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EVALUATION OF THE ELASTIC PROPERTIES OF COMPOSITE MATERIALS BY THE RULE OF MIXTURES AND COMPUTER PROGRAM

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Abstract. *From the elastic properties of the individual constituents of composite materials, it is possible to predict the final properties of the composite before its manufacture, through traditional formulations such as the rule of mixtures (ROM). This work deals with a comparison of the elastic properties of composite materials through classical equations and computational simulation. The classical equations are the direct and inverse rule of mixtures, the Halpin-Tsai semi-empirical model, and the computational simulation performed in ANSYS R17.2®. The elastic properties evaluated in this research are longitudinal and transverse modulus of elasticity and the Poisson ratio. The finite element analysis considered a representative volume element, with dimensions in the order of microns. Each constituent was modeled with the isotropic properties of the modulus of elasticity and Poisson ratio for the matrix of epoxy and fiberglass. The results of the simulation were consistent with the values from ROM, except for transverse module that presented high error unless for Halpin-Tsai model, exhibiting 6% of error.*

Keywords: *Element representative volume; composite materials; fiberglass; epoxy.*

1. INTRODUCTION

The application of composite materials has become frequent due to the great advantages presented in its use. Some factors may interfere with the properties of the composite, such: adhesion between matrix and fibers, matrix properties, orientation and fiber geometry. Among the various methods of evaluation of the mechanical properties of composites, the modeling and computational analysis method show great results.

In this work, an evaluation was made through finite element analysis of the mechanical properties of epoxy composites reinforced with fiberglass, using rule of mixtures (ROM) and the Halpin-Tsai model to obtain these properties. For this, the composite was considered as a Representative Volume Element (RVE) with circular section fiber.

2. THEORETICAL REFERENCE

2.1 Representative volume element (RVE)

From a unidirectional lamina, representative volume element is the smallest part of the material and represents the material as a whole. RVE consists of a fiber surrounded by the matrix, as it seen in Fig. 1, in the shape of cubic, cylindrical or square elements that are applied in numerical approximations, facilitating the prediction of micromechanical behavior of the fiber and matrix. (Kaw, 2006)

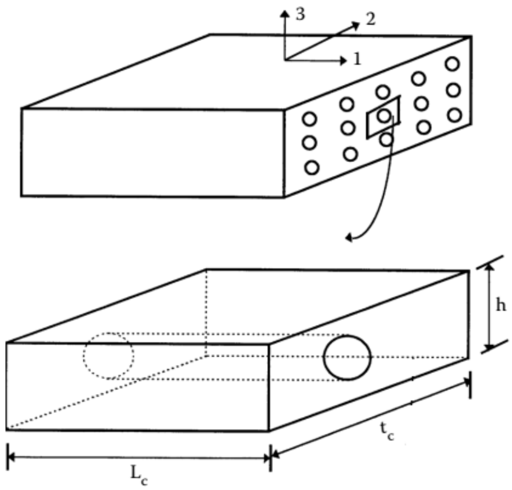

Figure 1. Representative volume element of a unidirectional lamina. (Kaw, 2006)

2.2 Rule of mixtures (ROM)

This method consists of mathematical expressions that provide some elastic properties of the unidirectional composite material as a function of the properties, quantity and arrangement of its constituents. The modulus of elasticity of the matrix, the fiber and the Poisson ratio, are defined as E_m, E_f and ν_f, respectively. The elastic properties of the composite are given by Eq. (1) to (3). Where E_1 is the longitudinal modulus, E_2 is the transverse modulus and ν_{12} is the major Poisson ratio.

$$E_1 = E_{f1} v_f + E_m v_m \tag{1}$$

$$\frac{1}{E_2} = \frac{v_f}{E_{F2}} + \frac{v_m}{E_m} \tag{2}$$

$$v_{12} = v_f V_f + v_m V_m \tag{3}$$

Equations (1) to (3) are validated among some assumptions: the bond between fibers and matrix is perfect; the elastic moduli, diameters, and space between fibers are uniform; the fibers are continuous and parallel; the fibers and matrix follow Hooke's law (linearly elastic); the fibers possess uniform strength and the composite is free of voids. (Kaw, 2006)

2.3 Semi-empirical of Halpin-Tsai

Halpin and Tsai developed their models through simple equations with curve fitting for the transverse modulus. The equations are of semi-empirical origin, as shown in Eq. (4) and (5). (Pal and Haseebuddin, 2015)

$$E_2 = E_m \frac{1+\xi\eta V_f}{1-\xi\eta V_f} \tag{4}$$

$$\eta = \frac{(E_f/E_m)-1}{(E_f/E_m)-\xi} \tag{5}$$

The term ξ is an empirical factor that measures the level of reinforcement of the matrix by the fibers and depends on the load and geometry of the fiber.

2.4 Semi-empirical model of Chamis

This model determines the elastic properties for transversely isotropic unidirectional composites and can be applied perpendicular to the direction of the fiber. Equation 6 presents the transverse modulus of Chamis model.

$$E_2 = \frac{E_m}{1 - \sqrt{V_f\left(1 - \frac{E_m}{E_f}\right)}} \qquad (6)$$

3. INPUT MATERIALS

They are cubic, cylindrical or square elements used in numerical approximations, illustrated at Fig. 2a, due to the facility to predict micromechanical behavior of the fiber and matrix.

Table 1. Elastic properties of the constituents. (Sudheer, 2015)

	Epoxy matrix	Fiberglass
Effective modulus (MPa)	3450	73100
Major Poisson ratio	0.35	0.22

The EVR is a cube with a side, L, of 8μm and the circular fiber has a diameter of 9 μm, according to Figure 1a (Gibson, 1994). The fiber volume fraction was 24.8%.

3.1 Boundary conditions

Three simulations were performed to analyze the stresses on each axis, applying displacements. Table 2 describes the restrictions and displacement conditions applied to each face of the RVE, according to Fig. 2b.

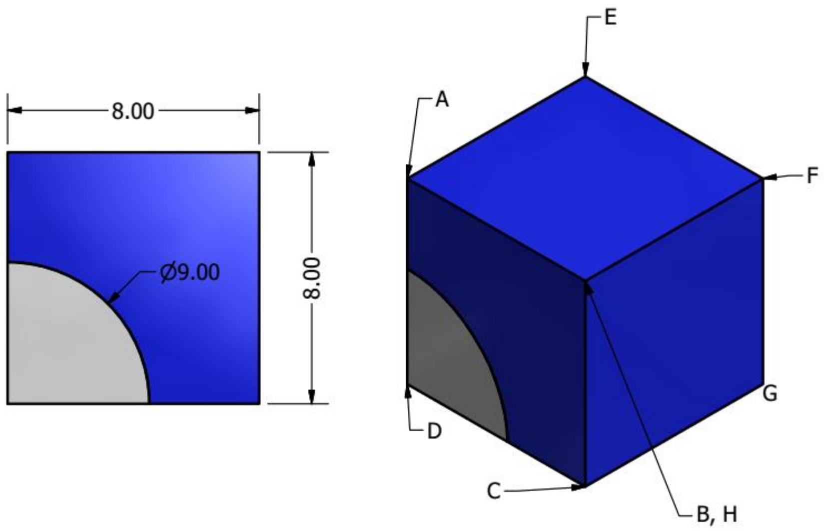

Figure 2. RVE. (a) Dimensions, (b) edge. (Barbero, 2014)

Table 2. Boundary conditions (Barbero, 2014)

Simulation	X	Y	Z
1			EFGH=0 ABCD=L
2	AEHD=0 FBCG=L		
3		DCGH=0 ABFE=L	

4. RESULTS AND DISCUSSION

Figure 3 expresses the results from each simulation, which they were obtained results of the normal stresses that act in each axis of the RVE. Table 3 shows the analytical results and the results from finite element simulation.

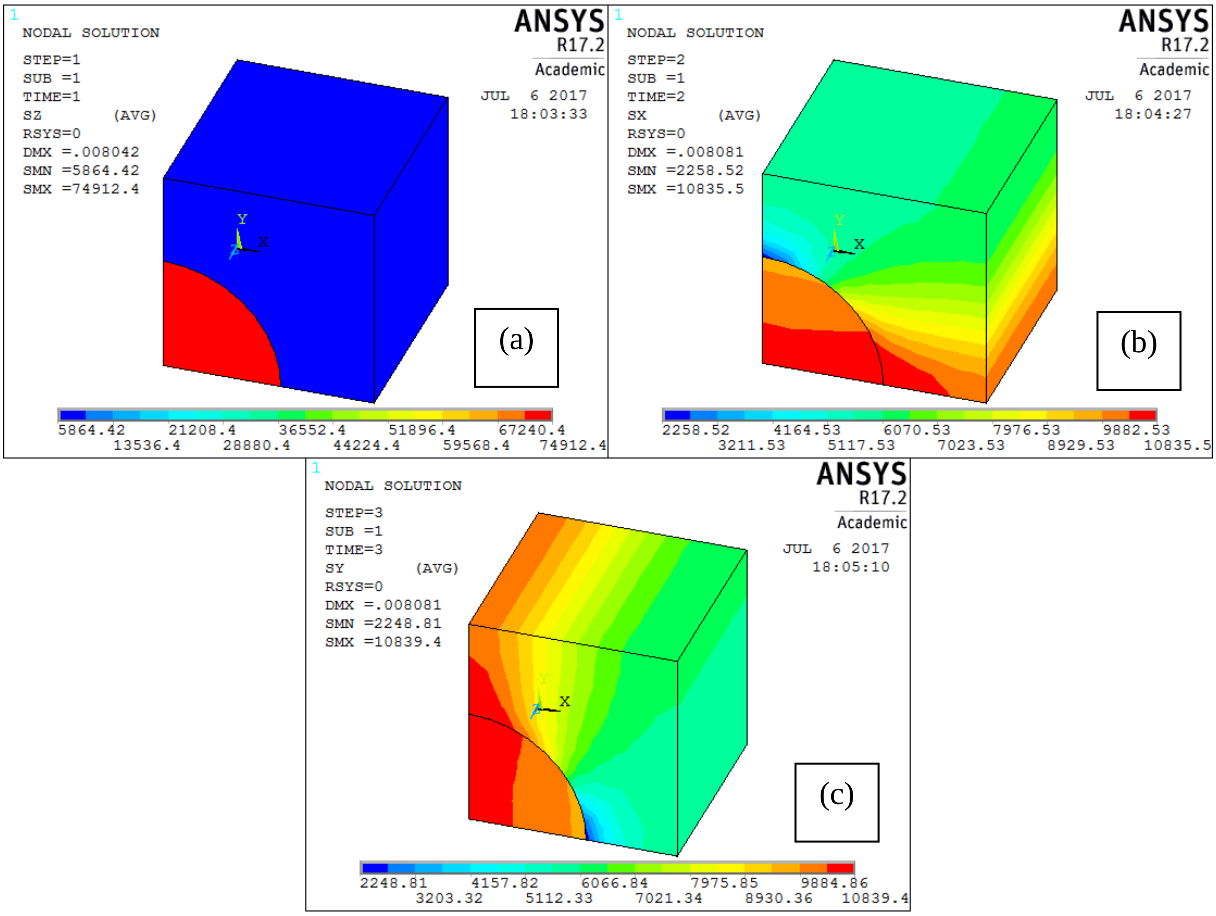

Figure 3. Normal stress (MPa). (a) Z-axis; (b) X-axis; (c) Y-axis.

Table 3. Analytical results and computational analysis.

Properties	Analytical results	FEA	Error
E_1	20758.37 MPa	20773.14 MPa	0%
E_2	4520.30 MPa	5953.93 MPa	32%
E_2 (Halpin-Tsai)	6307.50 MPa	5953.93 MPa	6%
E_2 (Chamis)	6719.86MPa	5953.93 MPa	11%
ν_{12}	0.32	0.31	2%

Table 3 notice a proximity between the analytical and computational results. The value of the longitudinal modulus and Poisson ratio presented an error of 0% and 2%, respectively. For the transverse modulus, the error was 32% for the FEA, which was the biggest error obtained between the results. For Halpin-Tsai model, the error was 6% due to the curve fitting previously established by the model. The Chamis model showed an error of 11%.

5. CONCLUSIONS

The present work carried out a study of the elastic properties of composite materials, comparing the analytical methods, such as ROM and the semi-empirical model of Halpin-Tsai Chamis, and computational. The FEA proved to be satisfactory for the study of the tensions between fiber and matrix, proving the study with the elastic properties from ROM.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

B. Pal and M. R. Haseebuddin, “Analytical Estimation of Elastic Properties of Polypropylene Fiber Matrix Composite by Finite Element Analysis”. American Journal of Materials Science, 5(3C): 162-168. 2015

E. J. Barbero, “Finite element analysis of composite materials using ANSYS®”. CRC Press Taylor & Francis Group. 2014.

A. K. Kaw, “Mechanics of composite materials”. 2nd edition, Boca Raton, FL, CRC Press - Taylor & Francis Group, 2006, 473.

R. F. Gibson, "Principles of composite material mechanics" McGraw-Hill series in mechanical, aeronautical and aerospace engineering. 1994.

Sudheer, M., Pradyoth, K. R., Somayaji, S., 2015 “Analytical and Numerical Validation of Epoxy/Glass Structural Composites for Elastic Models”. American Journal of Materials Science 2015, 5(3C): 162-168.

8. RESPONSIBILITY NOTICE

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