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ON NONLINEAR WIND TUNNEL AEROELASTIC TESTS AND APPLICATION OF 0-1 TEST FOR CHAOS

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Abstract. *Nonlinear aeroelastic phenomena are continuously investigated in aeronautical researches. The nonlinearity nature can be aerodynamic or structural. This work will investigate a very flexible wing with high aspect ratio subjected to unsteady flow. A flutter analysis is proceeded in order to evaluate the error between the computational results and the experiment. Since the linear flutter theory considers small disturbances, nonlinear phenomena are expected. The experiment time series shall be analyzed and the evaluation if the system presents chaotic behavior will be performed through the 0-1 test.*

Keywords: *flutter, 0-1 test, chaos, wind tunnel, aeroelasticity*

1. INTRODUCTION

This work shall present an application of 0-1 test for chaos in an aeroelasticity context. The present investigation scope is especially the experimental flutter analysis of high aspect ratio wing with wing tip slender body using subsonic wind tunnel. Nonlinear flutter tests were conducted using a wind tunnel at Aeronautics Institute of Technology (ITA). The intention was to obtain the acceleration time histories from a wing model subjected to a flutter test. To achieve this objective it is necessary to design a wing in which flutter occurs within the wind tunnel speed range. The flexible wing design is very important to provide models that can be used in further researches, like limit cycle oscillation or chaos.

Nonlinearities in aeroelastic systems might be from different natures, like aerodynamic and structural (Lee et.al., 1999). For instance, HALE (high altitude long endurance) aircraft is a current important topic of investigation. With the advance of materials and manufacturing process evolution, it became possible to produce more optimized aircraft structures, lighter and with high aspect ratio. This improvement in aspect ratio is interesting in the aerodynamic point of view (it reduces the induced drag, which, consequently, reduces fuel consumption).

This system's nonlinearity is structural, since this is a high aspect ratio aluminum flat plate model, as presented in Fig. 1:

Figure 1: Wind tunnel model (Westin, 2010)

Jinwu et.al. (2013) presented a nonlinear aeroelasticity review, which high aspect ratio wings are mentioned and presented different ways for structural and aerodynamic modeling and different software which those models are implemented. Lee et.al. (1999) presented a similar review, but also treating aeroservoelasticity in a nonlinear context. A 0-1 test for chaos (Gottwald and Melbourne, 2004) was performed to verify if this system has chaotic behavior. The 0-1 test is the most modern tool to investigate if a phenomenon has chaotic behavior from the time history of an experiment. Chaotic dynamics is not desirable, due to its high sensibility to initial conditions. It is possible to suppress the chaotic behavior through targeted energy transfer (Vakakis et.al., 2008), but this will not be treated in this paper.

2. THE AEROELASTIC DYNAMICAL SYSTEM

Flutter is an aerodynamic auto-excited phenomenon which occurs due to the coupling of two or more different vibration modes. This coupling results from the interaction between aerodynamic, elastic and inertial forces. Since flutter is usually a catastrophic phenomenon, it must be avoided for the entire aircraft flight envelope (Bisplinghoff, 1955).

For some special cases, nonlinear phenomena might occur, such as limit-cycle oscillations due to control surface freeplay, storages or HALE (high altitude long endurance) aircrafts (Lee et.al., 1999). In those cases, even when the velocity is increased, the amplitude is the same. If the system presents non-periodic behavior, with high sensitivity to initial conditions, the system presents chaotic behavior (Nayfeh and Balachandran, 2004). As will be presented later, there is a test to verify this special case.

First, the flutter prediction methodology based on the g-method was calculated for the model to be tested in wind tunnel. In order to predict the flutter mechanism evolution, the software ZAERO® was used. The wing structural dynamic model are computed using the finite element method implemented in NASTRAN® solver (solution 103), in order to provide ZAERO® with a modal base. Subsequently, unsteady aerodynamic loading is computed through a lifting surface interference method known as ZONA 6, also implemented in ZAERO®. Then, ZAERO® software is employed to compute the aeroelastic model, which results in the Vgf (velocity-damping-frequency) curves.

These curves show each aeroelastic mode evolution and, when a damping curve crosses the x-axis, the flutter velocity is defined. The frequency evolution for each aeroelastic mode is important to visualize the coupling between two, or more, different structural modes. With this information one could avoid flutter, actively or passively.

The g-method formulation, which is implemented in ZAERO®, is demonstrated in Westin (2010). This method introduces a first order damping perturbation in the flutter equation. So, the equation of motion used to determine flutter velocity is given by (Westin, 2010):

$$\left[\left(\frac{V}{L}\right)^2 Mp^2 + \left(\frac{V}{L}\right) Bp + K\right]\{\xi(p)\} = q[Q(ik) + gQ'(ik)]\{\xi(p)\} \tag{1}$$

where, V is the undisturbed flow velocity, L is a reference length, M is the generalized mass matrix, B is the generalized damping matrix, K is the generalized stiffness matrix, $\xi(p)$ is an eigenvector with the generalized coordinates, $Q(p)$ is the generalized aerodynamic force matrix and its derivative related to k (reduced frequency) and p is defined as:

$$p = g + ik \tag{2}$$

where g is the damping perturbation introduced in the system. The first step is to substitute Eq. (2) into Eq. (1) and then put the equation of motion in state-space, like this:

$$[\tilde{A} - gI]\{\bar{\xi}\} = 0 \tag{3}$$

where:

$$\tilde{A} = \begin{bmatrix} 0 & I \\ -\bar{M}^{-1}\tilde{K} & -\bar{M}^{-1}\tilde{B} \end{bmatrix} \tag{4}$$

$$\bar{M} = \left(\frac{V}{L}\right)^2 M \tag{5}$$

$$\tilde{B} = 2ik\left(\frac{V}{L}\right)^2 M - \frac{1}{2}\rho V^2 Q'(ik) + \left(\frac{V}{L}\right) Z \tag{6}$$

$$\tilde{K} = -k^2\left(\frac{V}{L}\right)^2 M + K - \frac{1}{2}\rho V^2 Q(ik) + ik\left(\frac{V}{L}\right) Z \tag{7}$$

Now, it is necessary to find the roots of this equation of motion in state-space form. For the g-method this is achieved by varying the reduced frequency from zero to a determined maximum value.

The flutter frequency and associated damping is, then determined by:

$$\omega_f = k\left(\frac{V}{L}\right) \tag{8}$$

$$2\gamma = 2\frac{Re(g)}{k} \tag{9}$$

From Eq. (8) and Eq. (9) is possible to determine the Vgf curves. These curves contain both damping and frequency evolution for each aeroelastic mode. Also, it is possible to investigate which structural modes couple, that is, the flutter mechanism, and provide solutions to avoid it. All flutter investigation is possible due to these two curves.

3. 0-1 TEST THEORETICAL BACKGROUND

The system's dynamics reconstruction using Takens' theorem (Takens, 1981), which is the usual method to verify if a system presents chaotic behavior, can lead to some difficulties, especially in determining the embedding dimension and delay parameter, which are fundamental for a reliable result and are not trivial to determine. A poor choice of these parameters leads to wrong attractor, so a wrong dynamic (Abarbanel, 1996).

A new test for chaos, called 0-1 test, applies directly to the experimental data, so the phase space reconstruction is no longer necessary (Gottwald and Melbourne, 2004). Also, it can be applied to all dynamical systems, e.g., continuous or discrete, governed by ordinary or partial differential equations, experimental data, maps, etc (Falconer et.al., 2007). It has presented a great performance when dealing with noisy data and it leads to a binary conclusion (does or does not have chaotic behavior are the only two possible results) (Gottwald and Melbourne, 2005).

Consider $\phi(j)$ the observed experimental data, for $j = 1,2,\dots,N$. For $c \in (0,\pi)$, where c is chosen randomly, the translation variables are:

$$p_c(n) = \sum_{j=1}^{n} \phi(j)\cos jc \tag{10}$$

$$q_c(n) = \sum_{j=1}^{n} \phi(j)\sin jc \tag{11}$$

The p_c versus q_c plot gives if the system behavior is diffused, like a brownian movement (chaotic) or bounded (periodic or quasi-periodic). This behavior can be obtained by analyzing the mean square displacement. If it results in a bounded function in time, the dynamics is regular, but if it scale linearly with time, the dynamics is chaotic. The mean square displacement is given by (Gottwald and Melbourne, 2009).

$$M_c(n) = lim_{N\to\infty}\frac{1}{N}\sum_{j=1}^{N}\{[p_c(j+n) - p_c(j)]^2 + [q_c(j+n) - q_c(j)]^2\} \tag{12}$$

In practice $n_{cut} = N/10$. The periodic component of the mean square displacement is given by (Gottwald and Melbourne, 2009):

$$V_{osc}(c,n) = (E\phi)^2\frac{1-cos(nc)}{1-cos(c)} \tag{13}$$

where the expectation $E\phi$ is (Gottwald and Melbourne, 2009):

$$E\phi = lim_{N\to\infty}\frac{1}{N}\sum_{j=1}^{N}\phi(j) \tag{14}$$

For better convergence properties, the periodic component is subtracted from mean square displacement, since this result will present the same asymptotic growth as the mean square displacement (Gottwald and Melbourne, 2009):

$$D_c(n) = M_c(n) - V_{osc}(c,n) \tag{15}$$

After calculating the Eq. 15 up to n_{cut},the asymptotic growth rate is calculated using a correlation method (Gottwald and Melbourne, 2009):

$$K_c = corr(\xi,\Delta) = \frac{cov(\xi,\Delta)}{\sqrt{var(\xi)var(\Delta)}} \in [-1,1] \tag{16}$$

ξ is the vector from one up to n_{cut} and Δ is the vector formed by the results from Eq. 15. Calculations in Eq. 10 to Eq. 16 are repeated for a batch of random values of c in the interval $(0,\pi)$. Usually $N_c = 100$ will suffice. The final result is obtained from the median of the N_c results of Eq. 16 (Gottwald and Melbourne, 2009):

$$K = median(K_c) \tag{17}$$

Equation 17 will result in either zero or one. If the result is zero, the system has no chaotic behavior (might be periodic or quasi-periodic). If the result is one, the system presents chaotic behavior. It is important to observe that this test is not to determine if the system presents a stochastic dynamics. This test is applied to deterministic systems.

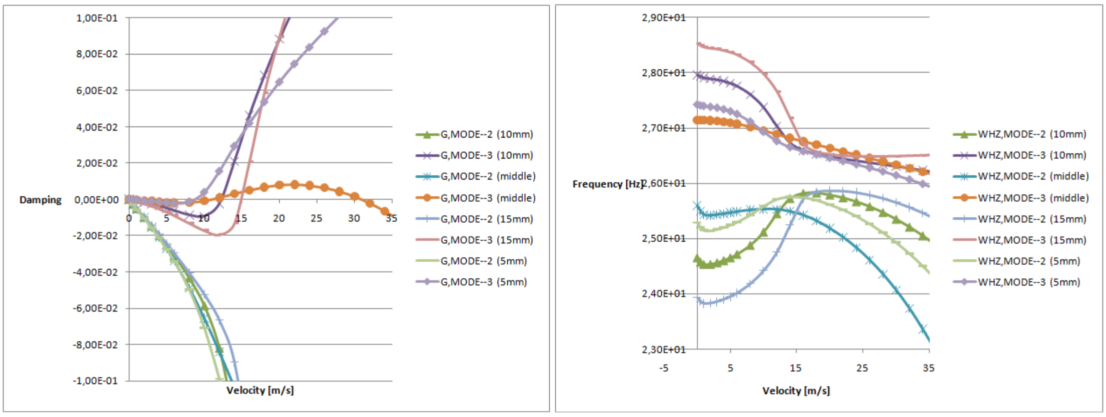
This is a very important test and also simple and fast to have a result. Its reliability was questioned once and the authors proved these questioning to be unfounded (Gottwald and Melbourne, 2008). They also presented mathematical results on the validity of the 0-1 test (Gottwald and Melbourne, 2009) and the test was applied for a bipolar motor which dynamics could be changed from periodic to chaotic (Falconer et.al., 2007).

The 0-1 test was performed in Matlab™ using the time history for each model modification (slender body center of gravity position).

4. DISCUSSION OF NUMERICAL FLUTTER RESULTS

The wing model is an aluminum flat plate with a brass slender body (see Figure 1). This construction allows the center of gravity (CG) to be changed, in order to study this effect in flutter velocity (Westin, 2010 and Westin et al., 2016).

The computational results, which were obtained from ZAERO® software using g-method, are presented below (Westin, 2010):



a) b)

Figure 2. a) Damping and b) Frequency versus velocity evolution curves

Figure 2 shows the frequency and damping evolution for each CG position. Since the structure is a flat plate, the elastic center is located at the center (middle chord line). The slender body CG position was varied from 5mm forward the middle until 15mm forward. The flutter velocities and frequencies are presented in Tab. 1 and compared with experimental results (Westin, 2010).

For the slender body CG located at the middle chord line, the computational result shows a hump mode (orange curve in Fig. 2a). However, this mode was not observed during the tests, since the structural damping usually shifts the velocity axis up by an amount given by ground vibration test (GVT), which was not conducted for these models. Then, these computational results should be more conservative when compared with the experimental results (Westin, 2010).

5. EXPERIMENTAL FLUTTER RESULTS

The model shown in Fig. 1 was tested in the wind tunnel located at Professor Kwei Lien Feng in Technological Institute of Aeronautics (ITA):

Figure 3. Wind tunnel used for the tests

This wind tunnel has square test section with 465mm each side, maximum velocity of 33m/s and power of 30HP. The model accelerometers scheme is shown bellow, as well as its support used in the tests (Westin, 2010):

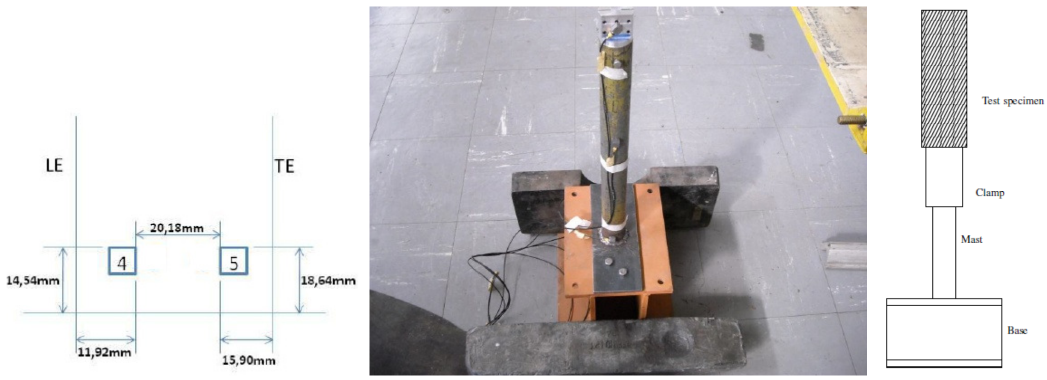


Figure 4. Accelerometers scheme (LE/TE - leading/trailing edges) and support

The support shown in Fig. 4 is rigid enough to simulate a clamp. The model was tested for 0º of angle of attack. The experiment procedure is as follows:

1) The model is clamped at the support shown in Fig. 4;
2) The wind tunnel is turned on and the velocity is slowly increased as the PSD is monitored on the laptop;
3) As the model starts to vibrate, the PSD (power spectral density) curve presents a sharp peak. This is the moment which flutter is observed. The PSD approach was employed as one way to identify flutter (Sheta et al., 2002). The output signal from an accelerometer, through its PSD computation, identifies the flutter onset condition and the corresponding undisturbed flow speed. The sharp peak represents the flow's energy that the model extract to flutter.
4) The slender body CG is changed and steps 2 and 3 are repeated.
5) The acceleration time history is recorded for 10 seconds for each experiment. These results are presented here (Fig. 5, 8 and 11). In order to obtain the velocity and displacement, shown in the same Figures, the acceleration result was numerically integrated once for velocity and twice for displacement.
6) The 0-1 test was performed for each acceleration time history.

The flutter did not diverge until the wing destruction for all cases. This means the system presents nonlinearities. The most probable reason is the presence of structural nonlinearities (Dowell et. al., 2003). The model is subject to high deformations during flutter and cannot be treated like a linear structure and both NASTRAN® and ZAERO® models, which were the software used for computational analysis, consider only the linear theory.

Table 1. Flutter results for each CG position

	CG at 5mm	
	Computational	Experimental
Velocity[m/s]	8.1	9.45
Frequency[Hz]	27.2	24.7
	CG at 10mm	
Velocity[m/s]	12.3	12.1
Frequency[Hz]	27.4	24.6
	CG at 15mm	
Velocity[m/s]	14.6	14.5
Frequency[Hz]	27.1	24.7

The flutter velocity and flutter frequency are considerably close from the theoretical results. From this result, it is possible to validate the aeroelastic theoretical model in a small disturbance context, i.e., the linear theory is valid until the flutter speed, in order to predict flutter frequency and velocity. From that point, nonlinear theory must be considered.

5.1 CG 5mm Forward Middle Chord Line

The acceleration time history and the integrated signal to obtain velocity and displacement are shown in Fig. 6 (Westin et al., 2016):

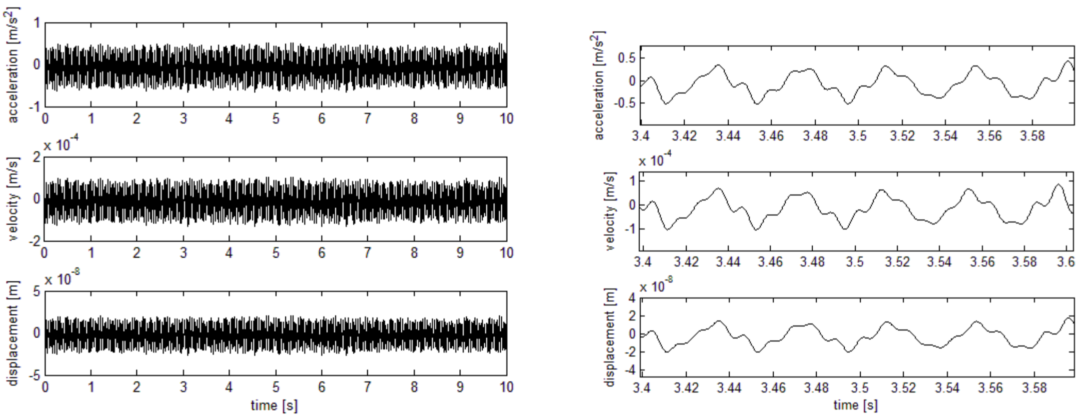


Figure 5. Time histories for CG at 5mm

The time history does not have a pattern and it is not possible to determine the movement amplitude or its frequency, for example. To get more information, the FFT and the PSD are important plots in signal analysis (Westin et al., 2016):

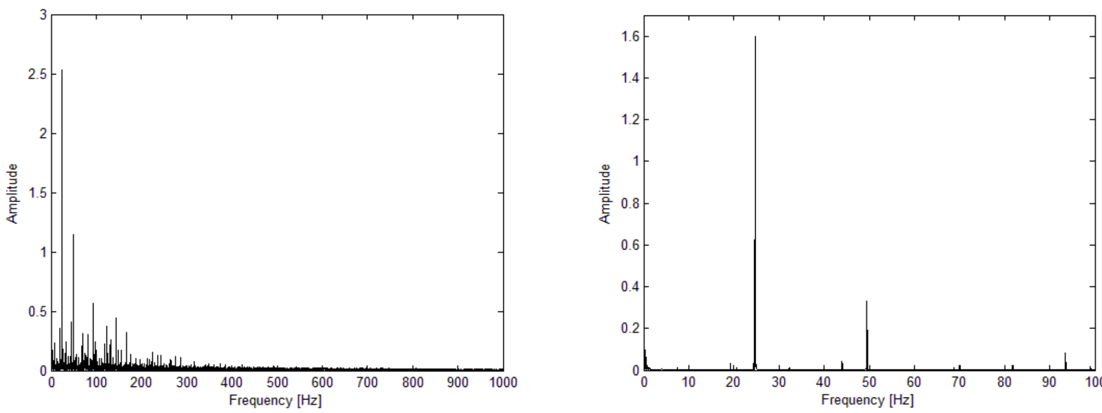


Figure 6. FFT (left) and PSD (right) for CG at 5mm

The FFT shows the presence of various peak of frequency, but the PSD shows, not only the flutter frequency determined computationally (24.7Hz), but also other frequencies. This might indicate chaos.

The 0-1 test result for this case is 0.9828, which indicates a chaotic dynamics. According to Gottwald and Melbourne (2009) and Bernardini and Litak (2016), if a system presents chaotic behavior, i.e. 0-1 test very close to one, then the phase portrait of the translation variables p and q, which were already defined in the section 3, will represent a brownian movement. However, if the system trajectory is bounded, the 0-1 test result might be very close from zero. This p versus q curve is also known as auxiliary trajectory (Bernardini and Litak, 2016). The auxiliary trajectory for this CG position is:

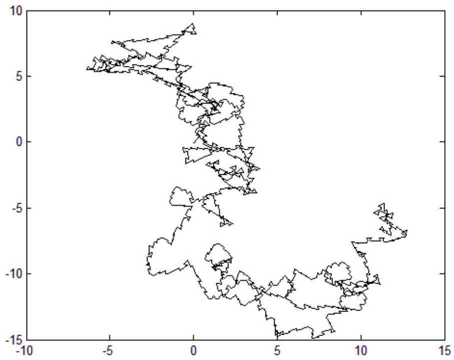


Figure 7. Auxiliary trajectory for CG at 5mm

Since the auxiliary trajectory represents a brownian movement and the 0-1 test result is very close to one, this system dynamics is chaotic.

5.2 CG 10mm Forward Middle Chord Line

The time histories when the CG is positioned at 10mm from middle chord line is shown in Fig. 9 (Westin et.al., 2016):

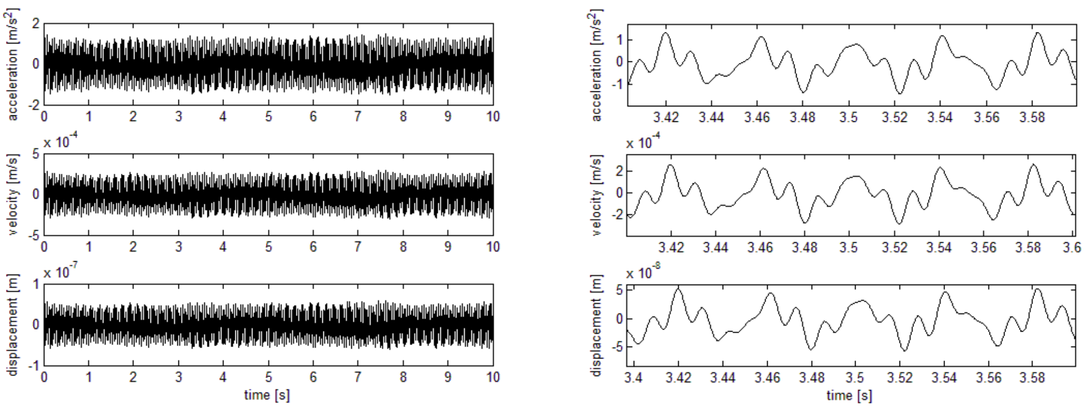


Figure 8. Time histories for CG at 10mm

Again, from time histories, it is not possible to observe a certain pattern of the data. The FFT and the PSD are presented next:

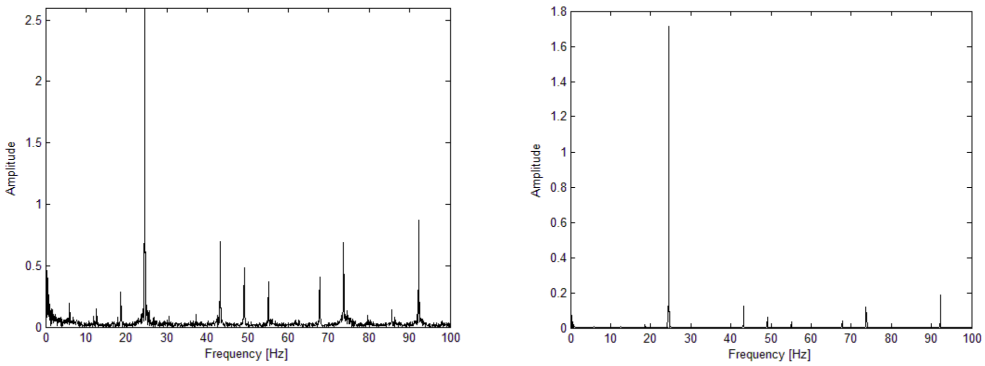


Figure 9. FFT (left) and PSD (right) for CG at 10mm

The flutter frequency (24.6Hz) is still there among many other frequency peaks, but it is not the only frequency peak as expected. But, by evaluating only these results, it is not possible to infer if the system has chaotic behavior.

The 0-1 yielded 0.9799, so, with all these information, it is possible to confirm that this system presents a chaotic behavior, which can be confirmed from the auxiliary trajectory plot.

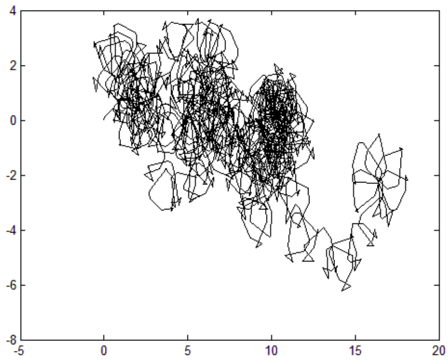


Figure 10. Auxiliary trajectory for CG at 10mm

5.3 CG 15mm Forward Middle Chord Line

Next, the CG is moved another 5mm and the accelerometer time history and the integrated signal for velocity and displacement are (Westin et.al., 2016):

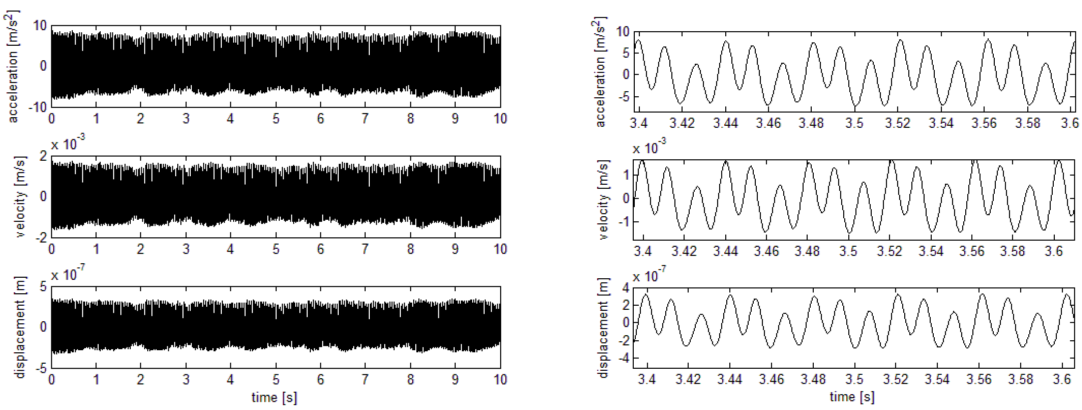


Figure 11. Time histories for CG at 15mm

From the time histories, one has a first clue that this system is periodic. Nevertheless, further analyses are necessary. The FFT and PSD are shown in the next Figure:

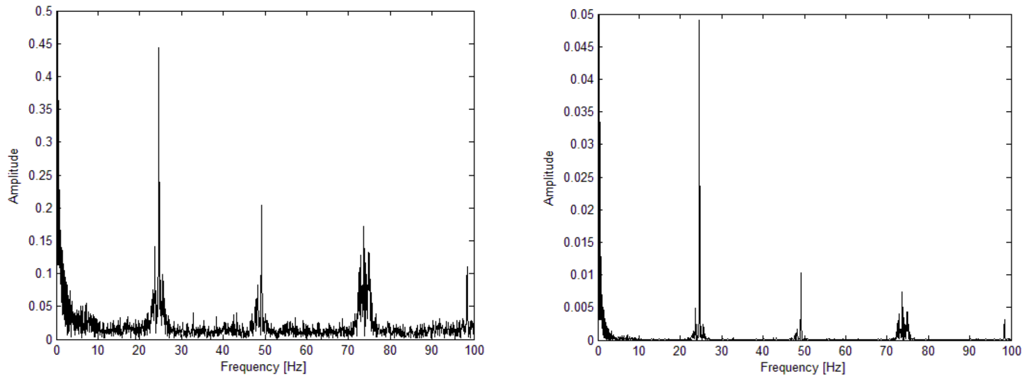


Figure 12. FFT (left) and PSD (right) for CG at 15mm

Both FFT and PSD shows, besides the flutter frequency, other peaks of frequency, but now they are periodic. All peaks are multiples of the first (flutter frequency). This is a nonlinearity related to model's flexibility (third order) and mode coupling (second order).

The 0-1 test for chaos result in -5.1993×10^{-4}. The 0-1 test result and other information given in this section indicates that this system, for this CG position, has periodic or quasi-periodic dynamics. The auxiliary trajectory shows a bounded movement (closed form, very similar to a circle):

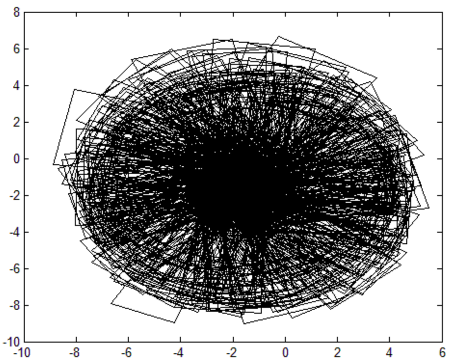


Figure 13. Auxiliary trajectory for CG at 15mm

6. CONCLUSIONS

The results presented here show the importance of the application of 0-1 test when a nonlinear system is investigated. Furthermore, all traditional signal analysis should be performed to find more evidences for chaos or other nonlinearities that might lead to periodic or quasi periodic results.

The 0-1 test is the most modern way to evaluate if the system has chaotic behavior. Initially, it is not necessary to reconstruct the system attractor (Takens, 1981) and then determine the Lyapunov exponent (Abarbanel, 1996). The 0-1 test is quicker, easier and very reliable for noisy data, as expected from any experimental result. Since the 0-1 test is easy to implement and has low computational cost (for 50000 data points it took a few seconds in a common laptop), it is recommended its application to all nonlinear aeroelastic tests. The 0-1 test was performed both for acceleration and velocity and no difference was observed, so it is recommended to use what was measured instead of the integrated signal. Also, one should always perform the oversampling test. The 0-1 test should not be performed for oversampling data in order to avoid mathematical issues.

For this wing model, the system dynamics for CG 5mm forward and CG 10mm forward were concluded as chaotic only using the 0-1 test. For CG 15mm forward the system dynamics is periodic or quasi-periodic. Also, it is important to build the bifurcation diagram, which might help to understand the system dynamics behavior with the variation of certain parameters.

As a future work, more data will be acquired for this model and a laser vibrometer will be used to determine the displacement at some points at the wing model instead of the wing root. Also, a carbon fiber model will be tested for different fiber orientations in order to perform this analysis.

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