24th ABCM International Congress of Mechanical Engineering
December 3-8, 2017, Curitiba, PR, Brazil

# COBEM-2017-0256
# REVISITING THE DUAL-SPIN SPACECRAFT DYNAMICS DURING PLATFORM SPINUP WITH LIMITED TORQUE SUPPLY

**J. M. Balthazar**
**E. Cereja**
**A. Nabarrete**
ITA - Aeronautics Technology Institute, 12228-900, São José dos Campos - SP, Brazil
jmbaltha@ita.br
cereja@ita.br
nabarret@ita.br

**E. Cereja**
INPE - National Institute for Space Research, 12227-010 - São José dos Campos - SP, Brazil

**A. M. Tusset**
**R. T. Rocha**
UTFPR – Federal University of Technology - Paraná, 84016-210, Ponta Grossa - PR, Brazil
a.m.tusset@gmail.com
digao.rocha@gmail.com

**J. L. P. Felix**
UFFS – Federal University of Fronteira Sul, 97900-000, Cerro Largo - RS, Brazil
jorge.felix@uffs.edu.br

**R. M. L. R. F. Brasil**
UFABC – Federal University of ABC, 09210-580, Santo André - SP, Brazil
reyolando.brasil@gmail.com

***Abstract.*** *This work comprises a review on the attitude dynamics of a gyrostat used as an idealized model for Dual Spin Satellites, as well as the analysis of some control strategies aimed at overcoming resonance traps occurring during satellite spinup, in particular considering maneuvers executed with limited power supply (non-ideal oscillations).*

***Keywords:*** *dual spin satellite, attitude, control, non-ideal oscillations.*

## 1. INTRODUCTION

A general Dual Spin Satellite (DSS) is a spacecraft composed of two main bodies, a platform (P) and a rotor (R), connected by common rotation axis (shaft). P and R are constrained to relative rotation about their common shaft, although the whole body is free to rotate in space.

A DSS is generally injected on orbit with zero relative angular velocity, in what is known as all-spun configuration. An attitude acquisition maneuver is then initiated in which an internal motor provides a torque that will spin up R and consequently spinning down P until it attains a near zero angular momentum. It is known that dynamical instabilities may generate attitude deviations in the despin phase. For example, a slender cylindrical DSS rotating about its minimum inertia axis may jump to rotation about the maximum inertia axis (*flat-spin*) when subjected to a limited-constant-torque maneuver. On the other hand, a despin maneuver with limited-variable-torque, in face of mass unbalances in either or both bodies, may give place to synchronization between the rotor's rotation and the spacecraft's precession around its angular momentum vector (*resonance-capture*). In any case, energy-spending attitude recovery maneuvers may be required to bring the DSS-attitude to the desired state. This is normally done either by determining approximate asymptotic or numerical solutions, which permits to identify and avoid those initial conditions that result in flat spin or captures, or by determining an adequate control strategy that produces a rotation regime onto the spacecraft that overcomes these traps, resulting in a pass-through rotational trajectory (Rand, *et al*., 1992; Haberman, *et al*., 1999; Kinsey, *et al*., 1995).

## 2. UNBALANCED FREE GYROSTATS AS DSS MODELS

Gyrostat models have been of academic and practical interest since Volterra (1899) used it to study the precession of Earth's equinoxes. Within the first applications to Earth's artificial satellites is the work of Masaitis (1961) analyzing the angular velocities of an axial gyrostat, a configuration in which the rotor spins around the spacecraft's principal (shaft) axis. Later on DSS models have been applied in the study of the attitude of other artificial satellites, for instance a communication or observation satellites that have to maintain sensors or antenna pointing to a target region on space or on the Earth's surface.

An idealized DSS model will be considered, consisting of two rigid bodies P+R connected by a rigid shaft that will be assumed as the spacecraft's rotation axis (Fig. 1). The rotor will be assumed axis-symmetric, while the platform will be assumed to have an asymmetric (unbalanced) fixed mass distribution. The geometry depicted in Fig 1 refers to an *oblate gyrostat*, that is, one with inertia axes $i_1 > i_2 \geq i_3$. Note that **h** is the angular momentum vector of P+R, $h_1$ is the angular momentum vector of R about $\mathbf{e_1}$ (shaft) and $g_1$ is the rotor torque about $\mathbf{e_1}$. From the assumption of a spacecraft free of external constant torques, $\|\mathbf{h}\| = h_0$ is a first integral of the motion.

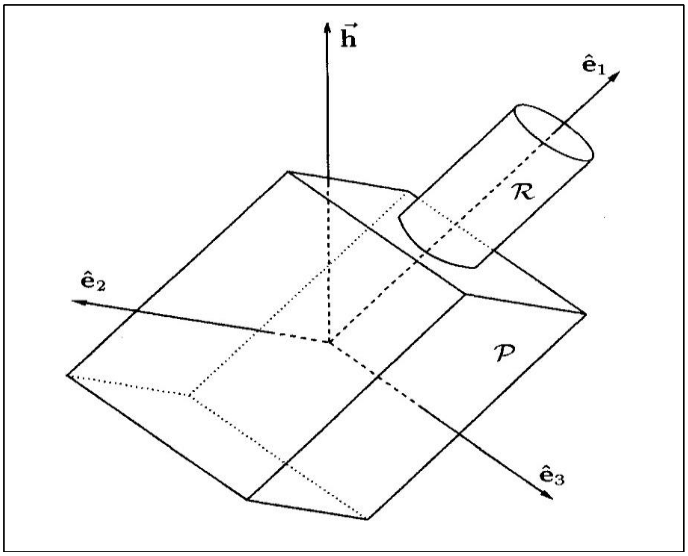


Figure 1 - Axial Gyrostat model (Hall, 1995).

The dimensionless equations of motion may be written as

$$\dot{x}_1 = (i_2 - i_3) x_2 x_3 \tag{1}$$

$$\dot{x}_2 = (i_3 x_1 - \mu) x_3 \tag{2}$$

$$\dot{x}_3 = -(i_2 x_1 - \mu) x_2 \tag{3}$$

$$\dot{\mu} = \varepsilon \tag{4}$$

$$x_1^2 + x_2^2 + x_3^2 = 1 \tag{5}$$

The standard *integrable* case, $\varepsilon = 0$, brings an extra integral of the motion and allows to describe the gyrostat's angular momentum dynamics as a one-dimensional trajectory depicted by the constant energy **x** vector over the momentum sphere *S* defined by Eq. (5). This problem has been analyzed analytically or numerically by several authors, as, for example, Cochran, *et al.*, 1982, Elipe, *et al.*, 2008 and Aslanov (2014).

Bifurcation diagrams show the existence of heteroclinic and homoclinic **x**-trajectories over *S*. Figure 2 shows some separatrices $S_\mu$ illustrating the shape of the stability regions over S as a function of μ.

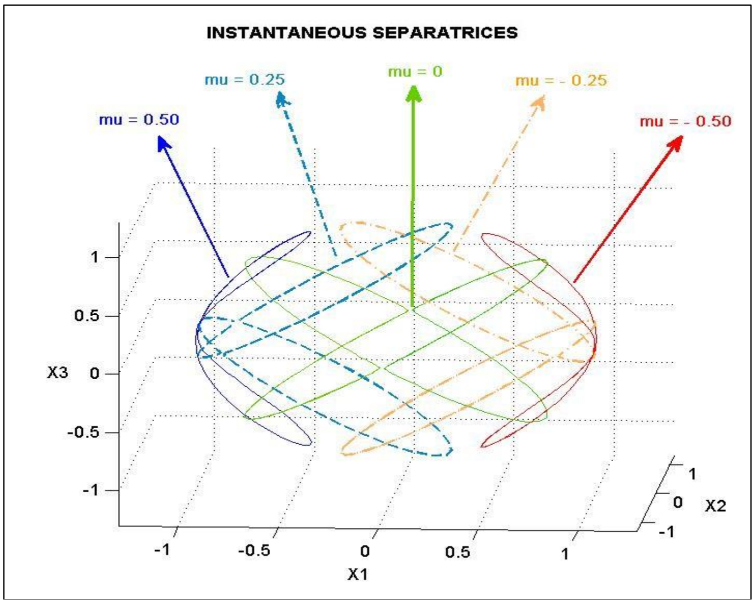


Figure 2 - Instantaneous separatrices ($S_\mu$)

When $\varepsilon \neq 0$, the rotor's rotational kinetic energy μ varies and the full spacecraft's rotational kinetic energy is given as

$$\dot{T} = \varepsilon\left(-x_1 + \mu / i_1\right) \tag{6}$$

As a result, the equilibrium points shown in Fig. 2 (except the points (±1, 0, 0), that are stable irrespective the μ value) start to drift over *S*, reshaping the instantaneous separatrices $S_\mu$ so that they drift as well, leading them to eventually be crossed by the instantaneous **x**-trajectory.

These separatrix-crossings are the ultimate cause of the spacecraft's attitude trapping mentioned earlier, as depicted in Fig. 3a-b. The trajectories shown in Fig.3a started at the respective initial **x**-positions, with initial rotatory energy $m_0$ = +0.25 and internal torque *e* = - 0.002, and evolved for 300 time units. At the same time, as the value of *m* changes according to Eq. (4) the corresponding instantaneous separatrix $S_m$ (see Fig. 2) floats over momentum sphere, eventually crossing the x trajectory, as happens with the rightmost and with the intermediate trajectories. Figure 3.b shows how differently may behave two **x**-trajectories whose starting points are near an instantaneous separatrix as they are trapped by different equilibria. One can see the trajectory starting at the left **x**-point initially crossing the neighboring separatrix at left, following multiple instantaneous middle separatrix-crossings, until a final crossing occurring at right. This sensitivity on initial conditions is a remarkable characteristic of chaotic dynamics, which comes up either as the effect of internal mass unbalances or from axis misalignments during attitude acquisition maneuvers.

## 3. UNBALANCED-OSCILLATOR ANALOGY

From the above, one can see that, albeit mechanically simple, the DSS model shown in Fig. 1 exhibits very complicated dynamics. Thus, as an accurate understanding/modeling of the dynamics is crucial to get an adequate control law to overcome the cited attitude traps. To accomplish this, Yee (1981), Rand, *et al.*, 1992, Kinsey, *et al*., 1995, among others, used the *unbalanced oscillator* (Fig. 4), a mathematically equivalent, however mechanically much simpler, as model for a DSS attitude dynamics. Indeed, from the references just cited one can see that, when spinning-up the rotor under assumption of small unbalanced mass ($M_2$) and applied torque (A), as the rotor's angular velocity gets close to the cart's spring-mass natural frequency, it can either remains trapped close to the cart's natural frequency (capture) or increases further (pass-through). This is just the phenomena observed in the DSS attitude dynamics.

This mimetic characteristic of the DSS and Unbalanced Oscillator models can also be analyzed in the framework of the *Non-ideal Vibrations*, that is, vibrations driven by devices with limited power in which the interaction driver-oscillator is considered in the model, as can be seen in Balthazar, *et al.,* 2017-a,b.

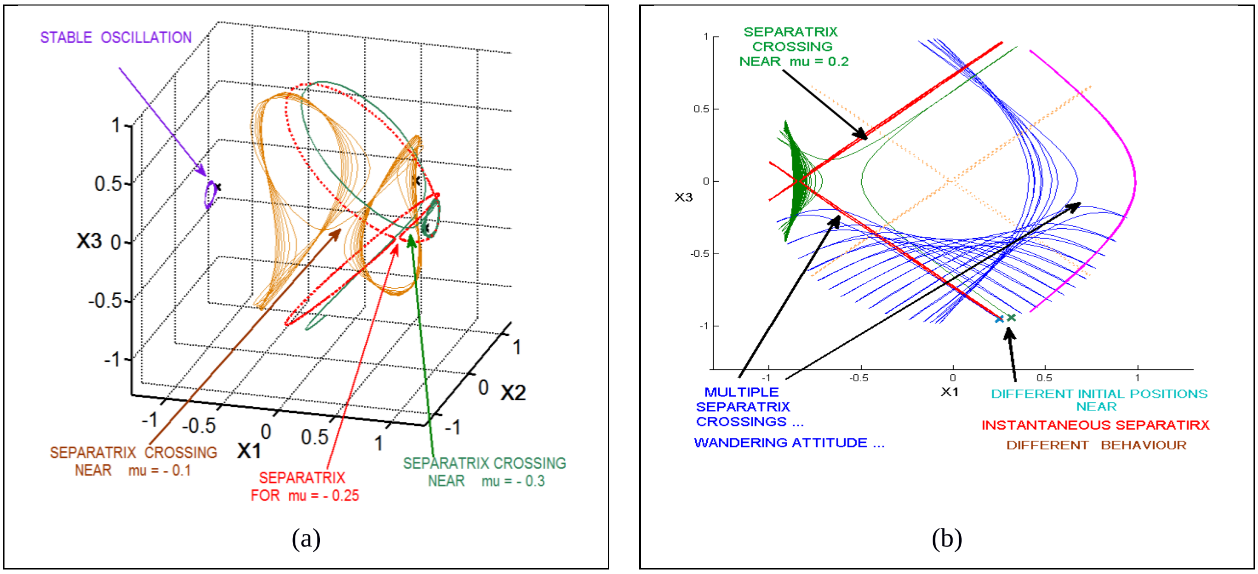


Figure 3 - Separatrix crossings: (a) single crossings; (b) single and multiple crossings.

Seen as a *Non-Ideal* device, as the DSS's rotor accelerates and reach a *near resonant condition* rotation/precession, part of its output energy is consumed (transferred) to the structure, generating large amplitude precession motions instead of increasing its own angular speed. Depending on the values of the system's parameters, the motor can get stuck at resonance, not having enough power to reach higher rotation regimes. Additionally, when some more power is available, a *jump phenomenon* may occur, that is, the rotor's angular velocity jump from a near resonance condition to a considerably higher value, no stable motions being possible between these two values.

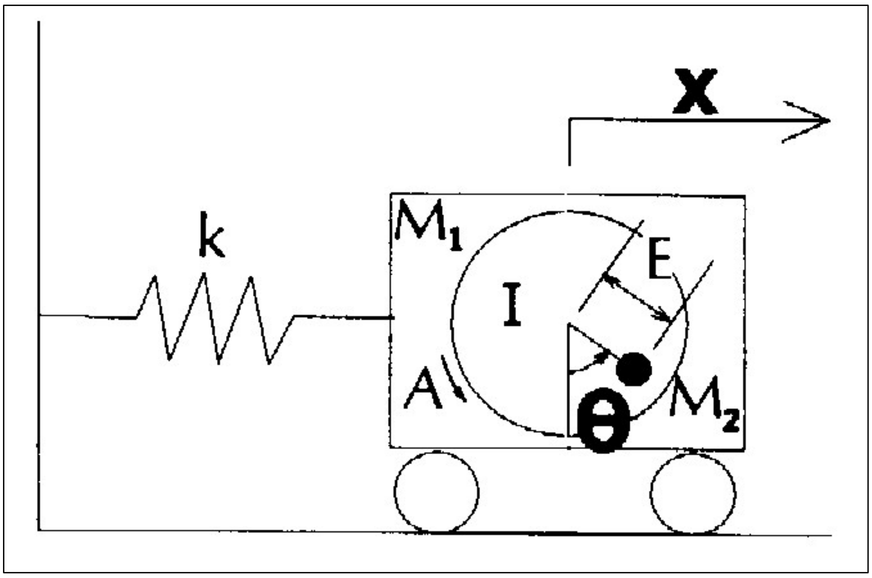


Figure 4 - Unbalanced oscillator (Rand, *et al.*, 1992).

## 4. CONCLUSIONS

Several strategies can be used to minimize the energy loss produced by attitude trappings in DSS attitude dynamics, either by determining approximate asymptotic or numerical solutions that allow identifying and avoiding those initial conditions resulting in attitude traps, or by determining an adequate attitude control strategy that overcomes these traps, resulting in a pass-through rotational trajectory. The resemblance within the shown dynamic models may stand as a test bad for analyzing the dynamics of several mechanical systems in the framework of the Non-Ideal Mechanical Vibrations, that is, vibrations produced by limited power actuators in which the interaction of the energy source with the structure has to be considered.

**5. ACKNOWLEDGEMENTS**

This research has been supported by FAPESP under contracts 15/09047-5 and 13/09723-5 and by CAPES under CAPES-ITA Edital 48/2014.

**6. REFERENCES**

Aslanov, V. S., "Integrable Cases of the Problem of the Free Motion of a Gyrostat", Journal of Applied Mathematics and Mechanics, 78(5) 445-453.

Balthazar J.M., Tusset A.M., Brasil, R.M.L.R.F., Rocha T.R., Janzen F., Nabarette A., De Oliveira C., 2017-a. "An overview on the appearance of the Sommerfeld effect and saturation phenomenon in non-ideal vibrating systems (NIS) in macro and mems scales", submitted to Nonlinear Dynamics.

Balthazar J.M., Tusset A.M., Felix, J.L.P., Cunha Jr., A., Brasil, R.M.L.R.F., Picirillo, V., Rocha, R.T., Nabarette A., Cereja, E., 2017-b. "Passive and Active Control Strategies Using Energy Transfer in a Non-Ideal Structure Coupled to an Essentially Nonlinear Oscillator Taking into Account Parametric Errors", In *3rd EMI International Conference* – March-2017, Rio de Janeiro.

Cochran,J.E., Ping-Huei,S., and Stephen, D., 1982. "Attitude Motion of Asymetric Dual-Spin Spacecraft", J*ournal of Guidenace, Control and Dynamics* 5(1) 37-42.

Elipe, A., Lanchares, V., 2008. Exact Solution of a Triaxial Gyrostat With One Rotor", Celetial Mechanics and Dinamical Astronomy, 101(1-2) 49-68.

Haberman, R., Rand, R., & Yuster, T., 1999. "Resonant capture and separatrix crossing in dual-spin spacecraft". *Nonlinear Dynamics*, *18*(2), 159-184.

Hall, C.D. (1995). "Spinup Dynamics of Gyrostats". *Journal of Guidance, Control and Dynamics*, 18(5), 1177-1183.

Kinsey, R. J., Mingori, D. L., and Rand, R. H., 1995. "Limited torque spinup of an unbalanced rotor on an elastic support". In *American Control Conference, Proceedings of the 1995* (Vol. 6, pp. 4368-4373). IEEE.

Masaitis, Č.,1961. "On the Motion of Two Linked Bodies". *Archive for Rotational Mechanics and Analysis,* 8(1), 23-35.

Rand, R. H., Kinsey, R. J., & Mingori, D. L., 1992. "Dynamics of spinup through resonance". *International Journal of Non-Linear Mechanics*, *27*(3), 489-502.

Volterra, V., 1899. "Sur la Theorie des Variations des latitudes". *Acta Mathematica,* 22(1), 201-357.

Yee, R.K.M. 1981. "Spinup Dynamics of a Rotating System with Limited Torque". *M.S. Thesis, UCLA*.

**7. RESPONSIBILITY NOTICE**

The authors are the only responsible for the printed material included in this paper.