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TWO-PHASE FLOW STABILITY ANALYSIS USING A LEVEL SET APPROACH

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Abstract. *Prediction methods for flow pattern transition within pipelines in industrial applications still relies on the famous one-dimensional two-phase model, which in turn owes its success to being calibrated with empirical correlations. Mathematical and physical models capable of doing such predictions from first principles exist only for two-dimensional channels. Extending such models for real three-dimensional cases remains as a challenge to the scientific community. Using a variable-properties, single-fluid formulation together with the level set method, as opposed to the traditional one which considers two separated fluid phases is a promising approach to accomplish a rigorous stability analysis for three-dimensional flows. This work presents the theoretical derivation and computational implementation of this methodology, and compares its predictions with those from the classic approach for a simple, two-dimensional channel flow. Linearization of the Navier-Stokes equations, interface condition terms and level set advection additional equation, and substitution of the modal instability Ansatz, leads to an eigenvalue problem (EVP). Arnoldi algorithm was applied to efficiently solve the EVP from which modal analysis was performed.*

Keywords: *flow, two-phase, stability, pattern, transition*

1. INTRODUCTION

Since the middle of the 20th century, many methods were developed attempting to predict multiphase flow mechanics. Today, industry still relies on one-dimensional models which consider uniform velocities for both phases and empirical friction factor correlations to model and compute multiphase flows within pipes. However, the stability analyses based on such concepts, as introduced by Taitel and Dukler (1977) and Barnea and Taitel (1993), that are employed for the transition of the phase pattern from stratified to intermittent slug regimes, in spite of their high level of maturity, still result in important inaccuracies. On the other hand, first-principles analysis exists since Yih (1967), that do not impose approximations to the spatial structure of the underlying flow.

Relatively recently, Boomkamp et al. (1997) performed modal analysis and South and Hooper (1999) performed a non-modal (transient growth) analysis of simple two-dimensional flows. Both approaches require numerical solutions and currently are solely applicable to two-dimensional channels. A promising approach for extending stability analysis to a real three-dimensional geometry is a variable-properties, single-fluid formulation together with the level set approach (Sussman et al. (1994)) instead of the traditional formulation considering the two phases independently and interfacial conditions. This research presents the derivation of the governing equations, and comprehensive comparisons with the results of the classic methodology (South and Hooper 1999), in order to validate the approach.

2. GOVERNING EQUATIONS AND SINGLE FLUID LEVEL SET APPROACH

An incompressible flow is governed by the Navier-Stokes equations:

$$\nabla . v = 0 \tag{1}$$

$$\rho \frac{\partial v}{\partial t} + (\rho v.\nabla)v = \rho g - \nabla p + \nabla.(2\mu D) \tag{2}$$

where ρ is specific mass, v is the velocity vector, t is time, g is the gravity acceleration vector, p is pressure, μ is dynamic viscosity and D is the stress tensor. The single fluid level set approach for modeling two co-current immiscible phases as presented by Susman et al. (1994) and Gada and Sharma (2009) consists of adding a level set function ϕ which represents a vertical position relative to interface ($\phi > 0$ in the upper phase, $\phi = 0$ at the interface and $\phi < 0$ in the bottom phase). A convection equation for this scalar is also introduced in the system of equations:

$$\frac{\partial \phi}{\partial t} + (v.\nabla)\phi = 0 \tag{3}$$

Then, a volumetric force term is added to account for the interfacial tensions, with σ representing the interfacial tension coefficient, k the local curvature of the interface, $\delta(j$) Dirac's delta function and n a unit vector normal to the interface:

$$\sigma k \delta n \tag{4}$$

$$n = \frac{\nabla \phi}{|\nabla \phi|} \tag{5}$$

$$k = \nabla.n \tag{6}$$

Finally, the variation of properties between phases is properly emulated by defining ρ and μ with a Heaveside function:

$$\rho = \rho_1 H(\phi) + \rho_2(1 - H(\phi)) \tag{7}$$
$$\mu = \mu_1 H(\phi) + \mu_2(1 - H(\phi)) \tag{8}$$

2.1. Dimensionless form

The density and viscosity ratios and the dimensionless parameters are defined with respect to the upper phase (index 1) as follows:

$$\chi = \frac{\rho_2}{\rho_1} \quad \text{and} \quad \eta = \frac{\mu_2}{\mu_1} \tag{9}$$

$$\mathrm{Re} = \frac{\rho_1 U_1 L}{\mu_1}, \quad We = \frac{\rho_1 {U_1}^2 L}{\sigma} \quad \text{and} \quad Fr = \frac{U_1}{\sqrt{gL}} \tag{10}$$

From this point, let us redefine the single fluid properties assuming they are converted to their dimensionless form:

$$\rho = H(\phi) + \chi(1 - H(\phi)) \tag{11}$$
$$\mu = H(\phi) + \eta(1 - H(\phi)) \tag{12}$$

The momentum conservation in Eq. (1) can also be rewritten in dimensionless form including the interface volumetric force term:

$$\rho \frac{\partial v}{\partial t} + (\rho v.\nabla)v + \nabla p = \frac{1}{\mathrm{Re}} \nabla.(2\mu D) + \frac{1}{We} k\delta n + \frac{\rho}{Fr^2} g \tag{13}$$

2.2. Smoothed functions

For numerical reasons, the interface is smoothed accordingly Gada and Sharma (2009). This leads to the definition of smoothed Heaveside and Dirac's delta functions:

$$H_\varepsilon(\phi) := \begin{cases} 0 & \phi < -\varepsilon \\ \dfrac{\phi+\varepsilon}{2\varepsilon} + \dfrac{1}{2\pi}\sin\left(\dfrac{\pi\phi}{\varepsilon}\right) & |\phi| < \varepsilon \\ 1 & \phi > \varepsilon \end{cases} \tag{14}$$

$$\delta_\varepsilon(\phi) := \begin{cases} \dfrac{1}{2\varepsilon} + \dfrac{1}{2\varepsilon}\cos\left(\dfrac{\pi\phi}{\varepsilon}\right) & |\phi| < \varepsilon \\ 0 & otherwise \end{cases} \tag{15}$$

The thickness of the interface is 2ε and it should be comparable to the grid spacing.

3. DERIVATION OF THE STABILITY EQUATIONS FOR THE LEVEL SET APPROACH

3.1. Cartesian coordinate system

A Cartesian coordinate system is defined as follows: x is the axial or longitudinal direction and y is the transversal direction in the gravity force plane. Similarly, u and v are the velocity components on the x and y directions, and i and j the respective unitary vectors.

3.2. Linearization

Let $q=[u,v,p,\phi]^T$ be a vector containing the variables of interest. The flowfield quantities are decomposed into a time-invariant base flow, which is also assumed to be homogeneous along the streamwise direction due to the parallel flow assumption, and small amplitude fluctuations:

$$q(x,y,t) = \bar{q}(y) + \varepsilon q'(x,y,t) \qquad \text{with} \qquad \varepsilon << 1. \tag{16}$$

Substitution of (16) in the governing equations and linearization about the base flow based on the smallness of the disturbances, results in the system of partial-derivative-based coupled equations:

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial x} = 0 \tag{17}$$

$$\bar{\rho}\frac{\partial u'}{\partial t} + \bar{\rho}\bar{u}\frac{\partial u'}{\partial x} + \bar{\rho}\frac{\partial \bar{u}}{\partial y}v' + \frac{\partial p'}{\partial x} = \frac{\bar{\mu}}{\mathrm{Re}}\left(\frac{\partial^2 u'}{\partial x^2} + \frac{\partial^2 u'}{\partial y^2}\right) + \frac{1}{\mathrm{Re}}\frac{\partial \bar{u}}{\partial y}\frac{\partial u'}{\partial y} + \frac{1-\eta}{\mathrm{Re}}\delta\left(\frac{\partial \bar{u}}{\partial y}\frac{\partial \phi'}{\partial y} + \frac{\partial^2 \bar{u}}{\partial y^2}\phi'\right) \tag{18}$$

$$\bar{\rho}\frac{\partial v'}{\partial t} + \bar{\rho}\bar{u}\frac{\partial v'}{\partial x} + \frac{\partial p'}{\partial y} = \frac{\bar{\mu}}{\mathrm{Re}}\left(\frac{\partial^2 v'}{\partial x^2} + \frac{\partial^2 v'}{\partial y^2}\right) + \frac{1}{\mathrm{Re}}\frac{\partial \bar{u}}{\partial y}\frac{\partial v'}{\partial y} + \frac{\delta}{We}\frac{\partial^2 \phi'}{\partial x^2} \tag{19}$$

$$\frac{\partial \phi'}{\partial t} + \bar{u}\frac{\partial \phi'}{\partial x} + v' = 0 \tag{20}$$

3.3. Matrix form of the temporal EVP

Modal perturbations are introduced following the homogeneity of the base flow in time and the streamwise direction of the form

$$q' \approx \hat{q}(y)\exp[i(\alpha x - \omega t)] \tag{21}$$

which transforms the equations (17-20) into the generalized matrix eigenvalue problem

$$\omega L\hat{q} = R\hat{q} \tag{22}$$

The EVP must be complemented with adequate boundary conditions. No-slip and compatibility conditions result into

$$\hat{u} = \hat{v} = [D_y]\hat{p} = [D_y]\hat{\phi} = 0 \tag{23}$$

A temporal instability framework is considered here, in which a real wavenumber α is prescribed and complex eigenvalues $\omega = \omega_r + i\omega_i$ are obtained as solution. The real part ω_r is a circular frequency while the imaginary part ω_i corresponds to a temporal growth rate. If the eigenvalue imaginary part is negative for all modes, then any linear perturbation will decay on its amplitude and it will return to base flow, which is said to be stable. However, if the eigenvalue imaginary part is positive for at least one mode, the perturbation amplitude will grow to a level which non-linear effects becomes relevant. In this case, the base flow is unstable and a transition to a different pattern will occur. Associated to each eigenvalue there is an eigenfunction describing the spatial structure of the fluctuations in the velocity, pressure and level set fields $\hat{q} = [\hat{u}, \hat{v}, \hat{p}, \hat{\phi}]^T$.

4. NUMERICAL SOLUTION AND ANALYSIS

The eigenvalue problem is discretized using 4th-order finite differences, then solved using a shift-and-invert implementation of the Arnodi algorithm. This numerical approach is more efficient than using the QZ algorithm as in Boomkamp et al. (1997) or South and Hooper (1999), which allows us to perform calculations with a very large resolution in few seconds. In the level set approach, large resolutions are required in the vicinity of the interface in order to converge results.

5. RESULTS

A gravity-stratified channel flow is considered here for the sake of the validation of the approach. Two immiscible phases flow co-currently due to a pressure gradient, the heavier one occupying the lower part of the channel. Laminar flow is considered for both phases, in order to ease the determination of adequate base flows. Under these circumstances, an analytical solution exists for the steady velocity field, that can be found in many other sources (e.g. Rodríguez 2017).

This section presents comparisons between the eigenvalue spectra computed for the same base flow and parameters, using the classic methodology based on two separated phases plus interface conditions, and the one proposed herein based on the level set approach. Implementation of the classic method is first validated with results by South and Hooper (1999), showing agreement as exemplified in figure 1.

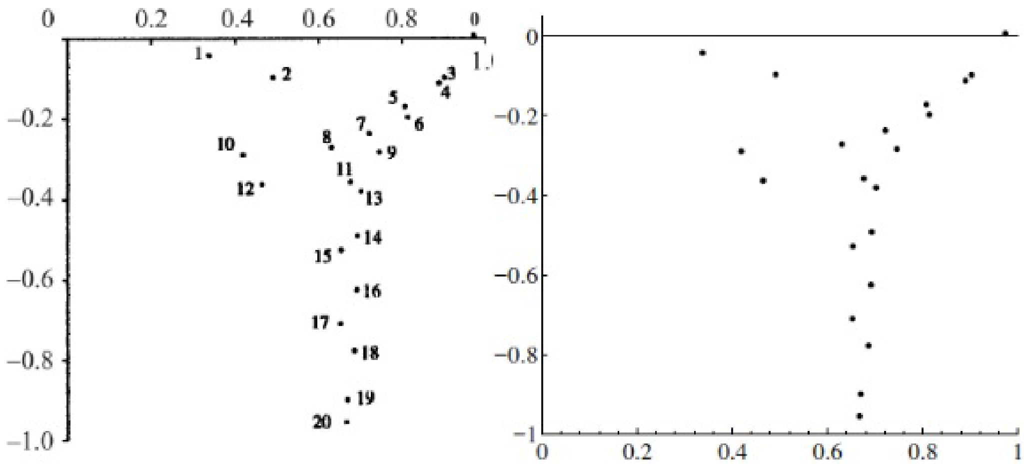

Figure 1. Eigenvalue spectrum (complex vs. real part). South and Hooper (1999) traditional two fluid modal stability analysis (left) and implemented routine (right). Reynolds number of 3000, wave number of 1, same specific mass for each phase and interface at half channel.

The following, results of the validated separate-phases approach are compared to those of the stability analysis based on the level set approach. Different density ratios and viscosity ratios are considered for the parameters $\mathrm{Re} = 5000$, $\alpha = 1$ and $h = 0.5$.

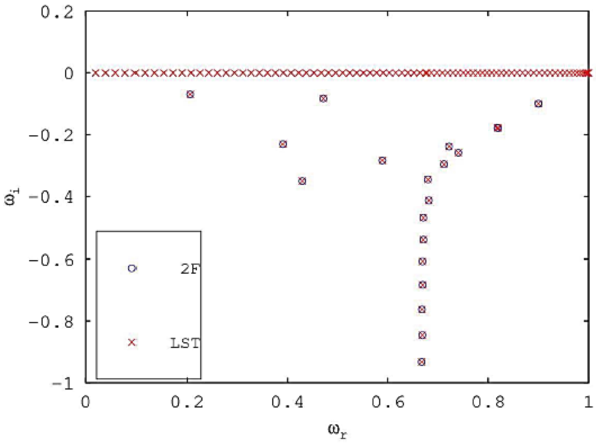

Figure 2. Eigenvalue spectrum (complex vs. real part). Two fluid (blue circles) and single fluid with level set (red crosses). Same specific mass and viscosity for each phase and interface at half channel.

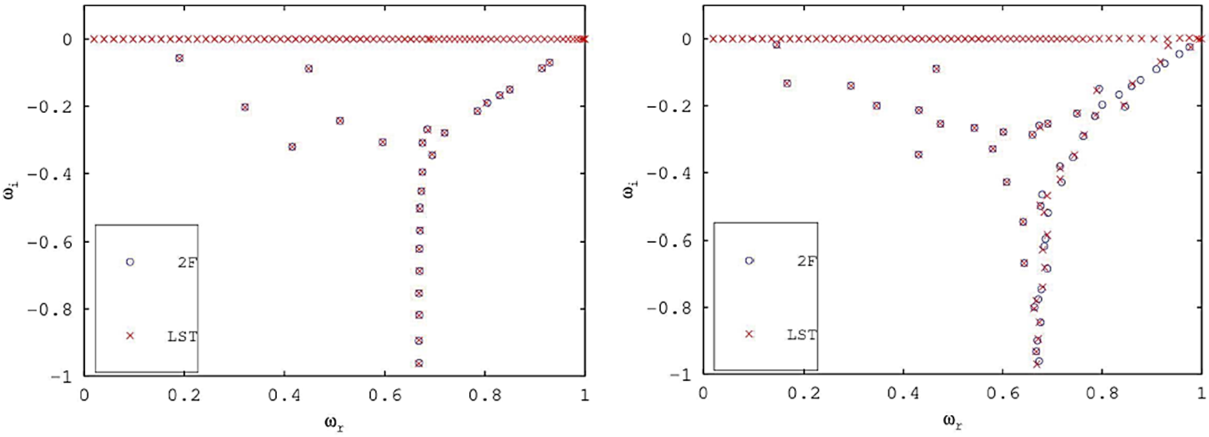

Figure 3. Eigenvalue spectrum (complex vs. real part). Two fluid (blue circle) and single fluid with level set (red crosses). Density and viscosity ratio of 2 and 1 respectively (left). Density and viscosity ratio of 10 and 1 respectively (right).

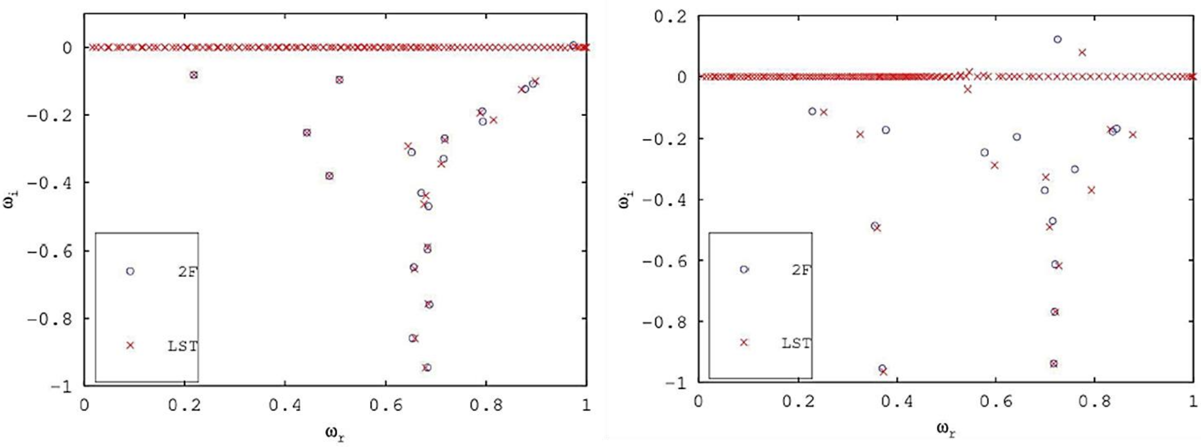

Figure 4. Eigenvalue spectrum (complex vs. real part). Two fluid (blue circle) and single fluid with level set (red x). Density and viscosity ratio of 1 and 2 respectively (left). Density and viscosity ratio of 1 and 10 respectively (right).

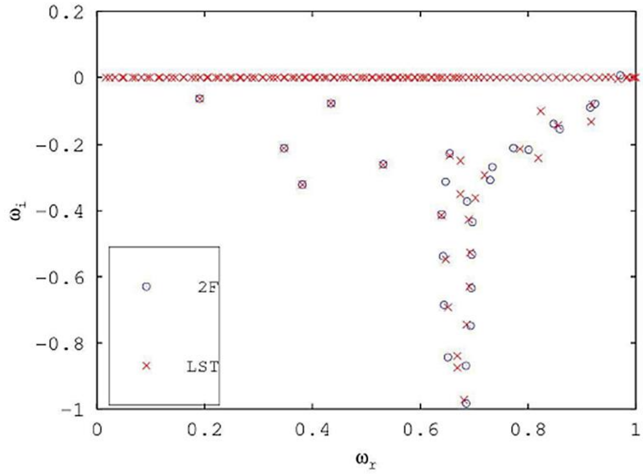

Figure 5. Eigenvalue spectrum (complex vs. real part). Two fluid (blue circle) and single fluid with level set (red crosses). Density and viscosity ratio of 1 and 0.5 respectively.

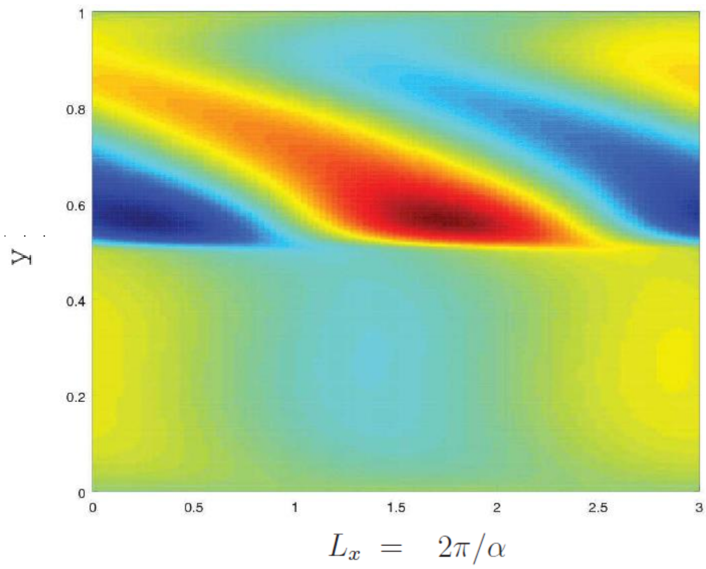

Figure 6. Perturbation velocity field for the unstable eigenvalue that appears for density and viscosity ratio of 1 and 10 respectively. The yellow level represents a null velocity, blue and red represent positive and negative magnitude respectively.

Preliminary analyses showed that the convection equation for the level set function generates a set of neutral instability solutions with spatial support localized in the interface. Also a harmonic mean was applied for the definition of viscosity, which improved the results as suggested by Patankar (1980). Figures 2 and 3 correspond to cases with viscosity ratio equal to one and density ratios equal to 1, 2 and 10, showing a very good agreement between the eigenvalues computed using the two methodologies. Density ratios below one are not considered, as the base flow with the heavier dense on top of the lighter one would be unrealistic.

Cases with density ratio equal to one and different viscosity ratios are shown in figures 4 and 5. Differences are visible in the eigenmode families when the viscosity ratio is varied. The Y-shaped eigenspectrum characteristic of the single-phase channel flow is separated in two Ys as the properties of the fluids vary. Two families of eigenmodes are related to the bulk velocity on each phase, and have constant phase speed. These eigenmodes are reasonably well recovered by the level set approach, but the agreement with the values of the classic approach is reduced as the viscosity ratio is varied. The spatial structure of these eigenmodes is concentrated in the interior of each phase. As we proceed in his branch towards more stable eigenmodes, the eigenfunctions decay faster towards the interface, and the effect of the viscosity ratio is reduced. In any case, these eigenmodes are always stable. Families with potentially unstable eigenmodes appear in a wider range of frequencies, and are related to fluctuations localized at the walls and at the interface. Larger differences are observed between calculations using the two methodologies for these eigenmodes.

Figure 6 presents the velocity field of the perturbation associated with the unstable eigenvalue of figure 4 (right eigenspectrum). As the viscosity ratio rises, the upper phase base flow grows in velocity compared to the lower phase. The importance of the pertubations at interface for the flow stability analysis, mainly for the less viscous phase (upper phase), becomes clear as shown in figure 6.

The results presented herein showed that a very good agreement between the classic separated-phases approach and the one proposed here, based on the level-set method, is achieved for immiscible fluids with different density ratios and comparable viscosity. When different viscosities are considered, the eigenmodes localized at the interface obtained by the two methodologies differ. The present results suggest this mismatch to be due to the treatment of the interface by the level set method. Separated-phases approach imposes directly the continuity and balance of tangential and normal stresses at the interface, which are proportional to the viscosity ratio. The level set method accounts for these terms indirectly, via the momentum conservation equations. An improved treatment of the viscosity and density gradients across the finite-width interface in the single-fluid approach is the object of current research.

The single-fluid formulation is a promising method for extending a rigorous stability analysis to real three-dimensional geometries, like multi-phase flow along pipes or ducts of arbitrary cross-sections. Once the developments and validations for simple canonical problems, like the one considered in this paper, are completed, it can be combined in a straightforward manner with multidimensional eigenvalue problems or parabolic stability equations.

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7. References

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