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LARGE SCALE EFFECTS IN THE TURBULENT KINETIC ENERGY BUDGET OF CHANNEL FLOWS

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Abstract. *Turbulent channels flows are investigated using numerical simulations. Incompressible, highly-resolved, large eddy simulations, LES, are performed to investigate straight channels. LES results are obtained using a semi-spectral formulation for low to moderate Reynolds numbers, $Re_\tau = 172$ and 617, based on the friction velocity at the channel inlet. The aim of this work is to analyze the turbulent kinetic energy, TKE,balances in wave-space for channel flows. The current study allows the investigation of the effects of large turbulent scales in the viscous wall region.*

Keywords: *Large scales, turbulence, boundary layer*

1. INTRODUCTION

Turbulent flows have been studied for decades and the development in this area remains extremely active. Wall-bounded turbulent flows with pressure gradients are found in several configurations of scientific and technological interest, for example, flows past wings and turbine blades, and convergent-divergent channels. In some cases, the presence of flow separation may occur adding complexity to the physical phenomena involved. The correct prediction of separation and re-attachment points in engineering problems is of paramount importance for understanding the loss of aerodynamic efficiency in wings, high-lift devices and gas and wind turbines. Moreover, it also has fundamental implications in pressure drag increase and noise generation.

In industrial applications, the solution of the Reynolds-averaged Navier-Stokes, RANS, equations remains the main flow simulation methodology due to the lower computational cost compared to higher-fidelity approaches such as LES and DNS. However, turbulence closure remains a problem far from being solved in a more universal way. Despite the increase in computational power, DNS and wall resolved LES are still non-viable for the simulation of flows with realistic Reynolds numbers typical of aeronautical and mechanical engineering applications. Second and higher order moment RANS closures have been developed due to the need to accurately predict complex flowsLaunder and Shima (1989); Kurbatskii and Poroseva (1999). These higher order closures have shown advantages and are potentially more universal than the lower order ones, but they require more equations to be solved Jeyapaul *et al.* (2015).

The studies presented in Refs. Jesus *et al.* (2014, 2015) indicate that two-equation eddy-viscosity models tend to either under-predict or over-predict flow separation. Reynolds stress transport models show improved results, especially with regard to the extension of the separated flow region. The Reynolds stress and the turbulent kinetic energy equations bring important information for scientists to improve the problem of turbulence closure. Kim *et al.* Kim *et al.* (1987); Mansour *et al.* (1988) pioneered the work with DNS that presented turbulent kinetic energy budgets as a function of the wall distance. Currently, such studies are presented for a wide range of Reynolds numbers Jiménez and Hoyas (2008); Lee and Moser (2015a) for straight channels flows.

In the present work, wall-resolved LES of incompressible turbulent flows are presented for wall-bounded turbulent flows. Simulations are performed for channels for $Re_\tau = 172$ and 617 based on the inlet friction velocity. An assessment of the spectral turbulent kinetic energy (TKE) budgets is performed for both Reynolds numbers analyzed in order to characterize how the processes are distributed in wavenumber-space. POD is also applied to reconstruct the TKE budgets with the most energetic modes.

Table 1. Details of the domain dimensions and mesh configurations for the current LES. The values of wall units correspond to the maximum value at the lower wall. L_x, L_y and L_z, and n_x, n_y and n_z correspond to the channel length and number of points in the x, y and z directions, respectively. N_f is the number of files used to compute the statistics, T is the time for which the statistics are collected, after discarding initial transients, and U is the mean velocity in the inlet centerline.

Re_τ	L_x/h	L_y/h	L_z/h	$n_x \times n_y \times n_z$	Δx^+	Δz^+	Δy^+	N_f	TU/L_x
172	4π	2	3π	$512 \times 97 \times 256$	4	6	0.1	401	15.9
617	4π	2	π	$512 \times 129 \times 256$	15	8	0.2	900	3.5

2. NUMERICAL MODEL

Straight channel flows with periodic boundary condition in spanwise and streamwise directions are investigated for Re_τ =172 and 617. In the current work, the simulation at $Re_\tau = 172$ considers a channel with $2h$ in height, $4\pi h$ in length, and $3\pi h$ in the spanwise direction, where h is the channel half height. For the simulations performed at $Re_\tau = 617$, the channel has dimensions of $2h$ in height, $4\pi h$ in length, and πh in the span. Details about the domain and the mesh configuration can be found at Tab. 2. It can be seen that both simulations have adequate values in terms of wall units, which indicates that a wall resolved LES is employed (Choi and Moin, 2012). The simulations are performed using the WALE subgrid model (Nicoud and Ducros, 1999).

The solver uses 8th and 4th order finite difference schemes in the streamwise direction, for the first and secound derivatives, respectively. Chebyshev polynomials are applied in the normal wall direction and a Fourier transform is applied in the periodic spanwise direction. The time integration is accomplished by a 2nd order implicit backward Euler for the viscous terms and a 2nd order Adams-Bashforth for the other terms. More details about the numerical formulation can be found in Marquillie *et al.* (2008)

3. RESULTS

The turbulent kinetic energy transport equation for incompressible flows can be written as

$$\frac{\partial k}{\partial t} = C + P + T + D + D_\rho - \epsilon \, . \tag{1}$$

This equation indicates the balance of convection, production, turbulent transport, viscous diffusion, pressure diffusion and pseudo-dissipation for turbulent kinetic energy. Clearly, for a statistically stationary flow, the time derivative must be zero. The terms in the right-hand side are defined as

$$A = -\langle u_j \rangle \frac{\partial k}{\partial x_j} \, , \; P = -\langle u_i^{'} u_j^{'} \rangle \frac{\partial \langle u_i \rangle}{\partial x_j} \, , \; T = -\frac{1}{2} \frac{\partial \langle u_i^{'} u_i^{'} u_j^{'} \rangle}{\partial x_j} \, ,$$
$$D = \nu \frac{\partial^2 k}{\partial x_j \partial x_j} \, , \; D_\rho = -\frac{1}{\rho} \frac{\partial \langle u_j^{'} p^{'} \rangle}{\partial x_i} \, , \; \epsilon = \nu \left\langle \frac{\partial u_i^{'}}{\partial x_j} \frac{\partial u_i^{'}}{\partial x_j} \right\rangle \, . \tag{2}$$

where $\langle \, . \, \rangle$ means a Reynolds average and $(\, . \,)^{'}$ means a fluctuation. Figure 1 shows a comparison between the current results for a straight channel validated against DNS data at Re_τ180 and 590 from Moser *et al.* (1999). It can be observed that the current simulations have a good agreement with literature, except by the dissipation term. The difference in the dissipation is expected since we perform a wall resolved LES and the budget is computed using filtered variables. The standard processes occurring in the budget are observed as expected for a wall bounded flow. Viscous effects predominate along the wall region and production and transport terms are more relevant in the buffer layer region.

Since our model in the physical space is validated, a similar analysis can be performed in the Fourier space. Considering the spanwise direction as homogeneous, a novel spectral turbulent kinetic energy equation as function of the spanwise wave number can be written as

$$\frac{\partial \widehat{k}}{\partial t} = \widetilde{C} + \widetilde{P} + \widetilde{T} + \widetilde{D_\nu} + \widetilde{D_p} - \widetilde{\epsilon} \, . \tag{3}$$

The equation above presents a balance between spectral convection, production, turbulent transport, viscous diffusion, pressure diffusion and pseudo-dissipation for turbulent kinetic energy in wavespace, where the turbulent kinetic energy is defined as

$$\widehat{k} = \frac{\left\langle \widehat{u_i{'}} \widehat{u_i{'}}^* \right\rangle}{2} \, . \tag{4}$$

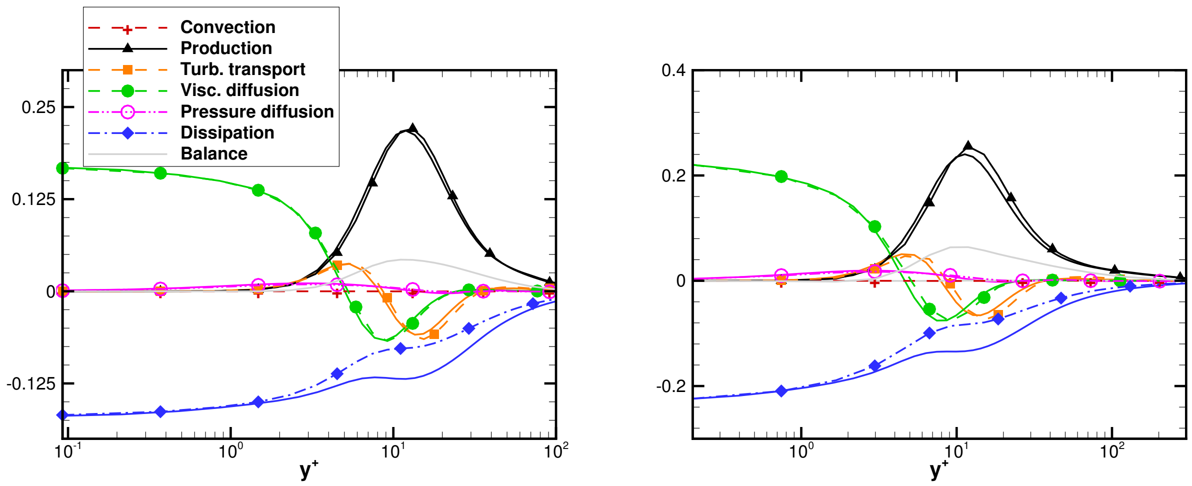

Figure 1. TKE budget for a straight channel normalized by u_τ^4/ν. The comparisons are presented between the current LES data (lines with symbols) for $Re_\tau = 170$ and 615 and DNS data (solid lines without symbols) from Moser *et al.* (1999) at $Re_\tau 180$ and 590. The budgets for the lower Reynolds number flows are shown in the left figure while those obtained for the higher Reynolds numbers are presented in the right figure.

The separate terms of the budget are defined as

$$\text{Convection} \qquad \widetilde{C} = -\langle u_1\rangle \frac{\partial \widehat{k}}{\partial x_1} - \langle u_2\rangle \frac{\partial \widehat{k}}{\partial x_2} \, , \tag{5}$$

$$\text{Production} \qquad \widetilde{P} = -\Re\left(\left\langle \widehat{u_i'}^{*}\, \widehat{u_j'}\right\rangle\right) \frac{\partial \langle u_i\rangle}{\partial x_j} \, , \tag{6}$$

$$\text{Turb. transport} \qquad \widetilde{T} = -\Re\left\langle \widehat{u_i'}^{*} \frac{\partial \widehat{u_i' u_1'}}{\partial x_1} + \widehat{u_i'}^{*} \frac{\partial \widehat{u_i' u_2'}}{\partial x_2} \right\rangle - \Im\left\langle i\kappa \widehat{u_i'}^{*}\, \widehat{u_i' u_3'}\right\rangle \, , \tag{7}$$

$$\text{Visc. diffusion} \qquad \widetilde{D_\nu} = \nu\left(\frac{\partial^2 \widehat{k}}{\partial x_1^2} + \frac{\partial^2 \widehat{k}}{\partial x_2^2}\right) \, , \tag{8}$$

$$\text{Pressure diffusion} \qquad \widetilde{D_p} = -\frac{1}{\rho}\Re\left(\frac{\partial\left\langle \widehat{p'}\,\widehat{u_1'}^{*}\right\rangle}{\partial x_1} + \frac{\partial\left\langle \widehat{p'}\,\widehat{u_2'}^{*}\right\rangle}{\partial x_2}\right) \, , \tag{9}$$

$$\text{Pseudo-dissipation} \qquad \widetilde{\epsilon} = \nu\left(\left\langle \frac{\partial \widehat{u_i'}}{\partial x_1}\frac{\partial \widehat{u_i'}^{*}}{\partial x_1}\right\rangle + \left\langle \frac{\partial \widehat{u_i'}}{\partial x_2}\frac{\partial \widehat{u_i'}^{*}}{\partial x_2}\right\rangle + 2\kappa^2 \widehat{k}\right) \, . \tag{10}$$

In the previous equation, $\widehat{(.)}$ represents a Fourier transform in the spanwise direction. A similar study was recently performed by Mizuno (2016) considering two homogeneous directions. One can observe that the distributions of the spectral components along λ_z^+ and y^+ are similar for both Reynolds numbers when they are normalized by the viscous scale. The peak of the spectral production occurs at $\lambda_z^+ \approx 100$, in the buffer layer region and, also along this region, the spectral transport has a large negative peak. Hence, the turbulent kinetic energy is transported from the production region in the buffer layer to the near-wall region, where it is dissipated. It is interesting to notice that the negative values of the transport term appear away from the wall, similarly to Fig. 2, and they are related to smaller spanwise scales. On the other hand, the transport term is positive closer to the wall and it is associated with larger spanwise scales.

The viscous diffusion and pseudo-dissipation occur in the viscous sub-layer and they are concentrated in scales within approximately 100 viscous lengths. These results agree with the behavior previously observed in the POD reconstructions and they are a strong indication of the lack of separation of scales in wall bounded flows, where production and dissipation are present in the larger, most energetic, turbulent scales (Schiavo *et al.*, 2016).

These results differ from the classical turbulence theory which states that production and dissipation predominate at different wavenumbers, the former at the lower wavenumbers and the latter at the higher wavenumbers. This behavior may be explained by the fact that the Reynolds numbers simulated are not high enough and, hence, dissipation is found in a close range of wavelengths as that found for the production term. Another explanation could be that the effects of turbulence anisotropy are changing the distribution of the turbulent processes along the length scales. It is important to mention that the behavior observed in the present results for the turbulent transport term was also recently seen by Mizuno

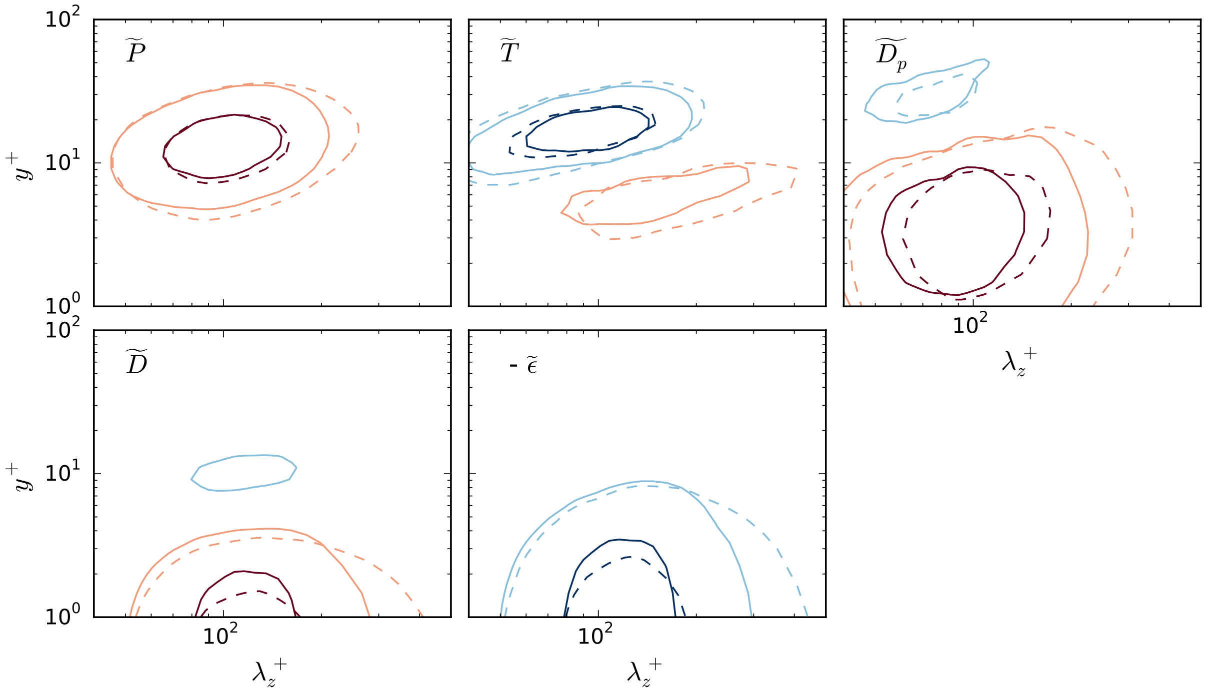

Figure 2. Components of the spectral turbulent kinetic energy budget for the straight channel flow. Results are presented for $Re_\tau = 170$ in solid lines and $Re_\tau = 615$ in dashed lines. Dark colors represent 0.7 of the maximum value of the component and light colors represent 0.3 of the maximum value. Positive values are shown in red and negative values in blue.

(2016) and Lee and Moser (2015b) for a higher Reynolds number straight channel flow.

4. CONCLUDING REMARKS

This work presents a study of the turbulent kinetic energy budgets for wall-bounded flows. Wall-resolved large eddy simulations (LES) are employed and the methodology is validated for straight channel flows at $Re_\tau = 170$ and 615. The TKE budgets in the physical space show a good agreement when compared with DNS data. The spectral formulation is employed to characterize the turbulent processes as a function of the wavelength in the channel spanwise direction. Results obtained by this formulation show that viscous processes, such as diffusion and pseudo-dissipation, occur for the same wavelengths where turbulence is produced. Therefore, a portion of the turbulent kinetic energy is transported to the near-wall region, being dissipated by large scale motion. These results indicate different trends when compared to the classical isotropic turbulence theory in which production and dissipation are related to low and high wavenumbers, respectively. We believe that the turbulence anisotropy is responsible for the changes in the distribution of the turbulent processes along the length scales at the range of Reynolds number analysed.

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