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A SIMILARITY LAW FOR TRANSPIRED TURBULENT FLOWS

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Abstract. *In this work a new expression for the characteristic velocity scale of transpired turbulent flows is derived. When scaled with the new velocity scale, experimental and DNS mean velocity profiles from several databases collapse onto one single curve in the near wall region, suggesting that similarity is possible when the proper scaling parameters are used. A new law of the wall that contain the classical logarithmic law of the wall as a particular case is proposed. Contrasting with the classical formulations, the new law of the wall has a power law functional form and all free parameters are constants independent of the transpiration rate.*

Keywords: *turbulence, wall transpiration, scaling law*

1. INTRODUCTION

Transpired turbulent flows are wall bounded flows where injection or suction (transpiration) of fluid is applied throw a porous wall. This includes boundary layers, pipe and channel flows. This kind of flow occur frequently in nature and in the industry and practical examples are drag reduction and boundary layer control, film cooling technique, atmospheric flows over vegetation canopies, geophysical flows in natural channels and rivers with permeable beds (seepage flows) and so on.

Many theoretical works concerning turbulent flows with wall transpiration focus on obtaining an expression for the stream-wise mean velocity profile close to the wall in the form of the so-called "law of the wall" . This expression is of great importance since an equation to evaluate the mean wall shear stress, an important parameter in engineering applications, can be obtained from it. Furthermore, the near wall laws can be used as boundary conditions for more sophisticated turbulence models (e.g. in high Reynolds number turbulence models) so better wall laws would improve the accuracy of this models.

Using Prandtl momentum transfer theory many authors (Kay, 1948; Dorrance and Dore, 1954; Rubesin, 1954; Clarke *et al.*, 1955; Van-Driest, 1957; Mickley and Davis, 1957; Black and Sarnecki, 1958; Townsend, 1961; Stevenson, 1963; Torii *et al.*, 1966; Simpson, 1967; Bradshaw, 1967; Nayak and Barden, 1972; Wilcox and Traci, 1976; Silva-Freire, 1988; Lin and Karunarathna, 2006; Kornilov and Boiko, 2015; Vigdorovich, 2016) derived a law of the wall that contains a logarithmic squared term, so that it has been labelled the "bi-logarithmic law of the wall". The most popular version of the bi-log law reads, after Stevenson (1963),

$$\frac{2u_\tau}{V_w}\left\{\left(1+\frac{V_w\bar{u}}{u_\tau^2}\right)^{\frac{1}{2}}-1\right\}=\frac{1}{\varkappa}\ln\left(\frac{yu_\tau}{\nu}\right)+A, \tag{1}$$

where V_w is the transpiration velocity, u_τ is the friction velocity, $\bar{u}$ is the stream-wise direction mean velocity component, y is the distance from the wall in the wall normal direction, ν is the fluid kinematic viscosity and $\varkappa = 0.41$ is the Von Karman constant. When plotted in non-dimensional coordinates accordingly to the bi-logarithmic law (scaled with the friction velocity u_τ and friction length scale ν/u_τ) experimental and DNS velocity profiles do not collapse onto a single curve (figure 1), suggesting, from the present point of view, that the relevant velocity and length scales are not the friction velocity u_τ and ν/u_τ. To obtain a better fit with experimental data, most theories assume that the constant of integration A in equation 1 is a function of the non-dimensional transpiration velocity V_w/u_τ. In most formulations, the functional form of this parameter has been obtained essentially by an empirical fit and has no real physical underpinning.

The other approach that have been taken by some authors to obtain the near wall laws is an extension of Millikan (1938) phenomenological theory of turbulence. Based fundamentally in simple dimensional analysis and similarity arguments

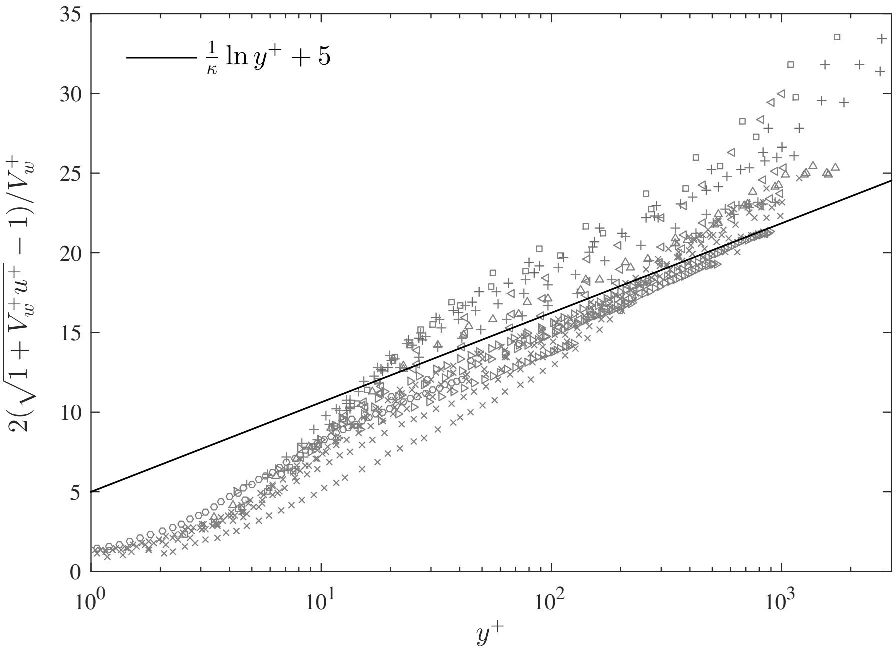

Figure 1: Experimental and DNS mean velocity profiles plotted in the bi-logarithmic coordinates. Dimensionless variables with the superscript $+$ are scaled with u_τ and ν/u_τ. Symbols as in figure 3.

this theory predicts a semi-logarithmic dependence in the law of the wall, i.e,

$$\frac{\bar{u}}{u_c} = C \ln\left(\frac{y}{y_c}\right) + B, \tag{2}$$

where u_c and y_c are characteristics velocity and length scales of the flow and C and B are parameters that usually depends on the value of the transpiration velocity. Here, we use the term "semi-logarithmic" to avoid confusion with the classic logarithmic law for solid walls. Several authors have proposed different expressions for u_c and y_c (Kay, 1948; Tennekes, 1964; Coles, 1972; Andersen *et al.*, 1972; Afzal, 1976; Nakagawa and Nezu, 1979; Avsarkisov *et al.*, 2014; Ferro *et al.*, 2017) but in all formulations the collapse of the profiles has not been obtained. Particularly, Tennekes (1965) proposed, based on semi-empirical grounds, that for "arbitrary suction or blowing rates" the characteristic velocity scale is given by $u_c = u_\tau + 9V_w$ and for "moderate suction rates" by $u_c = -0.06u_\tau^2/V_w$.

2. PROPOSED THEORY

2.1 Characteristic scales of the flow

A new expression for the characteristic velocity scale of the flow u_c, a crucial concept in turbulence modelling, will be determined here through some order of magnitude considerations. At the bottom of the overlap region the approximated equation of motion reads,

$$V_w\bar{u} = \nu\frac{\partial \bar{u}}{\partial y} - \overline{u'v'} - u_\tau^2, \tag{3}$$

where $-\rho\overline{u'v'}$ is the turbulent shear stress. In this region, is assumed that the characteristic length scale is given by ν/u_c and that the turbulent fluctuations are of the order of the characteristic velocity scale, u_c. Considering that the viscous term can be approximated by,

$$O\left(\nu\frac{\partial \bar{u}}{\partial y}\right) = O\left(\frac{\tau_w}{\rho}\right), \tag{4}$$

it results from simple order of magnitude arguments that the characteristic velocity can be estimated from the algebraic equation,

$$u_c^2 - \alpha V_w u_c - u_\tau^2 = 0, \tag{5}$$

where α is a proportionality coefficient of order one. Equation 5 has a positive real root given by

$$u_c = \frac{\alpha V_w + \sqrt[2]{\alpha^2 V_w^2 + 4u_\tau^2}}{2}. \tag{6}$$

In the blow-off condition ($u_\tau = 0$, $V_w > 0$)(Coles, 1972), expression 6 gives a non-zero velocity scale, $u_c = \alpha V_w$, and when the transpiration velocity is zero it reduces to $u_c = u_\tau$, the proper velocity scale for non-transpired flows. Now it will be shown that when two asymptotic cases are considered the velocity scale given by equation 6 recovers the expressions obtained by Tennekes (1965). Considering the case with "arbitrary suction or blowing rates" first and writing equation 6 as,

$$u_c = \frac{\alpha V_w + 2u_\tau \sqrt[2]{\alpha^2 V_w^2/(4u_\tau^2) + 1}}{2}, \tag{7}$$

its clear that in the limit $V_w/u_\tau << 1$ equation 7 can be approximated by,

$$u_c \approx \frac{\alpha}{2} V_w + u_\tau, \qquad \frac{V_w}{u_\tau} << 1, \tag{8}$$

recovering Tennekes velocity scale for "arbitrary suction or blowing rates". Now, in the case of "moderate suction", one may write equation 6 as,

$$u_c = \frac{\alpha V_w - \alpha V_w \sqrt[2]{1 + 4u_\tau^2/(\alpha^2 V_w^2)}}{2}, \tag{9}$$

where the minus sign before the square root appears because $V_w < 0$ for suction− so $\sqrt{V_w^2} = -V_w$. Expanding the square root term in equation 9 in a two terms Taylor series for small u_τ/V_w one might obtain,

$$u_c \approx \frac{\alpha V_w - \alpha V_w \left[1 + 2u_\tau^2/(\alpha^2 V_w^2)\right]}{2} = -\left(\frac{1}{\alpha}\right)\frac{u_\tau^2}{V_w}, \qquad \frac{|V_w|}{u_\tau} >> 1, \tag{10}$$

recovering Tennekes velocity scale for "moderate" suction rates. Here a curious result is noticed if one sets the value of $\alpha = 18$, in that case even the numeric coefficients in equations 8 and 10 are approximately the same obtained by Tennekes, i.e., $\alpha/2 = 9$ and $-1/\alpha^2 = -0.06$. Equation 10 was deduced for sucked flows with $|V_w| >> u_\tau$, where it is expected that the flow might be in a state of reversal to laminar. If this is the case, equation 10 is consistent with the laminar sub-layer solution,

$$\frac{V_w \bar{u}}{u_\tau^2} = \exp\left(\frac{V_w y}{\nu}\right) - 1, \tag{11}$$

where velocity profiles are self similar when scaled by u_τ^2/V_w (Tennekes, 1964).

2.2 A new law of the wall

To derive an expression for the stream-wise mean velocity profile in the fully turbulent region close to the wall, some considerations about the mean shear stress τ in that region will be done first. If the viscous contribution can be neglected, it will be considered that τ is affected essentially by two distinct mechanisms. One is momentum transport induced by eddies associated with the turbulence of the flow. This mechanism is enhanced when the flow is subjected to injection of fluid at the wall and is suppressed when suction is applied. With that considerations, is assumed that τ can be written as the sum of two components, τ_e and τ_{v_w}, associated with this two mechanisms respectively,

$$\tau = \tau_e + \tau_{v_w}, \tag{12}$$

where the subscript e reefers to eddy, and v_w the wall transpiration. Similar decompositions of the turbulent shear stress have been proposed in the past by different authors (Manes *et al.*, 2012; Mendoza and Zhou, 1992). Assuming the Boussinesq hypothesis (Boussinesq, 1870), an analogy with Maxwell kinetic theory of gases (Tennekes and Lumley, 1972) yields,

$$\tau_e = \rho u_c \ell \frac{\partial \bar{u}}{\partial y}, \tag{13}$$

where $\ell = \varkappa y$ is the mixing length.

To write an expression for the component of the mean shear stress associated with the extra momentum transport caused by the wall transpiration, τ_{v_w}, a cruder assumption will be made. This component must be zero in the case of a flow with no wall transpiration. Furthermore, in accordance with well known empirical information (Andersen *et al.*, 1972), it should be positive in the case of blowing and negative in the case of suction. It should also be somehow related

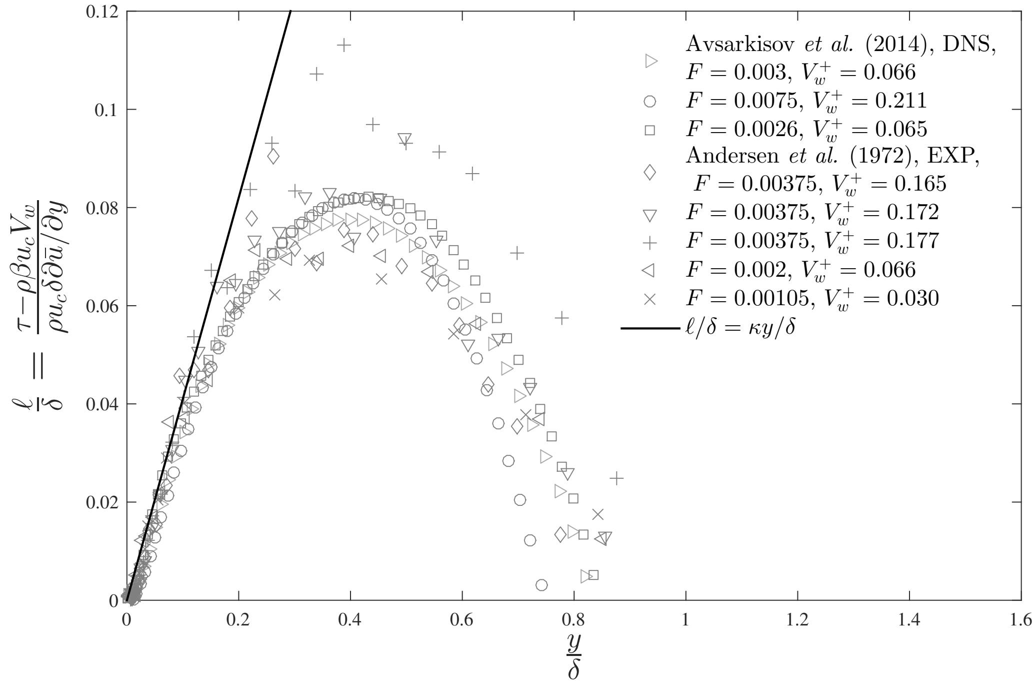

Figure 2: Experimental and DNS mixing length profiles accordingly with the proposed theory. $F = V_w/U_\infty$, where U_∞ is the free stream velocity, is the dimensionless transpiration parameter and δ is the boundary layer thickness.

to the characteristic velocity of the flow, u_c. With that considerations in mind, one of the simplest assumption that one can made is that τ_{v_w} is proportional to the transpiration velocity V_w and the characteristic velocity u_c,

$$\tau_{v_w} = \beta u_c V_w, \tag{14}$$

where β is a proportionality constant of order one. Figure 2 shows that equations 12 to 14 provide reasonable agreement with the data in the 15% (approximately) inner region of transpired flows with zero (or negligibly small) pressure gradients. Integrating expression 12 to 14 together with the approximated equation of motion,

$$\tau = \rho V_w \bar{u} + \tau_w, \tag{15}$$

and writing the result in an appropriate non-dimensional form yields,

$$\frac{\bar{u}}{u_c} = \frac{u_\tau^2}{u_c V_w}\left\{\left(A\frac{y u_c}{\nu}\right)^{\frac{V_w}{\varkappa u_c}} - 1\right\} + \beta, \tag{16}$$

where A is a constant of integration. Equation 16 is the new law of the wall for turbulent flows with wall transpiration. Now it will be shown that in the particular case of a flow with zero wall transpiration, i.e. in the limit $V_w \to 0$, equation 16 reduces to the classic logarithmic-law of the wall. To evaluate equation 16 in that limit, it is useful to use the following property of the logarithms,

$$\ln z = \lim_{w \to 0} \frac{1}{w}(z^w - 1), \tag{17}$$

with $z = Ayu_c/\nu$ and $w = V_w/\varkappa u_c$. Noticing that in the limit $V_w \to 0$ equation 6 for the characteristic velocity scale of the flow gives $u_c = u_\tau$, the new law of the wall equation 16 when evaluated in this limit gives,

$$\frac{\bar{u}}{u_\tau} = \frac{1}{\varkappa}\ln(Ay^+) + \beta. \tag{18}$$

From equation 18 its clear that the classical logarithmic-law of the wall is recovered if the following equality is satisfied,

$$\frac{1}{\varkappa}\ln(A) + \beta = 5, \tag{19}$$

giving an formula to express the constant A as a function of β or vice versa.

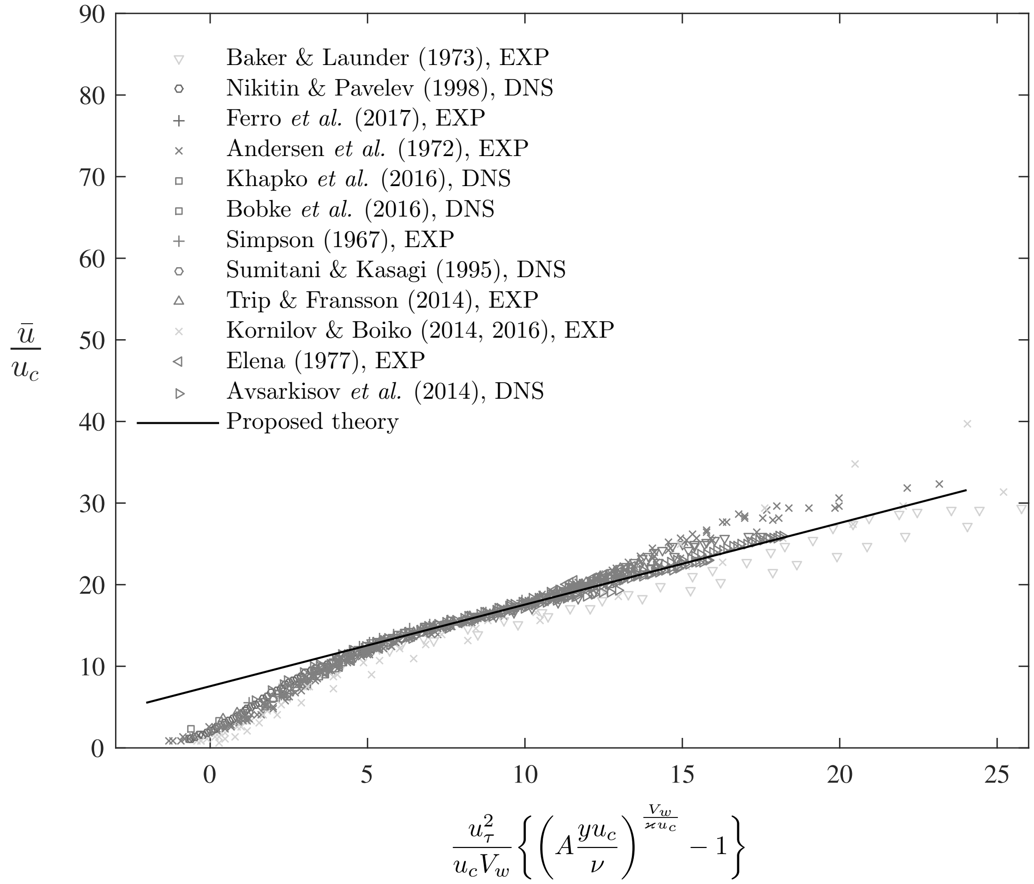

Figure 3: Thirty five (35) experimental (EXP) and DNS mean velocity profiles plotted with similarity coordinates. The values of the transpiration parameters are in the range, $-0.0035 \leq F \leq 0.0164$, and, $-0.065 \leq V_w^+ \leq 0.68$.

With equation 19, the new law of the wall has two constants that could not be obtained from theory and must be calibrated in order to give a good fit to the data. Convenient values was found to be $A = 0.35$ and $\alpha = 3.15$. In the authors opinion, a theory that contain empirically calibrated constants are superior, in some sense, to a theory that contain empirical functions. From this point of view, the present formulation has an advantage over most versions of the bi-logarithmic or semi-logarithmic formulas. When plotted in non-dimensional coordinates suggested from the new law of the wall, experimental and DNS mean velocity profiles from several databases collapse onto one single curve in the near wall region (figure 3). The excellent collapse of the profiles suggests that they can be self-similar if scaled with the proper velocity scale, u_c. The data shown in figure 3 include boundary layer flows with wall injection (Andersen *et al.*, 1972; Kornilov and Boiko, 2014; Baker and Launder, 1974; Kornilov and Boiko, 2016) and suction (Simpson, 1967; Trip and Fransson, 2014; Bobke *et al.*, 2016; Khapko *et al.*, 2016; Ferro *et al.*, 2017), pipe flow with wall suction (Elena, 1977) and closed channel flows with wall injection (Nikitin and Pavel'ev, 1998; Sumitani and Kasagi, 1995; Avsarkisov *et al.*, 2014).

Figure 4 shows a comparison between four different formulations and some data. Clarke *et al.* (1955) version of the bi-log law and Coles (1972) version of the semi-log law were chosen because they are the ones who provide the best fit to the data in most of the profiles analysed by the authors. The numerical solution of Cebeci (1970) turbulence model is also shown. Coles formula doesn't predict the correct slope of the profiles very accurately and Cebeci solution gives a slightly worst fit to the sucked flows data. For all profiles shown, the proposed theory seems to fit the data slightly better than the other formulations do but, considering the experimental uncertainties, it's difficult to tell if its fit is superior when compared to the bi-log law. A more complete comparison between the different theories and the data can be found in one of the authors thesis (Guimarães, 2018/unpublished).

3. DISCUSSION AND CONCLUSIONS

In this work, new expressions for the characteristic velocity scale and law of the wall for a turbulent flow with wall transpiration were derived. When plotted in coordinates suggested from the new formulation, experimental and DNS mean velocity profiles from several databases collapse onto one single curve suggesting, form the present point of view,

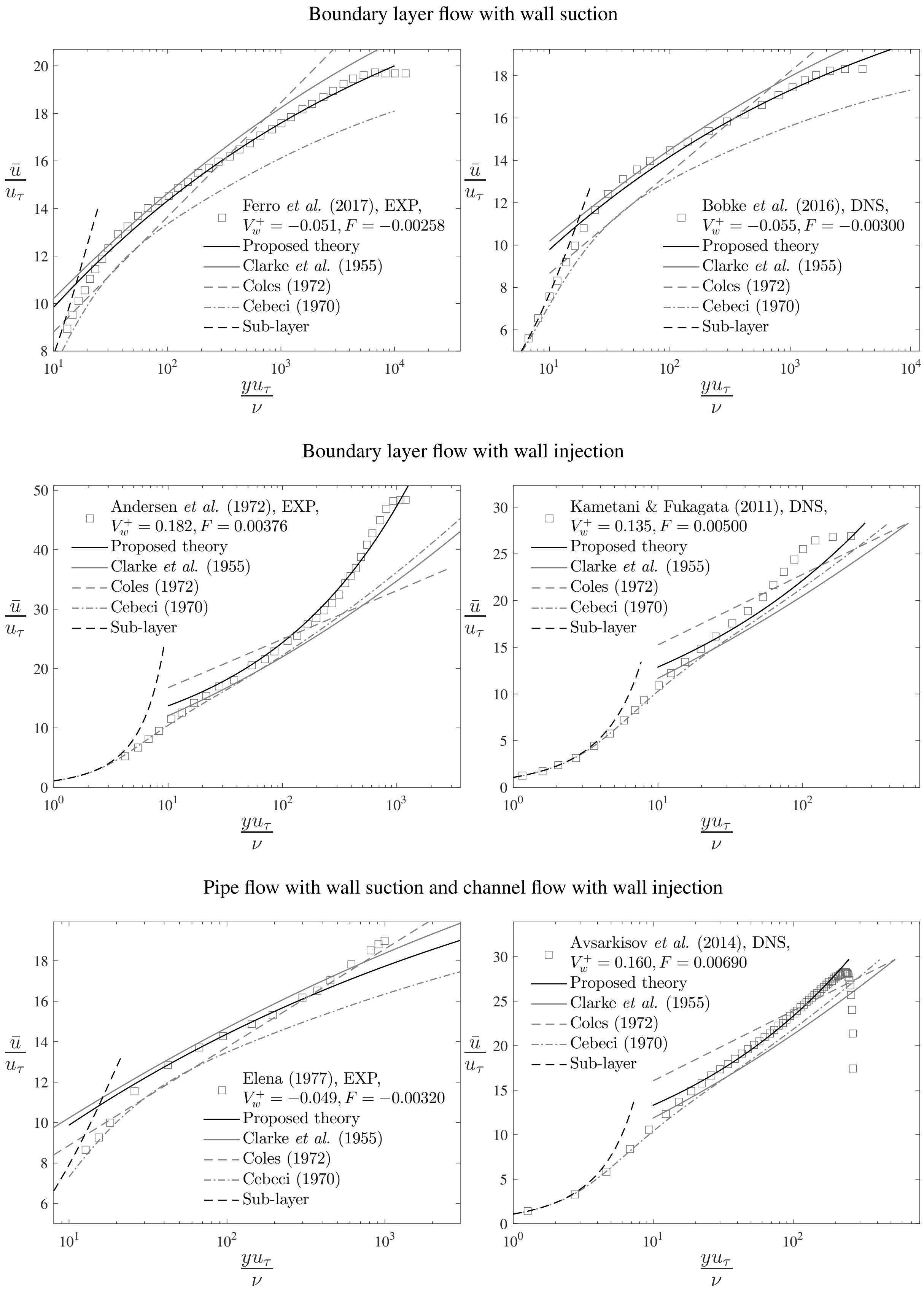

Figure 4: Comparison between different theories and the data.

that similarity is possible when the proper scaling parameters are used.

The turbulence model proposed in this work has a simpler mathematical form compared to Prandtl momentum transfer theory so it can be adapted to complex flows (e.g. flows with wall transpiration and separation) where conventional mixing length theory does not provide analytical solutions. This was carried out by the authors and a manuscript is being prepared for a future publication (Guimarães, 2018/unpublished).

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