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CONTROLLER BY MIXED-SENSITIVITY $\mathcal{H}_\infty$ DESIGN (S/KS/T) FOR LURIE TYPE SYSTEMS

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Abstract. *The goal of this work is to present a methodology for the design of controllers by mixed-sensitivity $\mathcal{H}_\infty$ (S/KS/T) for Lurie type systems. The Lurie problem originally searched for necessary and sufficient conditions for absolute stability. In this work, on the other hand, we are not only concerned with absolute stability problem, but also with performance and robustness of performance of the system. In this sense, we show how to get a controller by the mixed sensitivity technique for Lurie type systems with a single input. Next, we present an example and perform numerical simulations that verify that the suggested controller provides robustness of stability and performance.*

Keywords: *Lurie problem, absolute stability, robust control, mixed-sensitivity, Hopfield neural network*

1. INTRODUCTION

Halfway through the 1940s, due to a problem of automatic control of an aircraft, the Lurie problem appeared, which is also known in the literature as absolute stability problem. Leaving important contributions to mathematics and engineering, it is considered that it laid the foundations for an important area of the control engineering, that is Robust Control.

In searching for answers to this problem, many researchers have spent years looking for necessary and sufficient conditions for the global asymptotic stability of systems of Lurie type. Among these researchers, we can mention: Aizerman (1947), with the well known Aizerman conjecture, Krasovskii (1953), Popov (1961), and Kalman (1963).

From the 80's, there was a significant jump in the number of publications about the problem, where it was verified the possibility of application in other areas, for example: neural networks (Forti *et al.*, 1994) and (Kaskurewicz and Bhaya, 1995); convex approach (Gapski and Geromel, 1994); and chaos and chaos synchronization (Liao and Yu, 2008).

Furthermore, the Lurie problem is a particular type of robust control problem. In this work, based on suggestions of Pinheiro and Colón (2017), we develop a linear controller by the mixed-sensitivity technique. The mixed-sensitivity (S/T/SK) technique (Zhou *et al.*, 1995; Skogestad and Postlethwaite, 2007) aims to obtain a controller that minimizes the norm $\mathcal{H}_\infty$ of a matrix. In general, techniques for designing robust controllers using parametric uncertainty and structured singular value (SSV) have been extensively explored since the 1980s (Doyle, 1982).

The Lurie problem is considered solved for systems with a single input and a single output (Popov, 1961), but for multiple inputs the problem remains open. On the other hand, important steps were given in this sense, for example the use of LMI techniques (Zeng *et al.*, 2014).

In this paper, we present the design of a mixed-sensitivity controller for a Lurie type system, that is an artificial neural network (see Pinheiro and Colón (2017)), thus relating to areas such as neurodynamics and neuroscience. In section 2, the theoretical formulation of the Lurie type problems is presented. In section 3, the mixed-sensitivity technique is presented. In section 4, a neural network example and its controller is presented, and in section 5, the corresponding simulations are presented. Finally, section 6 concludes the paper.

2. THE LURIE PROBLEM

In 1944, Lurie was imbued to solve a problem of stability for the automatic control system of an aircraft (Lurie and Postnikov, 1944). Based on this system and the analysis of the stability of the null solution of the system (that is the equilibrium point), the objective was to find necessary and sufficient conditions for the global asymptotic stability of the system.

In Fig. 1, it is shown the Lurie type system in block diagram, where the linear dynamics is represented by the system

L, and the function $f(\sigma)$ represents a nonlinearity.

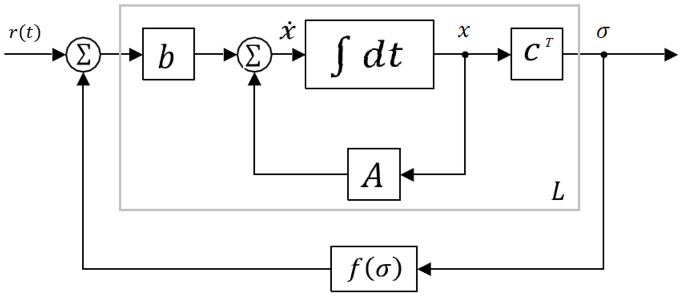

Figure 1. Block diagram of the Lurie type system.

In general, the nonlinearity $f(\sigma)$ is a continuous function restricted to the first and third quadrants of the plane, that belongs to one of the following families:

$$
\begin{gathered}
F_{(0,k]} := \{f|f(0) = 0, 0 < f(\sigma) \leq k\sigma, \sigma \neq 0\}, \\
F_{(0,k)} := \{f|f(0) = 0, 0 < f(\sigma) < k\sigma, \sigma \neq 0\}, \\
F_{[k_1,k_2]} := \{f|f(0) = 0, k_1\sigma \leq f(\sigma) \leq k_2\sigma, \sigma \neq 0\}, \\
F_\infty := \{f|f(0) = 0, f(\sigma) > 0, \sigma \neq 0\}.
\end{gathered}
$$

In Fig. 2, we have some graphical representations of this kind of nonlinearity. Considering $r = 0$, the diagram in Fig. 1 is expressed by the following system of differential equations, which was known in the literature as a Lurie type system:

$$
\begin{cases} \dot{x} = Ax + bf(\sigma) \\ \sigma = c^T x \end{cases} \tag{1}
$$

where $x \in R^n$ is the state vector, and $b, c \in R^n$, $A \in R^{nxn}$ are fixed matrices and $\sigma f(\sigma) > 0$.

We can formulate the Lurie's problem with the following question: What are the necessary and sufficient conditions for the equilibrium point of the system of Eq. (1) be globally asymptotically stable? In a more formal way, we can use the following definition:

Definition 1 *If the zero solution of the system of the Eq.(1) is globally asymptotically stable for $f(\sigma) \in F_\infty$, then we say that the zero solution of the system of the Eq. (1) is absolutely stable for F_∞.*

Thus, considering the system in Eq.(1) and definition 1, we have the formulation for the Lurie Problem with a single control, than can also be formulated for the cases of $F_{(0,k]}$, $F_{(0,k)}$ and $F_{[k_1,k_2]}$.

Finally, the Lurie problem can be extended to the case of multiple controls (inputs), i.e. with m nonlinear controls:

$$
\begin{cases} \dot{x} = Ax + \sum_{j=1}^{m} b_j f_j(\sigma_j) \\ \\ \sigma_j = c_j^T x = \sum_{i=1}^{n} c_{ij} x_i, \quad j = 1, ..., m, \end{cases} \tag{2}
$$

where $A \in R^{nxn}$, $x = (x_1, ..., x_n)^T$, $b_j = (b_{1j}, ..., b_{nj})^T$, $c_j = (c_{1j}, ..., c_{nj})^T$, $f_j \in F$, and $Re\lambda(A) \leq 0$.

Notice that the functions f_j can be not exactly known, and belonging to an uncertain family, which explains why the Lurie problem can be considered one of the founders of robust control.

3. MIXED-SENSITIVITY $\mathcal{H}_\infty$ (S/KS/T) METHOD

Consider the following plant:

$$
y = Gu + G_d d, \tag{3}
$$

where y is the vector of outputs, u is the vector of control signals, d is the vector of disturbances, and G and G_d are matrices of transfer functions. The block diagram of the control system to be designed is presented in Fig. 3. The output y can be expressed by:

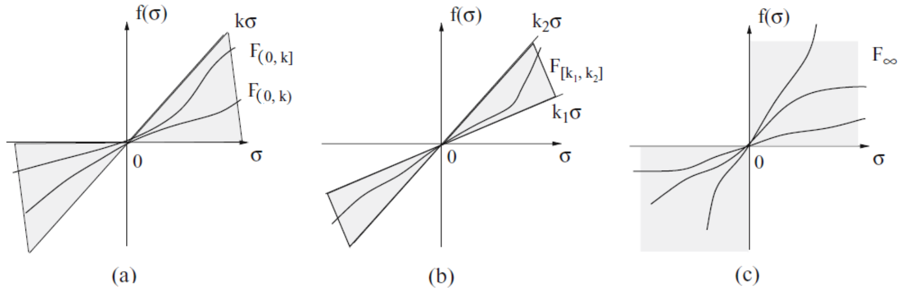

Figure 2. Functions types f: a) $F_{(0,k]}$, $F_{(0,k)}$; b) $F_{[k_1,k_2]}$; c) F_∞.

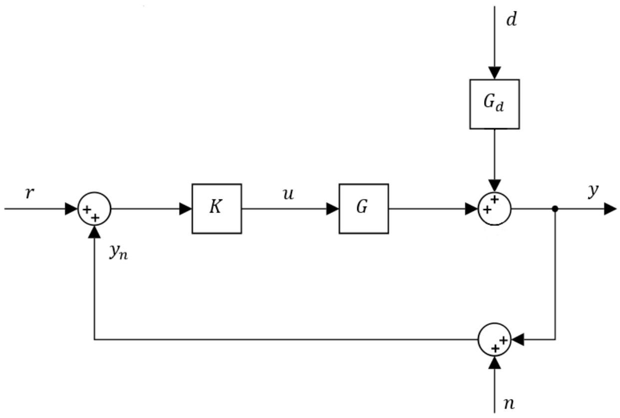

Figure 3. Control problem block diagram.

$$y = (I + GK)^{-1}GKr + (I + GK)^{-1}G_d d - (I + GK)^{-1}GKn. \tag{4}$$

From Eq. (4), $S = (I+GK)^{-1}$ is the sensitivity function and $T = (I+GK)^{-1}\,GK$ is the complementary sensitivity function. We intend to design a controller by mixed-sensitivity $\mathcal{H}_\infty$ (S/KS/T) which the goal is to minimize the $\mathcal{H}_\infty$ norm of Eq. (5):

$$N = \begin{bmatrix} W_p S \\ W_u KS \\ W_I T \end{bmatrix}, \tag{5}$$

where W_p, W_u and W_I are weight functions. The weight W_I is used for modeling the uncertainty and is given by the uncertainty model of the family of plants. The weights W_u and W_p are chosen by the designer in order to reflect the desired specifications for the closed loop system. W_u is used for penalize high controls. The weight W_p was chosen to be (Skogestad and Postlethwaite, 2007):

$$W_p = \frac{s/M + w_b}{s + w_b A}. \tag{6}$$

where:

ω_b: minimum bandwidth frequency (defined as the frequency where $S(j\omega)$ crosses from below).

A: parameter for $S(s)$ to be small in low frequency, this causes a small stationary error, doing $A \leq 1$.

M: maximum peak magnitude of S, $||S(j\omega)||_\infty$. Normally $M = 2$.

For the modeling of uncertainty, Skogestad and Postlethwaite (2007) provides several options for the designer to choose the type of uncertainty that is most convenient for the project. In the Fig. 4 we have the uncertainty model known as inverse multiplicative input, according with Eq. (7).

$$G_p(s) = \frac{G(s)}{1 + W_I(s)\Delta_I(s)}, \quad ||\Delta_I||_\infty \leq 1. \tag{7}$$

The transfer function $G(s)$ is the nominal model of the plant and $G_p(s)$ represents the disturbed plant, that is, with the uncertainty inserted. Δ denotes the normalized perturbation with the $\mathcal{H}_\infty$ norm less or equal than 1.

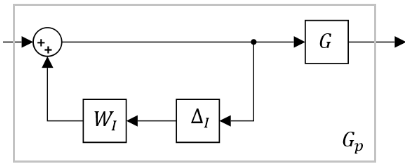

Figure 4. Inverse multiplicative input uncertainty.

In Fig. 5 we present the extended plant, including the weights W_p, W_I e W_u.

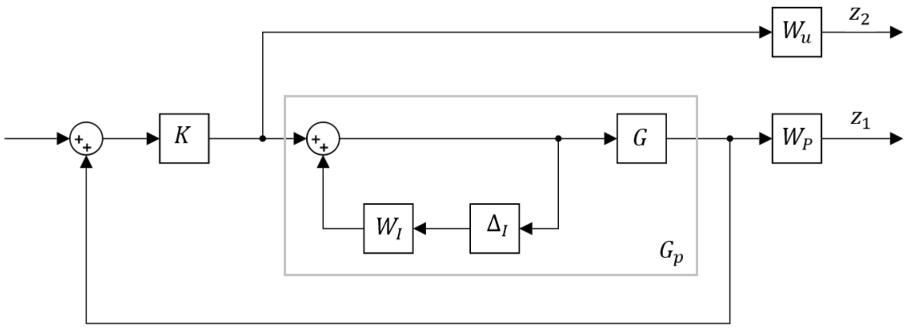

Figure 5. Control diagram with weights.

To obtain the robustness stability condition, consider $L = GK$ and analyze Fig. 5. We have

$$L_p = G_p K = \frac{G(s)K}{1 + W_I(s)\Delta_I(s)}$$

We assume that L_p is stable, and nominal stability for the closed-loop system. We will have robust stability if we do not have encirclements by the Nyquist diagram of L_p around the point -1 (see Fig. 6), and since L_p is in a norm-bounded set, one has

$$|1 + L_p| > 0, \quad \forall L_p, \quad \forall\omega \quad \Rightarrow \quad |1 + L(1 + W_I\Delta_I)^{-1}| > 0, \quad \forall|\Delta_I| \leq 1, \quad \forall\omega,$$

$$\Rightarrow \quad |1 + W_I\Delta_I + L| > 0, \quad \forall|\Delta_I| \leq 1, \quad \forall\omega.$$

The last condition is most easily broken (the worst case) when Δ_I is selected at each frequency so that $|\Delta_I| = 1$ and the term $1 + L$ and $W_I\Delta_I$ has opposite signs. Like this

$$|1 + L| - |W_I| > 0, \quad \forall\omega,$$

$$|W_I S| < 1, \quad \forall\omega.$$

Therefore, the robust stability condition is

$$|S| < \frac{1}{|W_I|} \tag{8}$$

We emphasize that we derived the condition of Eq.(8) for inverse multiplicative input uncertainty. However, for other types of uncertainty we also can have the condition:

$$|T| < \frac{1}{|W_I|}. \tag{9}$$

To derive the nominal performance condition , we observe Fig. 7. $|1 + L|$ represents at each frequency the distance of $L(j\omega)$ from the point -1 in the Nyquist diagram, so $L(j\omega)$ must be at least a distance of $|W_p(j\omega)|$ from -1. Thus, conform Fig. 7, we see that for NP, $L(j\omega)$ must stay outside a disc of radius $|W_p(j\omega)|$ centered on -1.

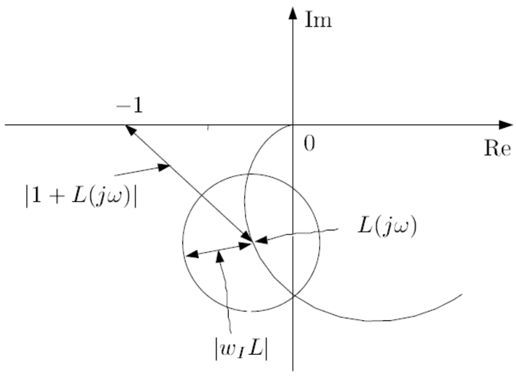

Figure 6. Nyquist diagram for robust stability.

Then:

$$|W_p| < |1 + L|, \quad \forall\omega, \Rightarrow \quad |W_p S| < 1, \quad \forall\omega.$$

Therefore, the condition of nominal performance is

$$|S| < \frac{1}{|W_p|} \tag{10}$$

In order to have robustness performance (RP), we must require that all possible $L_p(j\omega)$ stay outside a disc of radius $|W_p(j\omega)|$ centered on -1. Since L_p at each frequency stays within a disc of radius $W_I L$ centered on L. We can see from Fig. 8 that the condition for (RP) is that the two discs, with radii W_p and W_I, do not overlap. Since their centers are located a distance $|1 + L|$ apart, we have

$$|W_p| + |W_I L| < |1 + L|, \quad \forall\omega, \Rightarrow \quad |W_p(1 + L)^{-1}| + |W_I L(1 + L)^{-1}| < 1, \quad \forall\omega.$$

Therefore, the robust performance condition becomes

$$|W_p S| + |W_I T| < 1. \tag{11}$$

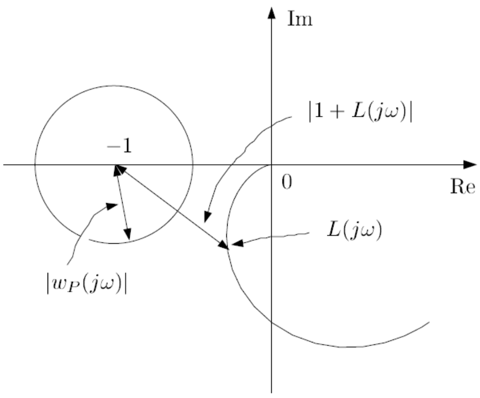

Figure 7. Nyquist for nominal performance.

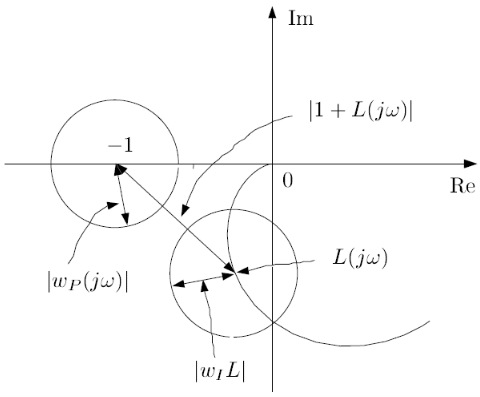

Figure 8. Nyquist for robust performance.

The nominal performance specification in Eq. (10) places an upper bound on the bandwidth. The conditions in Eq. (8) or Eq. (9) put an upper bound on the magnitude to make sure that L rolls off sufficiently fast at high frequencies. It is also possible to place a specification, $1/|W_u|$, on the magnitude of KS to restrict the size of the input signals, $u = KS(r - G_d d)$. Thus, we have a mix of combinations, which is the reason for the name mixed-sensitivity problem. The transfer matrix relating the inputs (reference and disturbance) to the performance outputs for the extended plant and its norm are:

$$||N||_\infty = \underset{\omega}{max}\, \gamma(N(j\omega)) < 1; \quad N = \begin{bmatrix} W_p S \\ W_u KS \\ W_I T \end{bmatrix}. \tag{12}$$

where γ is a superior limit to the norm, used here to measure the size of the matrix N at each frequency. For SISO systems, N is a vector and $\gamma(N)$ is the usual Euclidean vector norm:

$$\gamma(N) = \sqrt{|W_p S|^2 + |W_I T|^2 + |W_u K S|^2}. \tag{13}$$

In this procedure, the specifications are like buttons for the engineers to select the best adjustment. The optimal controller $\mathcal{H}_\infty$ is obtained by solving the problem

$$\min_K ||N(K)||_\infty, \tag{14}$$

where K is a stabilizing controller.

4. CONTROLLER FOR LURIE TYPE SYSTEMS

Over the years, many necessary and sufficient conditions for the absolute stability of type Lurie have been obtained (Pinheiro, 2015; Liao and Yu, 2008). Now suppose that we want to improve the stability and performance of a Lurie type system. The block diagram of the closed loop system is represented in Fig. 9.

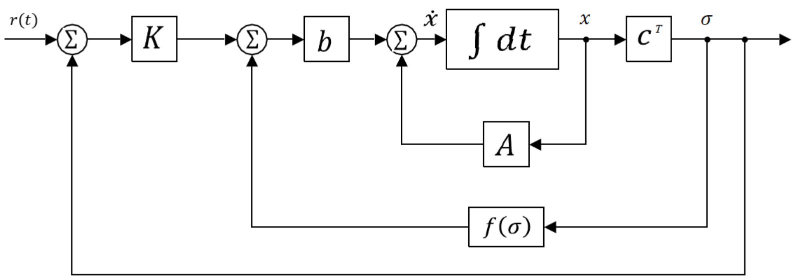

Figure 9. Block diagram of the type Lurie system.

In order to transform this block diagram in the form of Fig. 5, we have to replace the nonlinearity f by a parametric uncertainty. The resulting system will be an equivalent family of plants G_p.

Let $f \in F(0, k]$, which is presented in Fig. 10. The Lurie idea is to replace f by a set of linear functions $k\sigma$, Fig. 11, k is the uncertainty.

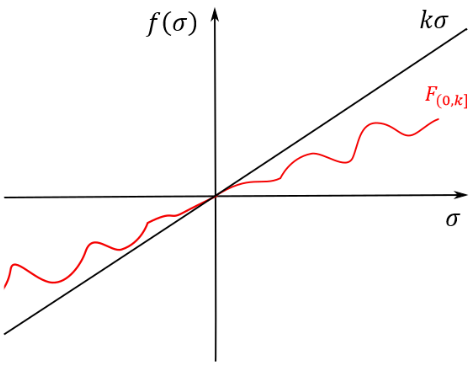

Figure 10. Nonlinear function $f(\sigma)$.

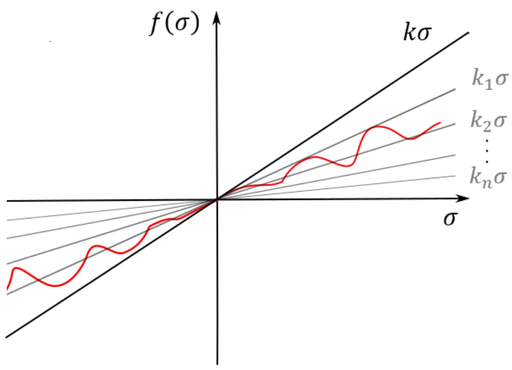

Figure 11. Mapping $0 < f(\sigma) \leq k\sigma$.

In fact, we are mapping any nonlinearity according to this constraint in the first and third quadrants. This makes sense for Lurie's problem, because the function $f(\sigma)$ is not known, only it is known that it is between the first and third quadrants. For more information on linear spaces and mapping see (Dullerud and Paganini, 2013).

In practice, the block $f(\sigma)$ can be replaced by the uncertain block gain α, which could assume any value between 0 and k, that is, $0 < \alpha \leq k$. The resulting block diagram is shown in Fig. 12.

From the diagram of Fig. 12, we obtain the perturbed transfer function $G_p(s)$, as follows:

$$\dot{x} = Ax + b(r_1 - \alpha c^T x) \quad \rightarrow \quad \dot{x} = Ax + br_1 - b\alpha c^T x \quad \rightarrow \quad \dot{x} = (A - b\alpha c^T)x + br_1$$

Making $A_p = (A - b\alpha c^T)$ and $\alpha = \frac{k}{2} + \frac{k}{2}\delta$, $|\delta| \leq 1$, we have:

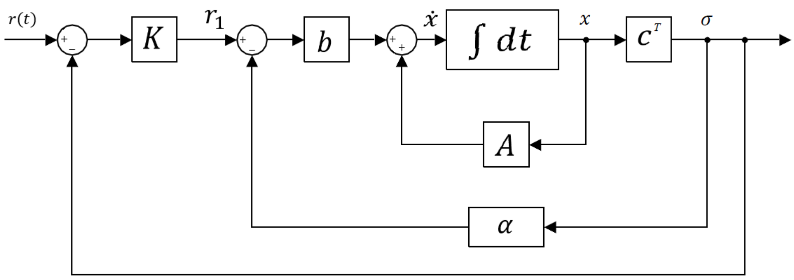

Figure 12. Block diagram of the Lurie type with α ranging from 0 to k.

$$\begin{cases} \dot{x} = A_p x + b r_1 \\ \sigma = c^T x \end{cases} \tag{15}$$

Therefore, the perturbed transfer function of the plant is given by:

$$G_p(s) = c^T(sI - A_p)^{-1}b. \tag{16}$$

The weight W_I can then be obtained and W_p and W_u are fixed by performance specification. For the synthesis of the controller we use the function of the Matlab $mixsyn(G, W_p, W_I, W_u)$. We are considering in the following example, a class of systems with one nonlinearity and the matrix A Hurwitz and $f(\sigma) \in F_{(0,k]}$.

Example 1 *Design a controller for the Lurie type system below by mixed-sensitivity $\mathcal{H}_\infty$ (S/KS/T).*

$$\begin{cases} \dot{x}_1 = -x_1 + f(\sigma) \\ \dot{x}_2 = -x_2 \\ \sigma = x_1 \end{cases}$$

In this example, we have:

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \textit{and} \quad c^T = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

By Eq. (16), we have $G_p(s) = c^T(sI - A_p)^{-1}b$, and A_p, is given by:

$$A_p = A - b\alpha c^T = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} \alpha \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

Replacing the uncertain parameter by $\alpha = \frac{k}{2} + \frac{k}{2}\delta$, $|\delta| \leq 1$, we have:

$$A_p = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} \frac{k}{2} + \frac{k}{2}\delta & 0 \end{bmatrix} = \begin{bmatrix} -1 - \frac{k}{2} - \frac{k}{2}\delta & 0 \\ 0 & -1 \end{bmatrix}.$$

Then:

$$(sI - A_p) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -1 - \frac{k}{2} - \frac{k}{2}\delta & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} s + 1 + \frac{k}{2} + \frac{k}{2}\delta & 0 \\ 0 & s+1 \end{bmatrix},$$

and,

$$(sI - A_p)^{-1} = \begin{bmatrix} \frac{1}{s+1+\frac{k}{2}+\frac{k}{2}\delta} & 0 \\ 0 & \frac{1}{s+1} \end{bmatrix}.$$

Finally,

$$G_p(s) = c^T(sI - A_p)^{-1}b = \frac{1}{s + 1 + \frac{k}{2} + \frac{k}{2}\delta}.$$

We then put $G_p(s)$ in the form of Fig. 4, which results in:

$$G_p(s) = c^T(sI - A_p)^{-1}b = \frac{1}{s + 1 + \frac{k}{2} + \frac{k}{2}\delta} = \frac{\frac{1}{s+1+\frac{k}{2}}}{1 + (\frac{\frac{k}{2}}{s+1+\frac{k}{2}})\delta} = \frac{G(s)}{1 + W_I(s)\Delta_I(s)}.$$

Where

$$G(s) = \frac{1}{s+1+\frac{k}{2}} \quad and \quad W_I(s) = \frac{\frac{k}{2}}{s+1+\frac{k}{2}}.$$

In the example, we adopt $k = 2$.

For $W_p(s) = \frac{s/M+w_b}{s+w_b A}$ we used the parameters $M = 2$, $A = 5.10^{-5}$ and $w_b = 10$. We chosen $A << 1$, for obtain a small stationary error. $M = 2$ was chosen according to the value suggested (Skogestad and Postlethwaite, 2007). We chose $w_b = 10$ in order to hav have a larger bandwidth, leaving the system response faster.

With the control u small, we have economy of control energy, and also does not cause disturbances in other parts of the system. Therefore, we chose $W_u = 1$, because in this problem it was not necessary to penalize very much the control in order to obtain the response desired.

Using the function $mixsyn(G, W_p, W_I, W_u)$ the controller obtained is

$$K = \frac{1054s^2 + 4172s + 4129}{s^2 + 6.447s + 0.003223} \tag{17}$$

The value of γ found was $\gamma = 1.2003$, where we can consider that it agrees with the specification of the Eq. (14).

5. SIMULATIONS

In simulations using the linear model with uncertainties, we have got the Bode diagrams of the conditions in Eq. (8) and Eq. (11), which can be checked in Fig. 13 and Fig. 14. We can see that the robustness of stability and performance are satisfied.

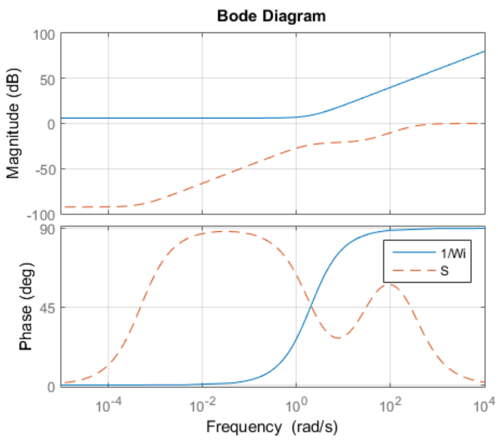

Figure 13. Robustness of Stability.

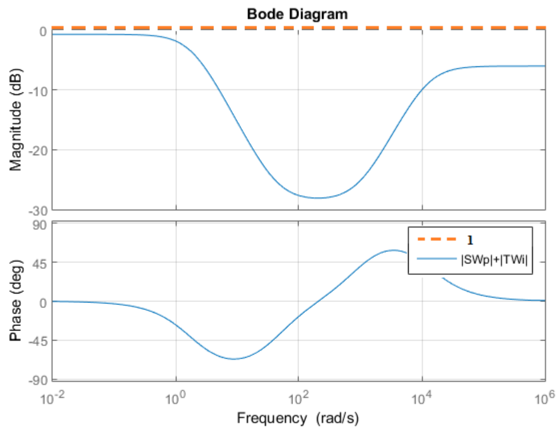

Figure 14. Robustness of Performance.

Figure 15 and Fig. 16 show the Bode diagram for the controller and controller times the sensitivity function.
Figure 17 shows the Bode diagram of the perturbed plant G_p for several randomly generated values for the uncertain parameters. In Fig. 18, we have the Bode diagram of KG_p (open loop).
A time simulation was performed using the nonlinear function as $f(\sigma) = tanh(\sigma)$ as presented in Fig. 19. In Fig. 20 the response to a sine reference with amplitude five is presented for the closed loop with unitary controller case (without controller - red dotted line) and the closed loop with the robust controller case (with controller - blue continuous line). We can see that the reference is not followed properly in the first case. A similar simulation was conducted for a pulse reference signal, which is shown Fig. 21. Again, the reference is not properly followed with unitary controller.

6. CONCLUSION

In this work, we developed a procedure for obtaining a controller by mixed-sensitivity $\mathcal{H}_\infty$ design (S/KS/T) for a class of SISO Lurie type systems. Simulations were conducted to validate the stability and performance robustness for the closed loop system.
In future works, we are planning to develop a procedure to obtain a controller for multiple inputs by means of more advanced $\mathcal{H}_\infty$ MIMO control techniques, like the μ synthesis. We intend to apply the method to a Hopfield neural

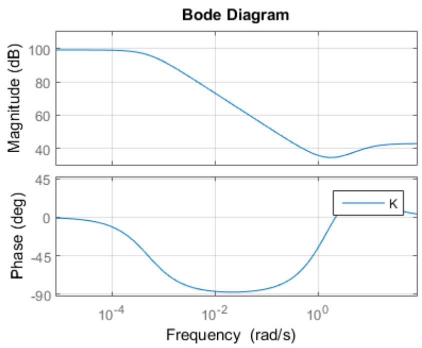

Figure 15. Bode diagram of the controller.

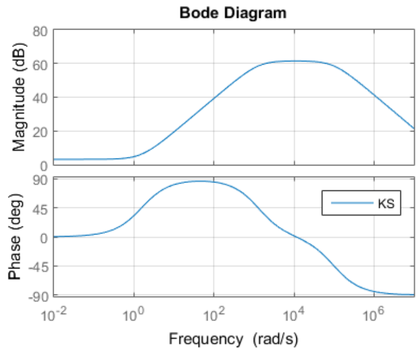

Figure 16. Bode diagram of the controller times the sensitivity function KS.

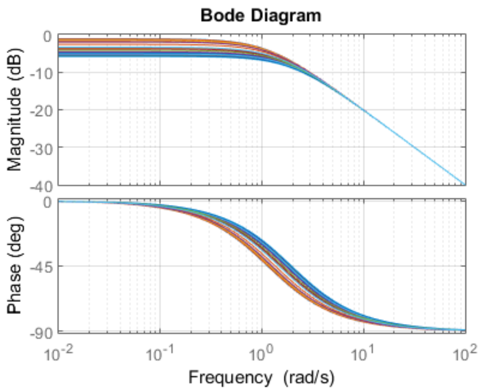

Figure 17. Family of plants G_p.

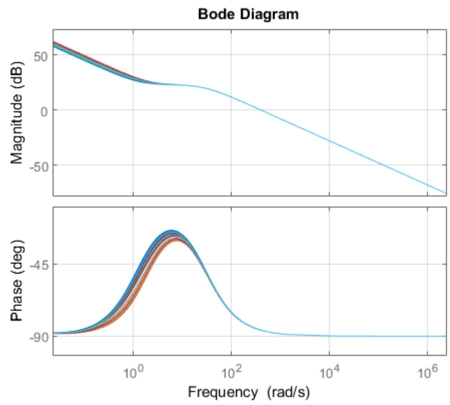

Figure 18. Bode diagram for plant G_p times the controller K.

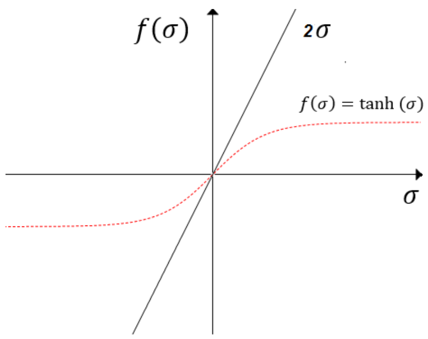

Figure 19. Nonlinearity function $f(\sigma) = tanh(\sigma)$.

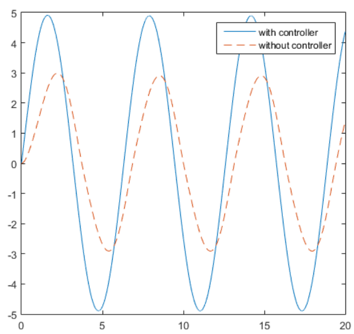

Figure 20. Response to sine wave.

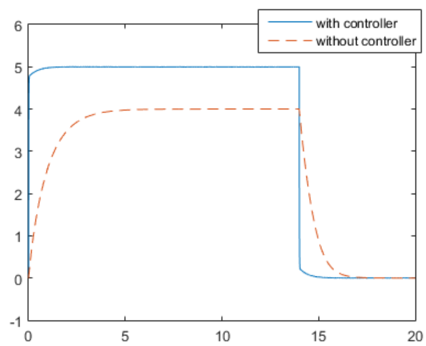

Figure 21. Response to pulse signal.

network, which is used to simulate associative memory. This memory can be considered analogue to the human memory, and so it can be used for modeling a neuropathology, for example the Alzheimer's disease. In this context, the controller could be used to assuage the effects of the disease.

7. REFERENCES

Aizerman, A., 1947. "On the effect of nonlinear functions of several variables on the stability of automatic control systems (in Russian)". *Autom. i Telemekh*, Vol. VIII, No. 1.

Doyle, J., 1982. "Analysis of feedback systems with structured uncertainty". *IEEE Proceedings*, Vol. D, No. 129, pp. 242–250.

Dullerud, G.E. and Paganini, F.G., 2013. *A Course in Robust Control Theory: a convex approach*, Vol. 36 of *Texts in Applied Mathematics*. Springer Science and Business Media, New York.

Forti, M., Manetti, S., Marini, M. and Liberatore, S., 1994. "On absolute stability of neural networks". *IEEE Trans. Circuits Syst.*, Vol. 6, pp. 241–244.

Gapski, P.B. and Geromel, J.C., 1994. "A convex approach to the absolute stability problem". *IEEE Trans. Automat. Contr.*, Vol. 25, pp. 613–617.

Kalman, R.E., 1963. "Liapunov functions for the problem of Lurie in automatic controls". *Proc. Natl. Acad. Sci*, Vol. 49, No. 8, pp. 201–205.

Kaskurewicz, E. and Bhaya, A., 1995. "Comments on necessary and sufficient condition for absolute stability of neural network". *IEEE Trans. Circuits Syst.*, Vol. 42, pp. 497–499.

Krasovskii, N.N., 1953. "On the stability of the solutions of a system of two differential equations (in Russian)". *Prikl. Mat. i Mekh.*, Vol. XVII, No. 6.

Liao, X. and Yu, P., 2008. *Absolute Stability of Nonlinear Control Systems*. 2nd edition. Springer.

Lurie, A.I. and Postnikov, V.N., 1944. "On the theory of stability of control systems (in Russian)". *Prikl. Mat. i Mekh.*, Vol. 8, No. 3, pp. 246–248.

Pinheiro, R.F., 2015. *The Lurie problem and applications to neural networks (in Portuguese)*. Master's thesis, University of São Paulo, Mathematics and Statistics Institute.

Pinheiro, R.F. and Colón, D., 2017. "An application of Lurie problem in Hopfield neural networks". In A.T. Fleury, D.A. Rade and P.R.G. Kurka, eds., *Proceedings of the XVII International Symposium on Dynamic Problems of Mechanics*, ABCM, São Paulo.

Popov, V.M., 1961. "Absolute stability of nonlinear systems of automatic control". *Remote Control*, Vol. 22, No. 8, pp. 857–875.

Skogestad, S. and Postlethwaite, I., 2007. *Multivariable Feedback Control: Analysis and Design*. 2nd edition. John Wiley and Sons.

Zeng, H.B., Ding, L., Xiao, S.P. and Yu, F., 2014. "Improved stabilization method for lurie networked control systems". *The Scientific World Journal*, Vol. 2014.

Zhou, K., Doyle, J.C. and Glover, K., 1995. *Robust and Optimal Control*. Pearson.

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