	Confidence interval Z 99%									
	Product A		Product B		Product C		Product D		Product E	
	LB	UB	LB	UB	LB	UB	LB	UB	LB	UB
1 Shift	-	-	10.884	26.856	20.614	32.738	7.322	46.181	0.652	44.067
2 Shift	-17.074	73.022	15.945	40.482	15.413	30.033	10.732	38.609	17.696	42.529
3 Shift	2.663	54.554	13.785	33.846	17.050	30.404	10.468	36.231	18.910	34.449

Table 4. Confidence intervals for the average loss function.

Loss Function	Confidence interval t 99%									
	Product A		Product B		Product C		Product D		Product E	
	LB	UB	LB	UB	LB	UB	LB	UB	LB	UB
1 Shift	0	0.618	0	16.784	0.313	1.002	0.107	0.986	0.638	2.449
2 Shift	0	5.662	0	3.313	0.284	1.861	0.039	1.690	0.296	2.132
3 Shift	0	8.715	0	7.903	0.134	1.635	0	2.623	0	1.581

In a general way, the work shift 2 presented a better average performance for signal-to-noise ratio and lower loss. The work shift 1 presented the worst performance (S/N) and higher loss.

After this first analysis, it was recommended a deeper evaluation of all the production processes and the optimization of the parameters and process.

The utilization of the app "*Experimento Taguchi*" facilitates all the calculus presented in this article, thus for future analysis is recommended its use.

6. ACKNOWLEDGEMENTS

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