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# AN EXPLICIT FORMULATION FOR THE THERMAL CONTACT CONDUCTANCE ESTIMATION USING TRUNCATED EIGENFUNCTION EXPANSIONS REGULARIZATION

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***Abstract.*** *This work aims to estimate the thermal contact conductance in a medium composed by two layers, using an explicit formulation for a transient inverse heat transfer problem. For the estimation, non-intrusive simulated measurements of temperature in one of the layers are used, with different levels of noise and controlled standard deviations. An integral transformation is employed to obtain a regularized version of the measurements. The results revealed better results for continuous functions than for non-continuous functions, as typical in function estimation problems. In comparison to other methods found in the literature, the proposed method is fast and quite satisfactory.*

***Keywords:*** *thermal contact conductance, integral transform, heat transfer problem, explicit formulation*

## 1. INTRODUCTION

The study of heat transfer in composite media is important because of the applications in different areas of engineering, physics, medicine as in the detection of tumors (Mital and Scott, 2007), in nuclear reactors (Pradere et al, 2006), in the detection of failures in composite media (Schöntag et al, 2010), among others. Interest in this area can be noted in the literature as long ago as in Huang and Ju (1995) who were able to estimate the thermal contact conductance using intrusive measurements. Another possible application is the identification of faults in composite media as in Abreu et al (2014). That work estimates the thermal conductance of contact by solving an inverse problem using the Markov Chain Monte Carlo method.

It can be found in other recent works, such as in Colaço and Alves (2015), the use of non-intrusive measurements for the estimation of thermal contact conductance varying with space in a permanent regime. In that same year, Abreu et al (2015), also analyzed a different approach of the detection of faults at the interface between two materials obtaining satisfactory and fast results. This method using reciprocity functional approach is a very fast method, but presents some instability with respect to the measurement data, and high sensibilities to Gibbs phenomena, once the solution is obtained by use of a Fourier basis functions representations (Colaço and Alves, 2015).

Inverse heat conduction problems are typically ill-posed, so they cannot be solved directly. The most well-known methods used to overcome this problem are: function specification methods, iterative methods, methods based on filtering properties and regularization techniques (Alifanov, 1994; Beck and Arnold, 1977; Beck, et al., 1985; Tikhonov and Arsenin, 1977). Recently, Knupp and Abreu (2016) proposed a regularization method based in truncated eigenfunctions expansions in order to recover a boundary heat flux using non-intrusive measurements. In this present work, that same regularization approach is employed for the inverse problem of thermal contact conductance estimate.

## 2. PHYSICAL PROBLEM AND MATHEMATICAL FORMULATION

In this work we consider a 1D transient heat transfer problem in a plate with two layers, with the thermal diffusivities $\alpha_1$ and $\alpha_2$. A prescribed heat flux *q(t)*, is applied on the top surface and the bottom surfaces of this plate exchanges heat by convection with the surrounding environment, as illustrated by Fig. 1. We consider the existence of a thermal contact conductance with time dependence, $h_c(t)$, at the interface $L_c$, between the layers.

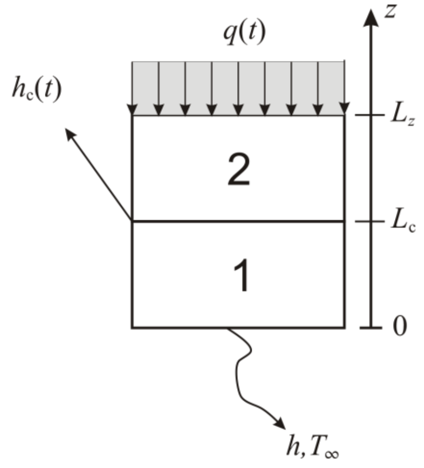



Figure 1 – Schematic of the physical problem

Representing the ambient temperature by $T_\infty$ and considering $k_1$ and $k_2$ as the thermal conductivities of the each material, the mathematical formulation of this problem, can be written as (Ozisik, 1993):

$$\frac{1}{\alpha_1}\frac{\partial T_1(z,t)}{\partial t}=\frac{\partial^2 T_1(z,t)}{\partial z^2} \qquad \text{in } 0<z<L_c \text{ and } t>0 \qquad (1.a)$$

$$\frac{1}{\alpha_2}\frac{\partial T_2(z,t)}{\partial t}=\frac{\partial^2 T_2(z,t)}{\partial z^2} \qquad \text{in } L_c<z<L_z \text{ and } t>0 \qquad (1.b)$$

$$-k_1\frac{\partial T_1(z,t)}{\partial z}+hT_1(z,t)=hT_\infty \qquad \text{at } z=0 \text{ and } t>0 \qquad (1.c)$$

$$k_1\frac{\partial T_1(z,t)}{\partial z}=h_c(z,t)[T_2(z,t)-T_1(z,t)] \qquad \text{at } z=L_c \text{ and } t>0 \qquad (1.d)$$

$$k_1\frac{\partial T_1(z,t)}{\partial z}=k_2\frac{\partial T_2(z,t)}{\partial z} \qquad \text{at } z=L_c \text{ and } t>0 \qquad (1.e)$$

$$k_2\frac{\partial T_2(z,t)}{\partial z}=q(t) \qquad \text{at } z=L_z \text{ and } t>0 \qquad (1.f)$$

$$T_1(z,t)=T_2(z,t)=T_\infty \qquad \text{in } 0<z<L_z \text{ and } t=0 \qquad (1.g)$$

Considering $L_1 = L_c$ and $L_2 = (L_z - L_c)$, constant thermal properties and assuming a plate sufficiently thin, the problem can be written in terms of the transversally averaged temperature field, using the Lumped analysis, defined as (Cotta and Mikhailov, 1997):

$$T_{m1}(t)=\frac{1}{L_1}\int_0^{L_c} T_1\left(z,t\right)dz \qquad (2.a)$$

$$T_{m2}(t)=\frac{1}{L_2}\int_{L_c}^{L_z} T_2\left(z,t\right)dz \qquad (2.b)$$

Applying the above operators in Eq.(1.a) for the temperature of the top plate, results in Eq.(3.a) and for the temperature of the lower top results in Eq.(3.b) (Cotta and Mikhailov, 1997).

$$\frac{1}{\alpha_1}\frac{dT_{m1}(t)}{dt}=\frac{1}{L_1}\frac{h[T_\infty-T_1(t)]}{k_1}-\frac{1}{L_1}\frac{h_c(t)[T_2(t)-T_1(t)]}{k_1}, \qquad t>0 \tag{3.a}$$

$$\frac{1}{\alpha_2}\frac{dT_{m2}(t)}{dt}=\frac{1}{L_2}\frac{h_c(t)[T_2(t)-T_1(t)]}{k_2}+\frac{1}{L_2}\frac{q(t)}{k_2}, \qquad t>0 \tag{3.b}$$

$$T_{m1}(t)=T_{m2}(t)=T_\infty, \qquad t=0 \tag{3.c}$$

Considering the Classical Lumped analysis, $T_2(L_c,t)=T_{m2}(t)$ and $T_1(L_c,t)=T_{m1}(t)$ (Cotta and Mikhailov, 1997), the equations (3a-c) can be written as:

$$\frac{1}{\alpha_1}\frac{dT_{m1}(t)}{dt}=\frac{1}{L_1}\frac{h[T_\infty-T_{m1}(t)]}{k_1}-\frac{1}{L_1}\frac{h_c(t)[T_{m2}(t)-T_{m1}(t)]}{k_1}, \qquad t>0 \tag{4.a}$$

$$\frac{1}{\alpha_2}\frac{dT_{m2}(t)}{dt}=\frac{1}{L_2}\frac{h_c(t)[T_{m2}(t)-T_{m1}(t)]}{k_2}+\frac{1}{L_2}\frac{q(t)}{k_2}, \qquad t>0 \tag{4.b}$$

$$T_{m1}(t)=T_{m2}(t)=T_\infty, \qquad t=0 \tag{4.c}$$

The direct problem is solved using an iterative method, using the Runge-Kutta Method and the Finite Differences Method. In the next section, will be discussed the method proposed for the estimation of the thermal contact conductance.

## 3. INVERSE PROBLEM FORMULATION

The inverse problem is formulated in order to recover the thermal conductance $h_c(t)$ using non-intrusive available at the top surface. Thus, from Eq.(4.b), the thermal contact conductance with temporal variation can be explicitly written as:

$$h_c(t)=\frac{\frac{k_2L_2}{\alpha_2}\frac{dT_{m2}(t)}{dt}-q(t)}{T_{m2}(t)-T_{m1}(t)} \tag{5}$$

where the bottom temperature is obtained with the equation (4.a). The thermal contact conductance will be obtained using an iterative form, considering an initial guess $h_c$ = 0. Therefore, assuming a vector containing the transient measurements taken on top surface, $\boldsymbol{\theta}=\{\theta_1, \theta_2,..., \theta_{Nt}\}$, and considering a central finite difference approximation to represent the derivative in Eq. (5), the following expressions can be obtained for the thermal contact conductance:

$$h_c(t_n)=\frac{k_2L_2\theta_{m2}(t_{n+1})-k_2L_c\theta_{m2}(t_{n-1})-2\alpha_2\Delta t q(t_n)}{2\Delta t\alpha_2[\theta_{m2}(t_n)-T_{m1}(t_n)]} \tag{6.a}$$

and

$$\frac{1}{\alpha_1}\frac{dT_{m1}(t)}{dt}=\frac{1}{L_1}\frac{h[T_\infty-T_{m1}(t)]}{k_1}-\frac{1}{L_1}\frac{h_c(t)[\theta_{m2}(t)-T_{m1}(t)]}{k_1} \tag{6.b}$$

with $n$ = 2, …, $N_t-1$, where $n$ is the number of transient measurements. The initial temperatures, at discrete time $n$=1, are:

$$T_{m1}(t_{n=1}) = T_{m2}(t_{n=1}) = T_\infty \tag{6.c}$$

The inverse heat conduction problems are ill-posed problems and the noise present in the experimental data are amplified into the estimative of the thermal contact conductance. In general, explicitly solutions like the present approach have great sensitivity to these input data variations (Alifanov, 1994; Beck and Arnold, 1977; Beck, et al., 1985; Tikhonov and Arsenin, 1977). In order to bypass these difficulties, many regularization methods, based on the processing of experimental data, have been suggested and validated in literature including the Tikhonov regularization, the truncated singular value decomposition, among others (Alifanov, 1994; Beck and Arnold, 1977; Beck, et al., 1985; Tikhonov and Arsenin, 1977). In this work, the integral transform method is employed for the regularization of non-intrusive measurements (Knupp and Abreu, 2016). Considering $t_f$ the total time of all measurements, the regularization is carried out from the use of eigenvalues and eigenfunctions according to Eq.(7a-c). Hence, consider the following eigenfunction expansion to represent the experimental data (Knupp and Abreu, 2016):

$$\hat{\theta}(t) = \sum_{m=1}^{M} \frac{1}{N_m} \bar{\theta}_m \psi_m(t) \tag{7.a}$$

with:

$$N_m = \int_0^{t_f} |\psi_m(t)|^2 dt \tag{7.b}$$

where the expansion coefficients, $\bar{\theta}_m$ , are:

$$\bar{\theta}_m = \int_0^{t_f} \psi_m(t)\theta(t)dt \tag{7.c}$$

The following 1D Sturm-Lioville problem is employed for obtain the eigenfunctions $\psi_m(t)$ :

$$\frac{d^2\psi_m(t)}{dt^2} + \mu_m{}^2\psi_m(t) = 0 \tag{8.a}$$

$$\psi'_m(0) = 0 ; \qquad \psi'_m(t_f) = 0 \tag{8.b,c}$$

which allows an explicit analytical solution. Using the discrepancy principle, we obtain the amount of suitable transformed modes that must be used to obtain an feasible approximation (Knupp and Abreu, 2016), according to Eq.(9).

$$\sigma^2{}_{reg} = \frac{1}{N_t} \sum_{i=1}^{N_t} [\hat{\theta}(t_i) - \theta(t_i)]^2 \tag{9}$$

After the measurements regularization, this new vector of the data is used of the solution of the inverse problem. Thus, rewritten the equations (6.a-c), using the filtered measures:

$$h_c(t_n) = \frac{k_2 L_2 \hat{\theta}_{m2}(t_{n+1}) - k_2 L_2 \hat{\theta}_{m2}(t_{n-1}) - 2\alpha_2 \Delta t q(t_n)}{2\Delta t \alpha_2 [\hat{\theta}_{m2}(t_n) - T_{m1}(t_n)]} \tag{10.a}$$

and

$$\frac{1}{\alpha_1}\frac{dT_{m1}(t)}{dt} = \frac{1}{L_1}\frac{h[T_\infty - T_{m1}(t)]}{k_1} - \frac{1}{L_1}\frac{h_c(t)[\hat{\theta}_{m2}(t) - T_{m1}(t)]}{k_1} \tag{10.b}$$

with $n = 2, \ldots, N_t - 1$, where n is the number of transient measurements. The initial temperatures, at discrete time $n$=1, are $T_{m1}(t_{n=1}) = T_{m2}(t_{n=1}) = T_\infty$ .

For the estimation of the desired parameter, an iterative process is made using a initial estimate of the coefficient $h_c(t_n)$ and using the Eq.(10.b) is calculated a temperature function for the bottom layer, in this case, $T_{m1}(t_n)$ . Then these function is used in Eq.(10.a) with the objective to find a new coefficient $h_c(t_n)$. This iterative process is done until a predetermined stopping criterion is found, so that the function of thermal conductance contact is found as close as possible to the true one.

## 4. RESULTS AND DISCUSSION

Experimental measurements were simulated by considering the addition of noise draw from a normal distribution with zero mean and constant standard deviation, $\sigma_e$. Considering the first case studied in Colaço and Alves (2015) in order to compare the proposed formulation with another analytical solution, the heat transfer coefficient at the bottom surfaces of the plate represented in Fig. 1 were taken as $10^{10}$ W/m$^2$K. We suppose a standard deviation of noise $\sigma_e$=0.2°C, and both layers of the plate are manufactured in AISI 1050 steel (thermal conductivity $k$=54W/mK and volumetric heat capacity $C_p$=3.66×10$^6$J/m3K). The imposed heat flux $q$ was set to 10,000W/m$^2$, the ambient temperature $T_\infty$ =25°C and the final time was $t_f$ = 600s. According to the discrepancy principle (Morozov, 1966), a good choice for the truncation order would be one such that (M=55, see Figure 2(a)). In Figure 2(b) are shown the results for the studied case using the regularization method proposed by Knupp and Abreu (2016).

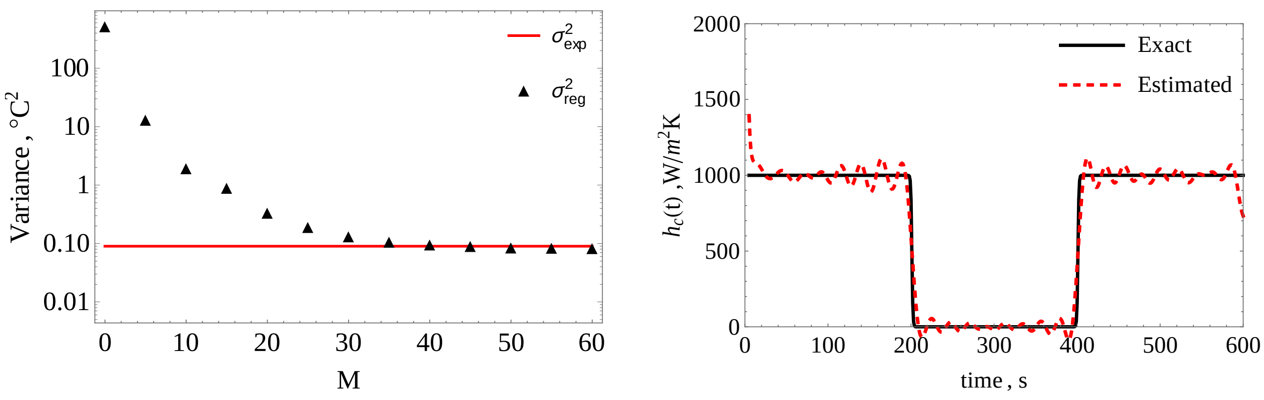
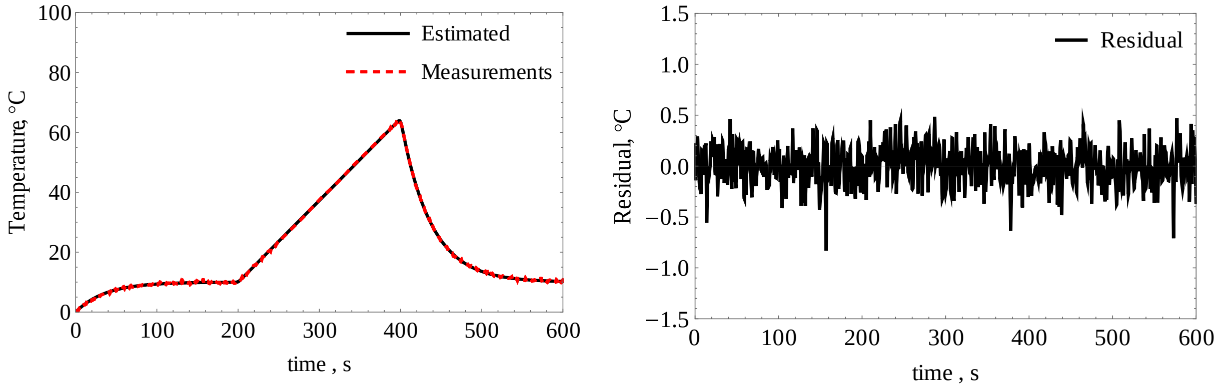



Figure 2: (a) Variance of the experimental measurements with respect to the regularized representation with $\sigma_e$ = 0.2°C; (b) Results of estimative of thermal contact conductance with $\sigma_e$ = 0.2°C .




(a) (b)

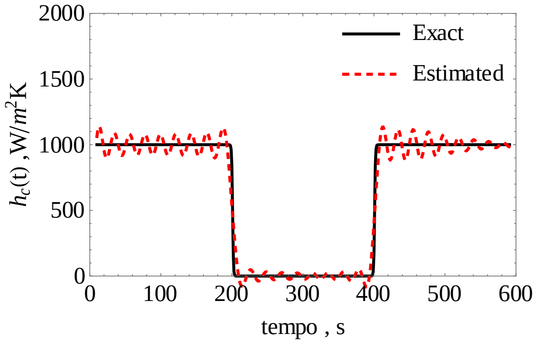
Figure 3: Residue of temperatures for $\sigma_e$ = 0.2°C.

Figure 3.a presents the temperature profile, estimated and experimental, confirming the model predictions are well fitted to the measurements and, confirming the low magnitude of the residuals and no significant signature in Fig 3.b. In this first case, was considered the problem studied by Colaço and Alves (2015), where the temperature at the bottom surface was prescribed and equal to the ambient temperature. Thus, in case 1, the method was capable to recover the thermal contact conductance on the first iteration. However, at the next case, we will consider the heat transfer coefficient at the bottom surfaces $h$ = 15 W/m$^2$K. For the case 2, two different profiles for the unknown thermal contact conductance were considered, according to Table 1.

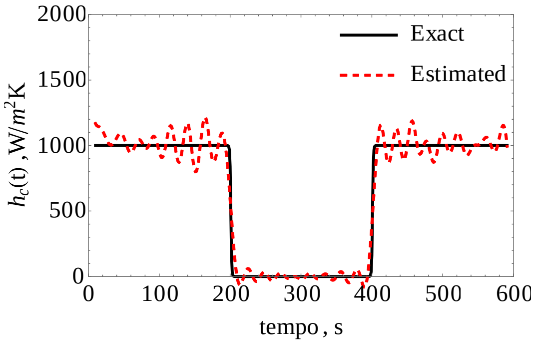
Table 1 – Functions used for the thermal contact conductance case 2.

| Profile | $h_c$ (W/m$^2$K) |
|---|---|
| 1 | $\begin{cases} 1000 & \text{for} \quad (t < t_f/3) \text{ and } t > (2t_f/3) \\ 0 & \text{for} \quad (t_f/3) < t < (2t_f/3) \end{cases}$ |
| 2 | $\begin{cases} 1000 & \text{for} \quad (0 < t < t_f/4) \\ 500 & \text{for} \quad (2t_f/4 < t < 3t_f/4) \\ 0 & \text{for} \quad \text{otherwise} \end{cases}$ |

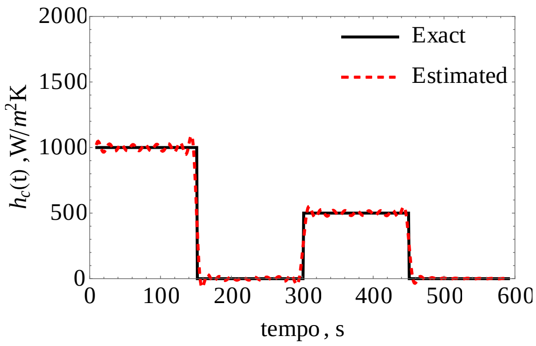
Considering the case 2, all results were obtained using 20 iterations and for all cases presented in this work the residuals were uncorrelated and very small. The Fig. 2 shows the estimated thermal contact conductance at the interface resulting from the three thermal contact conductance presented in Table 1, considering measurements without noise and with a noise considering a standard deviation equal to σ=0.2°C. The results are considered very good in all results, although some oscillations are present, mainly due to the Fourier series used on the regularization method. This methodology can also be extended in the future to other more complex physical problems, with spatial variations and where the thickness is large.



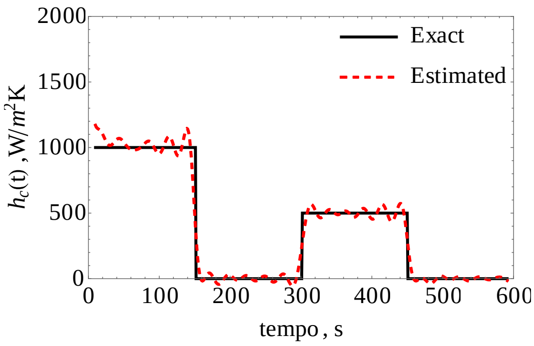

(a) Profile 1




(b) Profile 1



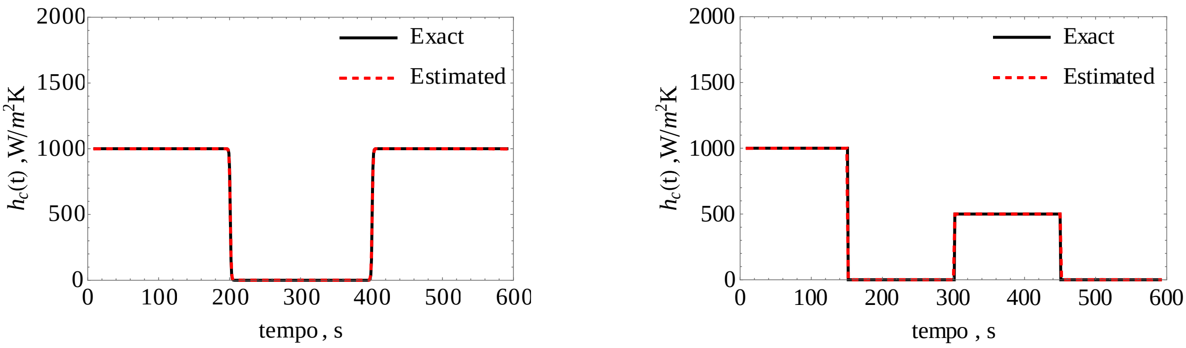

(c) Profile 2




(d) Profile 2

Figure 4 - Exact and estimated variation in thermal contact conductance (W/m$^2$K) at $L_c$ for errorless measurements (left) and for measurements containing noise with σ=0.2°C (right).

Figures 4.a and 4.b, shows the results when the standard deviation of the measurements tends to zero. In these cases, considering the Morozov Principle, the exact function can be recovered if an infinite number of transformed modes can be used. The results in Figures 4.a and 4.b were obtained using only 50 transformed modes, M=50. The Fig. 5 shows a case using M=300.




(a) (b)

Figure 5 - Exact and estimated variation in thermal contact conductance (W/m$^2$K) at $L_c$ for errorless measurements: (a) for the profile 1 and (b) for profile 2.

## 5. CONCLUSION

An explicit inverse problem solution for thermal contact conductance reconstruction in thermally thin plates was developed and verified. A regularization approach, using truncated eigenfunction expansions, was successful applied and analyzed in: two boundary cases, two different functions and considering different noise levels. The good results obtained with very little computational effort demonstrate the feasibility of the method in a composite medium with thin layers.

## 6. ACKNOWLEDGEMENTS

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