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ANALYSIS OF TEMPERATURE PROFILE AND EFFICIENCY OF CONVECTIVE STRAIGHT FINS WITH TEMPERATURE DEPENDENT THERMAL CONDUCTIVITY AND INTERNAL HEAT GENERATION

Luiz Carlos Chaves de Oliveira
Luiz Fernando Silva Ferreira
Marcos Filardy Curi
Centro Federal de Educação Tecnológica Celso Suckow da Fonseca, Department of Mechanical Engineering
Rodovia Mário Covas, lote J2, quadra J, Rio de Janeiro, CEP: 23812-101, Brazil
marcos.curi@cefet-rj.br

Abstract. *Fins are of great importance in many applications of mechanical engineering, and are widely studied for optimizes the heat transfer between a surface and the ambient. In many cases a variable thermal conductivity or/and internal heat generation performs strong effects on performance of extended surfaces. In this paper, we analyze with a routine from Wolfram Mathematica software the temperature distribution within the fin, to estimate the efficiency of convective straight fins with temperature dependent internal heat generation and thermal conductivity. In order to show the effectiveness of the computational method, the results obtained from a direct study of the effects of some physical parameters are demonstrated in this work, presenting the temperature distribution for a range of parameters values appeared in the mathematical formulation. Initially is studied constant thermal conductivity and temperature dependent heat generation. To verify the accuracy of the proposed method, this work shows that the implemented procedure is very effective and convenient for convective fin problems with an excellent agreement when compared with benchmark solutions and numerical solutions from literature for the efficiency and temperature profile analysis, achieving suitable results of such problems.*

Keywords: *Convective Straight fins; Fin Efficiency; Variable Heat generation, Variable thermal conductivity*

1. INTRODUCTION

A problem for improving heat transfer can be solved by using the extended surface called fin. These extended surfaces are extensively used in various conventional industrial applications, such as air conditioning, refrigeration, internal combustion engines, heat exchangers. Fins also confirm efficacious in heat rejection systems in space vehicles and in cooling of electronic components.

The heat transfer mechanism of fin is to conduct heat from heat source to the fin surface by its thermal conduction, and then dissipate heat to the air by the effect of thermal convection. Fins are the most effective instrument for increasing the rate of heat transfer on a surface, rectangular fin is widely used among them, due to simplicity of its design and its easy manufacturing process, but various types of fins could be optimize. As we know, they increase the area of heat transfer and cause an enhances the amount of heat transfer between primary surface and its surrounding medium, as well as fin assembly is commonly used to achieve the same objective.

As Kern and Kraus (1972), there are several techniques to optimize the performance of fins and minimize their costs of production. For example, fins with parabolic concave profile performs the maximum heat dissipation for a given area, but can be relatively expensive to produce this fins format (Yu and Chen, 1998, 199) Finned surfaces have enhanced heat transfer mechanism between the primary surface and the environment. Kern and Kraus (1972) made an extensive review on this issue.

Change (2005) studied the Adomian decomposition method analyzing the thermal behavior of a straight rectangular fin many types of heat transfer. The local heat transfer coefficient is modeled with a power-law function of temperature. This method gives an explicit expression of temperature distribution for each point within the fin, in the form of an infinite power. Thus all analysis in this field of the extended surfaces was calculated directly without the need of iteration, converging rapidly with high accuracy.

Babaelahi, *et.al*, 2009, using a simulation method called the Differential Transformation Method (DTM), studied the effects of some physical applicable parameters in this problem such as thermo-geometric and thermal conductivity parameter and the efficiency of convective straight fins with temperature-dependent thermal conductivity solved. The accuracy of the proposed method obtained good agreement with results from exact and numerical solutions form literature.

In Patra and Ray (2016), shows the temperature distribution, fin efficiency, efficacy of convective straight fins with constant and temperature-dependent thermal conductivity solved by homotopy perturbation sumudu transform method (HPSTM). The fin efficiency and the temperature distribution have been attained as a function of thermo-geometric fin parameter. It can be noticed that the thermal conductivity parameter has a strong influence over the fin efficiency.

For fins common problems, we set constant thermal conductivity and heat generation, but when the fin temperature gradient between the tip and base is large, the effect of the temperature, mainly, on thermal conductivity (for some applications the heat generation) should not be neglect. Due this fact, an extensive majority of problems and scientific phenomena like heat transfer by extended surfaces involve the function of nonlinearly.

In this work we study the fin temperature distribution with variable temperature-dependent properties, *Wolfram Mathematica* (Wolfram, 1999) software is used to solve this problem with temperature dependent on physical properties. The results obtained by the simulation is compared to results of Ganji, *et al.*, 2014.

Temperature distribution within the fin, to estimate the efficiency of convective straight fins shows the effectiveness of the computational method. The results obtained from a direct study of the effects of some physical parameters are demonstrated in this problem, presenting the temperature distribution for a range of parameters appeared in the mathematical formulation, showing the competence of this present approach.

2. MATHEMATICA FORMULATION

The mathematical procedure follows the ideas of Ganji, *et al.*, 2014, to build a code which uses Methods of Lines to solve the final differential equation. It is considered a longitudinal fin with a rectangular cross section, length L, perimeter P, section area A constant, thermal conductivity k and heat generation q_g. The fin is fixed on basis with temperature T_b and loses heat to ambient with temperature T_{inf}. We can write the differential equation and the boundary conditions according to Incropera, *et al.*, 2008, as:

$$\frac{d^2T}{dx^2} - \frac{hP}{kA}\left(T - T_{inf}\right) + \frac{q_g}{k} = 0 \tag{1}$$

$$x = 0 \rightarrow \frac{dT}{dx} = 0 \tag{2}$$

$$x = L \rightarrow T = T_b \tag{3}$$

A schematic fin representation is shown on Fig. 1:

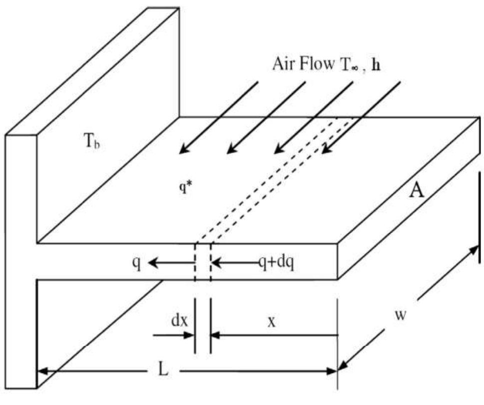

Figure 1: Rectangular fin with internal heat generation and convective air flow.

Initially, the thermal conductivity k is constant and the fin heat generation can be expressed by:

$$q_g = q_{inf_g}\left(\left(T(x) - T_{inf}\right)\epsilon + 1\right) \tag{4}$$

For dimensionless variables, the following expressions are used:

$$\theta(x) = \frac{T(x) - T_{inf}}{T_b - T_{inf}} \tag{5}$$

$$M = \sqrt{\frac{hL^2P}{Ak}} \tag{6}$$

$$X = \frac{x}{L} \tag{7}$$

$$G = \frac{Aq_{inf_g}}{hP(T_b - T_{inf})} \tag{8}$$

$$\epsilon_g = \epsilon(T_b - T_{inf}) \tag{9}$$

Where q_{inf_g} is the internal heat generation at temperature T_{inf}, so Eq. 1 can be rewritten, with their respective boundary conditions. Once the variables involved vary only with x and T, these mathematical simplifications can be disregarded for the obtained solution.

$$\frac{d^2T}{dX^2} - M^2\theta + M^2G(1 + \varepsilon_G\theta) = 0 \tag{10}$$

$$X = 0 \rightarrow \frac{d\theta}{dX} = 0 \tag{11}$$

$$X = 1 \rightarrow \theta = 1 \tag{12}$$

A more complete analysis considers the thermal conductivity varying with temperature, as well as the internal heat generation. A linear variation of thermal conductivity can be written as:

$$k = k_0[1 + \beta(T(x) - T_{inf})] \tag{13}$$

What gives us the following dimensionless expression:

$$\frac{k}{k_0} = [1 + \beta_g\theta] \tag{14}$$

Where $\beta_g = \beta(T_b - T_{inf})$. Finally, with these considerations, we have the following ODE and boundary conditions expressions:

$$\frac{d}{dX}\left[(1 + \beta_g\theta)\frac{d\theta}{dX}\right] - M^2\theta + M^2G(1 + \varepsilon_G\theta) = 0 \tag{15}$$

$$X = 0 \rightarrow \frac{d\theta}{dX} = 0 \tag{16}$$

$$X = 1 \rightarrow \theta = 1 \tag{17}$$

Fin efficiency η, which will also be studied for various cases, is given by the expression:

$$\eta = \frac{Q}{Q_{ideal}} = \frac{\int_0^L hP(T - T_\infty)dx}{hPL(T_b - T_\infty)} = \int_{X=0}^{X=1} \theta(X)dX \tag{18}$$

To get results consistent with the literature, you must specify the values of the variables before use the computational routines. In the case of constant thermal conductivity, it is possible to obtain an analytic solution, which increases the accuracy of the simulation, what does not occur to the opposite, where the software uses a numerical routine to best suited to the considered range.

Mathematica uses an NDSolve subroutine, which implements the methods of the lines as numerical solution for nonlinear equations, the method can present more or less accuracy according to the discretization parameter chosen in the implementation. The results obtained are shown in the next topic.

3. RESULTS AND DISCUSSION

In this topic, we show the results obtained by the computational implementation of the proposed problem, through the parameters given by the equations (6), (8) and (9). The temperature distribution in Figure 2, where $M = 0.5$ assumed, as in the case of compact heat exchangers project:

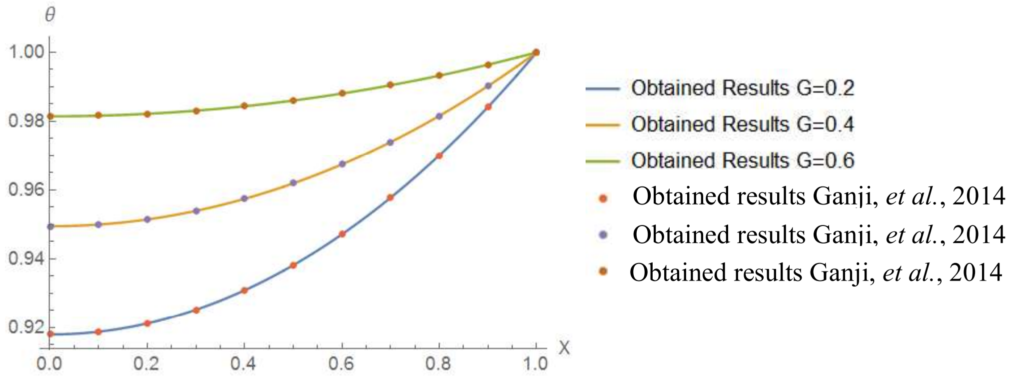

Figure 2: θ x X. Fin temperature distribution with internal heat generation dependence, assuming $M = 0.5$ and $\varepsilon_G = 0.4$, compared with Ganji, *et al.*, 2014.

We could note that the results from the Mathematica outcome numerical subroutine, presents excellent agreement with the results obtained by Ganji, *et al.*, 2014. Some analyzes above the parameters can be made in the above analysis, as G increases we notice a temperature gradient decreasing inside the fin, note when G = 0.6 the temperature gradient is small, keeping the temperature in the entire fin next to the temperature of the base, which does not happen in G = 0.2, where the temperature in the fin falls inside.

This decrease of the fin temperature gradient over all its extension with the increase of G can be explained on by two factors initially, the first one is due to the temperature difference between the base and the convection which decreases, approaching the entire extended surface with a temperature close to the temperature of the base since this gradient is small, another factor may be a decrease in fluid velocity leading to a lower h causing a difference of the temperature between the fin surface and the convection fluid the largest possible, along the fin, keeping it at a temperature close to the base.

For the case of non-compact heat exchangers, where $M = 1$, shown in Figure 3, which is common in the fins projects. The variation of the parameters cited in the two cases demonstrates a non-strong dependency in compact heat exchangers cases. From Fig. 3 we note that the increases of G and ϵ_g parameters causes an increase in fin temperature, once the heat generation also increases.

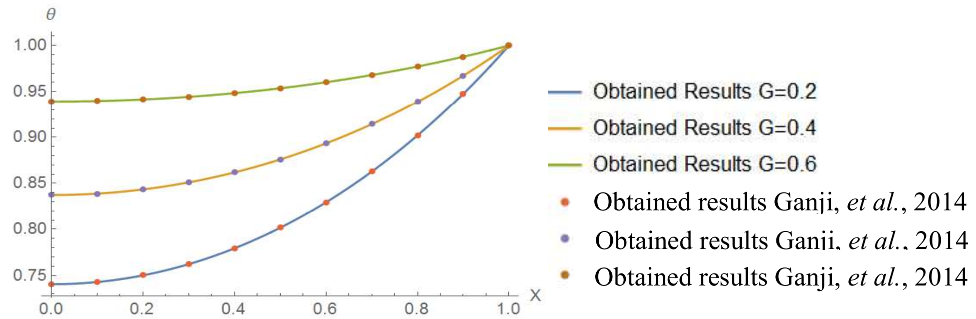

Figure 3: θ x X. Fin temperature distribution with internal heat generation dependence, assuming $M = 1$ and $\varepsilon_G = 0.4$, compared with Ganji, *et al.*, 2014.

Figure 3 shows that the same analysis performed for Fig. 2 (M = 0.5) is still valid, but with a more evident factor for analysis, M, which shows higher temperature gradients for the same variations of G, ie with the increase in M, the thermal conductivity tends to fall, thus decreasing the heat conduction inside the fin preventing it reaching values near the maximum temperature (base temperature), as well as a possible longer fin which would also cause a larger temperature drop, which is somewhat intuitive in the study of long fins, tending mathematically to infinite.

Figure 4 shows the temperature distribution, in several possible cases within the proposed theme. We could note when analyzing the case without generation and constant thermal conductivity, we obtain a higher temperature gradient than in the case of conductivity varying linearly with the temperature (aiding the conduction of heat proportionally).

A curious analysis become attractive in what refers to the fin with heat generation varying with temperature and constant thermal conductivity, which has a much lower gradient than the case without generation analyzed before. It is possible to note the substantial contribution that the heat generation plays: increases the temperature in the entire fin and helps in the final heat exchange with the medium, since it has a smaller gradient throughout his extension, which would be the ideal case for a fin. And a final analysis is the evident lower temperature drop on the extended surface with heat generation and conductivity as linear temperature functions, aiding the transport of thermal energy inside the fin.

For fixed values M=1 e G= $\boldsymbol{\epsilon_g}$= β_g =0.4,, we note that the temperature drops to 30% at the tip of the fin, indicating which modeling is more suitable for specific applications.

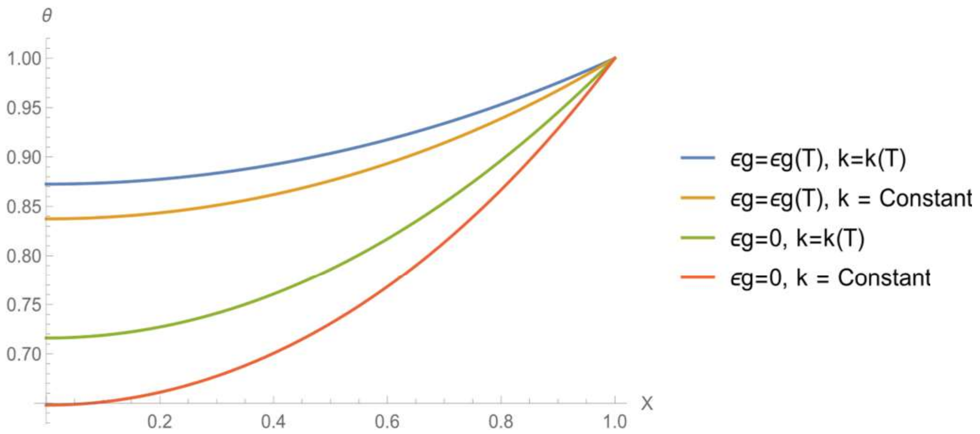

Figure 4: θ x X. Fin temperature distribution assuming M=1 e G= ϵ_g= β_g =0.4.

The figure below, Fig. 5, shows the temperature distribution, varying parameters of the thermal conductivity and setting the parameters of the heat generation (still function of the temperature), and like all the previous analyzes, we have the confirmation of all the hypotheses studied previously, indicating a solid confirmation of the problems physics, when the thermal conductivity gradually increases with temperature, the temperature gradient within the fin decreases which indicates a more intense heat conduction along it. The results obtained by the NDSolve routine of Mathematica, show good agreement with the data obtained by Ganji, *et al.*, 2014

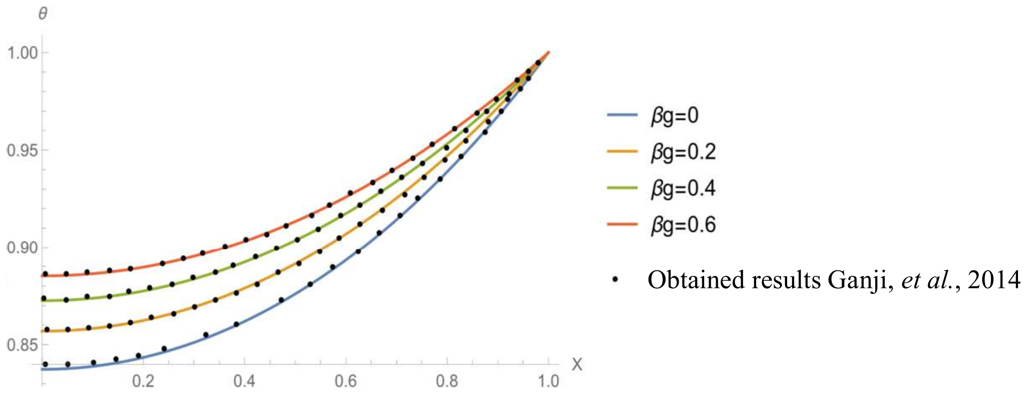

Figure 5: Temperature distribution for M=1 e G= ϵ_g=0.4 varying thermal conductivity β_g

Figure below, Fig. 6, shows the efficiency as a function of the parameter M, varying G and fixing $\epsilon_g = 0.6$. It is noted that the efficiency falls drastically with the decrease in G, which corroborates with the physical analysis made in all previous figures, especially Figs. 2 and 3. With smaller G, the gradient is larger within the fin, distancing it from an ideal model, which would be a negligible gradient inside it, which occurs with the gradual increase of G, then the fall in efficiency with G decreasing has the same physical basis of previous analyzes. As well as the decrease in efficiency with the increase of M, physically explained in Fig 3, fitting perfectly to this efficiency analyses .

With a lower thermal conductivity the transport or conduction of thermal energy becomes more inefficient, because of that the fin does not maintain smaller gradients inside it, distancing of the temperature of the base. The analysis of these parameters shows an efficiency around 45% for M = 2 and G = 0.2 and 80% for M = 2 and G = 0.4.

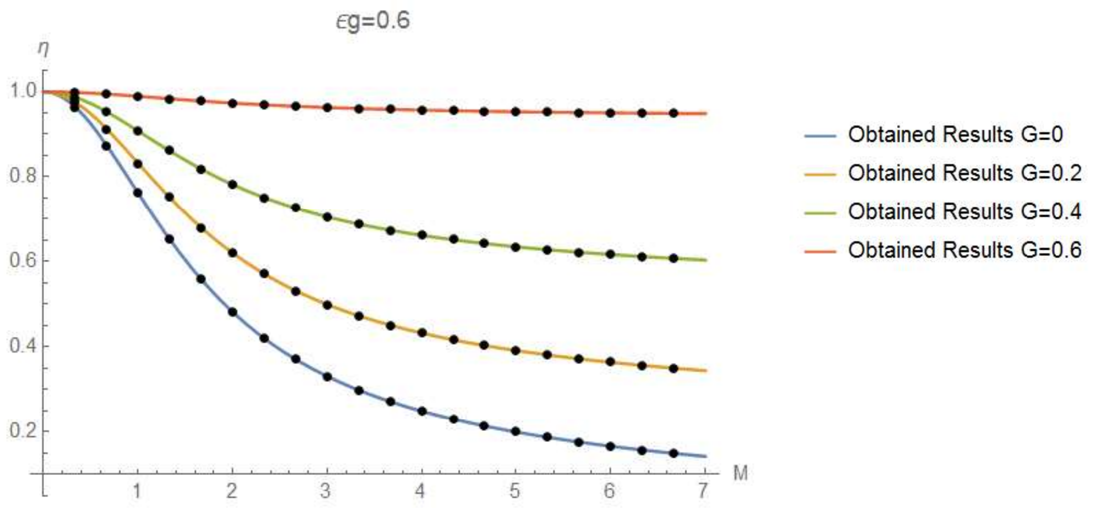

Figure 6: η xM. Fin efficiency for $\varepsilon_G = 0.6$, compared with Ganji, *et al.*, 2014 (black dots).

4. CONCLUSIONS

In this work commands of *Wolfram Mathematica* software are used to solve the problem of temperature distribution in a fin with temperature-dependent heat generation and constant thermal conductivity. The results were compared with references and found that it is possible to model with a minimum amount of computational requirements, as well as results were obtained with effectiveness and precision. Future works, the aim is to analyze the temperature distribution in other fin formats, as well as other forms of thermal conductivity variation due to various effects, such as soot, corrosion and aging of finned structures, where both thermal conductivity and heat generation parameters vary with temperature.

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6. RESPONSIBILITY NOTICE

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