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A STUDY OF THE BOUNDARY CONDITION APPLIED IN DESCRIPTION OF CONVECTIVE DRYING OF GINGER

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Abstract. *The water quantity withdrawn from an agricultural product in a drying process is an important indicator. This importance is due to the fact that, depending on the fate that the industry intends for this product, high values of this indicator can be positive or negative. For an adequate study of the mass transfer in the drying process, a good description of this process is necessary. However a good description is directly related to applied mathematical modeling. The migration of mass from the product to the medium can be modeled by the diffusion equation. The present work presents a mathematical modeling that considers 20 of terms of the series that represents the analytic solution for the diffusion equation, besides admitting a boundary condition of the third kind. This type of approach is uncommon in the literature. The proposed modeling was applied in the description of ginger drying at temperatures of 40 to 70°C, with air velocity of 1.5 ms^{-1}. The results indicated a good fit of the proposed modeling to the experimental data. In addition, it was possible to conclude that the most adequate boundary condition to describe this type of process is of the third type.*

Keywords: *Optimization, Diffusivity, Diffusion equation, Analytical solution*

1. INTRODUCTION

In the agricultural industry, when it is intended to store a product or produce new products therefrom, drying becomes an important stage of processing. This importance is due to the fact that agricultural products have a high water content, especially when it comes to fruits and vegetables, which contain between 75 and 95% of water. Although the high water content promotes benefits for consumer, there is one downside: the acceleration of product deterioration. This deterioration is due to the fact that water is responsible for the metabolism of bacteria and fungi (unicellular and multicellular) that promote the decomposition of organic matter. Thus, when it is desired to increase storage time, and consequently decrease losses, it is necessary to remove a fraction of the water present. Losses become even more important when countries such as Brazil, of continental dimensions, are considered. Due to the large distances to be traveled between producing regions and consuming regions, the risk of product loss before reaching the consuming region is high. This factor is also highlighted by Fioreze (2004), where it is reported that drying had a great development in World War II due to the need to transport food between different continents.

The water quantity withdrawn from the product is an important indicator in the study of the drying process, since, depending on the destination that the industry intends for the product, high values of this indicator can be positive or negative. In addition, the time required to reach the required values for the water quantity is an important factor, since the drying time is directly related to the energy used in the process.

For an adequate study of the above mentioned indicators, a good description of the process is necessary, and this description is directly related to applied mathematical modeling. The mass migration (water) of the product to the medium can be modeled by the diffusion equation, and some works have been published considering this modeling (Sharma et al., 2003; Rastogi and Raghavarao, 2004; Ruiz-López et al., 2010; Uribe et al., 2011; Shi et al., 2013; Akipnar and Toraman, 2016). However, generally only one term of the series that represents the analytical solution is used. Silva et al. (2012) have shown that the use of few terms of the series that represent the analytical solution can generate significant errors, especially at initial instants. Even more specifically, the greater the number of Biot, the greater is the error generated when considering few terms of the series. Still according to Silva et al. (2012), for a Biot number value of 35; for example, it would be necessary to use more than 16 terms of the series. Therefore, studies that model drying processes considering appropriate numbers of terms of the series are important.

Another factor that must be considered in the description of drying processes is the surface resistance to mass flow. This resistance can be described by considering a boundary condition of the third kind for the diffusive model. Silva et al. (2014) reported that few papers that use diffusive model consider boundary condition of the third kind, however, this condition makes it possible to investigate some resistance to mass flow promoted by the surface of the product, which provides a greater rigor in the description of mass migrations.

According to what has been exposed, it is possible to note the importance of new studies that consider a more general mathematical modeling in order to minimize possible errors generated by simplified modeling. The objective of this research is to present a diffusive model that considers an adequate number of terms of the series that represents the analytical solution for diffusion equation and considers a boundary condition of the third type. Finally, apply this modeling in the description of ginger drying.

2. MATERIAL AND METHODS

The present work considered a diffusive model to describe the mass transfer in the process of drying slices of ginger. The considered models admitted the geometry of an infinite slab. In order to analyze the resistance to the mass flow in the surface of the product, a boundary condition the third kind was admitted. Thus, an analytical solution was used for the diffusion equation, and an optimizer was coupled to this solution in order to obtain optimum parameters for the process.

2.1 Experimental data of ginger drying

The experimental data used in the simulations of this work were obtained by Akipnar and Toraman (2016). These data refer to the drying of slices of ginger with 4mm thickness, 30mm diameter and initial mass of 150g. The experiments were carried out in a cyclone dryer with air at temperatures of 40, 50, 60 and 70°C. These temperatures were combined with air velocities of 0.8, 1.5 and 3 ms^{-1}. This research used only the experimental data of the air velocity of 1.5 ms^{-1}. Further details about the experiment can be found in Akipnar and Toraman (2016).

2.2 The diffusion equation

The diffusion equation for the case of mass transfer in problems such as convective drying is given by (Luikov, 1968; Crank, 1992):

$$\frac{\partial M}{\partial t} = \nabla.(D\nabla M), \tag{1}$$

where M represents the moisture content (dry basis), and D is the effective water diffusivity.

As the mass flow in the direction of x (thickness of the ginger slices) is the main flow, it is possible to consider the slices as an infinite slab (LUIKOV, 1968). In this case, Eq. (1) can be written as follows (SILVA, 2009):

$$\frac{\partial M}{\partial t} = \frac{\partial}{\partial x}\left(D\frac{\partial M}{\partial x}\right), \tag{2}$$

where D is the effective water diffusivity (m^2s^{-1}), t is the time (s) and x is the Cartesian coordinate of the position (m).

In order to apply this solution, the following hypotheses were considered:

- The product is homogeneous and isotropic;
- The distribution of the water content at the beginning of the process is uniform;
- The only mechanism of water transport within the product is the liquid diffusion;

- Shrinkage is negligible;
- The effective water diffusivity and the convective mass transfer coefficient remain constant throughout the process.

The convective boundary condition is expressed as follows:

$$-D\left.\frac{\partial M(x,t)}{\partial t}\right|_{x=\pm L_x/2} = h\left(M(x,t)|_{x=\pm L_x/2} - M_{eq}\right), \tag{3}$$

In Eq. (3) h is the convective mass transfer coefficient (ms^{-1}), $M(x, t)$ is the value of M at a position x at time t, M_{eq} is the equilibrium value of M, while L_x is the thickness of the infinite slab (m).

Considering the above hypotheses, Eq. (2) can be solved with the boundary condition defined by Eq. (3). The analytical solution of Eq. (2) is given by (Luikov, 1968, CRANK, 1992):

$$M(x,t) = M_{eq} + \left(M_0 - M_{eq}\right)\sum_{n=1}^{\infty} A_n \cos\left(\mu_n \frac{x}{L_x/2}\right) e^{\left(-\frac{\mu_n^2}{(L_x/2)^2}Dt\right)}, \tag{4}$$

in which the origin of the x-axis is located at the central point of the slab. In Eq. (4), the coefficient A_n is given as

$$A_n = \frac{4\,sen\,\mu_n}{2\mu_n + sen\,(2\mu_n)}, \tag{5}$$

where μ_n is the root of the characteristic equation

$$\cot\mu_n = \frac{\mu_n}{Bi}, \tag{6}$$

In Eq. (6), the mass transfer Biot number is given by

$$Bi = \frac{hL_x/2}{D}, \tag{7}$$

The average value of $M(x, t)$ denoted by $\bar{M}(t)$ is expressed as follows:

$$\bar{M}(t) = M_{eq} + \left(M_0 - M_{eq}\right)\sum_{n=1}^{\infty} B_n e^{\left(-\frac{\mu_n^2}{(L_x/2)^2}Dt\right)}, \tag{8}$$

where the parameter B_n is given by

$$B_n = \frac{2Bi^2}{\mu_n^2\left(Bi^2 + Bi + \mu_n^2\right)}. \tag{9}$$

Eq.(6) is a transcendental equation that can be solved for the specified mass transfer Biot number. In order to find the roots of Eq.(6), a program were developed in FORTRAN on Windows platform based on the Newton Method.

2.3 Optimization

In order to obtain the process parameters (D and h) using experimental datasets, an optimizer was coupled to the analytical solution. In this optimizer the initial values are assigned to the parameters, and then these are corrected in order to minimize the objective function (here, the chi-square function) (BEVINGTON e ROBINSON, 1992), which is defined as follows:

$$\chi^2 = \sum_{i=1}^{N_p}\left[M_i^{exp} - M_i^{ana}(D,h)\right]^2 \frac{1}{\sigma_i^2}, \tag{10}$$

where M_i^{exp} is the average moisture content of the i-th experimental point, M_i^{ana} is the average moisture content calculated by the analytical solution as a function of *D* and *h*, N_p is the number of experimental points, and $1/\sigma_i^2$ is the statistical weight for the i-th experimental point. Thus the objective function (Eq. (10)) depends on the effective water diffusivity and the convective mass transfer coefficient.

3. RESULTS AND DISCUSSION

Akipnar and Toraman (2016) used a solution of Eq. (2) to determine the effective water diffusivity for each case studied. In their work a boundary condition of the first kind was admitted, and only one term of the series that represents the analytical solution was used. For the description of drying kinetics, the Midilli and Kucuk model was used (MIDILLI et al., 2002), which is an empirical model. Empirical equations have a good description of the diffusive process, however, they are only able to represent data under conditions similar to those developed.

As previously mentioned, the use of few terms of the series that represents the analytical solution can generate significant errors, especially at initial instants. In this way, optimizations were made in order to verify if the model used by Akipnar and Toraman (2016) was the most adequate. These optimizations considered 20 terms of Eq. (8) and the boundary condition of the third kind (Eq. (3)). The results obtained in the optimizations are arranged in Tab. 1.

Table 1. Values obtained by optimization processes in the present work

Temperature (°C)	*D* (m^2s^{-1})	*h* (ms^{-1})	Bi	R^2	χ^2
40	5.613×10^{-10}	4.123×10^{-7}	1.469	0.9940	2.9391×10^{-1}
50	9.596×10^{-10}	6.065×10^{-7}	1.264	0.9922	2.5086×10^{-1}
60	1.198×10^{-9}	7.027×10^{-7}	1.173	0.9911	2.4835×10^{-1}
70	1.452×10^{-9}	9.133×10^{-7}	1.258	0.9800	5.0275×10^{-1}

From the statistical indicators presented in Tab. 1, it can be seen that the model proposed in this paper was adequate to describe the process. In addition, according to the values obtained for the Biot number, the surface of the product presents a resistance to mass flow that should not be neglected in mathematical modeling. Therefore, the boundary condition of the first kind is not the most adequate to describe this process.

In Tab. 2 are presented values of the diffusividades obtained by Akipnar and Toraman (2016).

Tabela 2. Values obtained by Akipnar and Toraman (2016)

Drying air velocity	Temperature (°C)	*D* (m^2s^{-1})
1.5m/s	40	3.342×10^{-10}
	50	4.495×10^{-10}
	60	5.306×10^{-10}
	70	6.702×10^{-10}

Comparing the data of Tab. 1 and Tab. 2, it can be seen that the values of effective water diffusivity were higher in Table 1. This is due to the observed surface resistance that forces the diffusive flux to be greater to overcome this resistance. In addition, to obtain the values observed in Tab. 2 only one term of the series was used. On the other hand, for the values of Tab. 1, 20 terms of the series were considered. This shows that the other terms not considered by Akipnar and Toraman (2015) contribute significantly to the values predicted by the analytical solution. Therefore, only one term of the series is not sufficient to adequately describe the process.

Also in Tab. (1) it is noticed that the results obtained for *D* and *h* are in agreement with the expected one. The temperature influenced both diffusive mass and convective transport. As the temperature increased the values of *D* and *h* increased also (Silva et al., 2012; TZEMPELIKO et al., 2015; DAI et al., 2015).

In Fig. 1 are the graphs with the fits of the analytical solution to the experimental data for the four temperatures studied. These graphs confirm the good fits noted in Tab. 1.

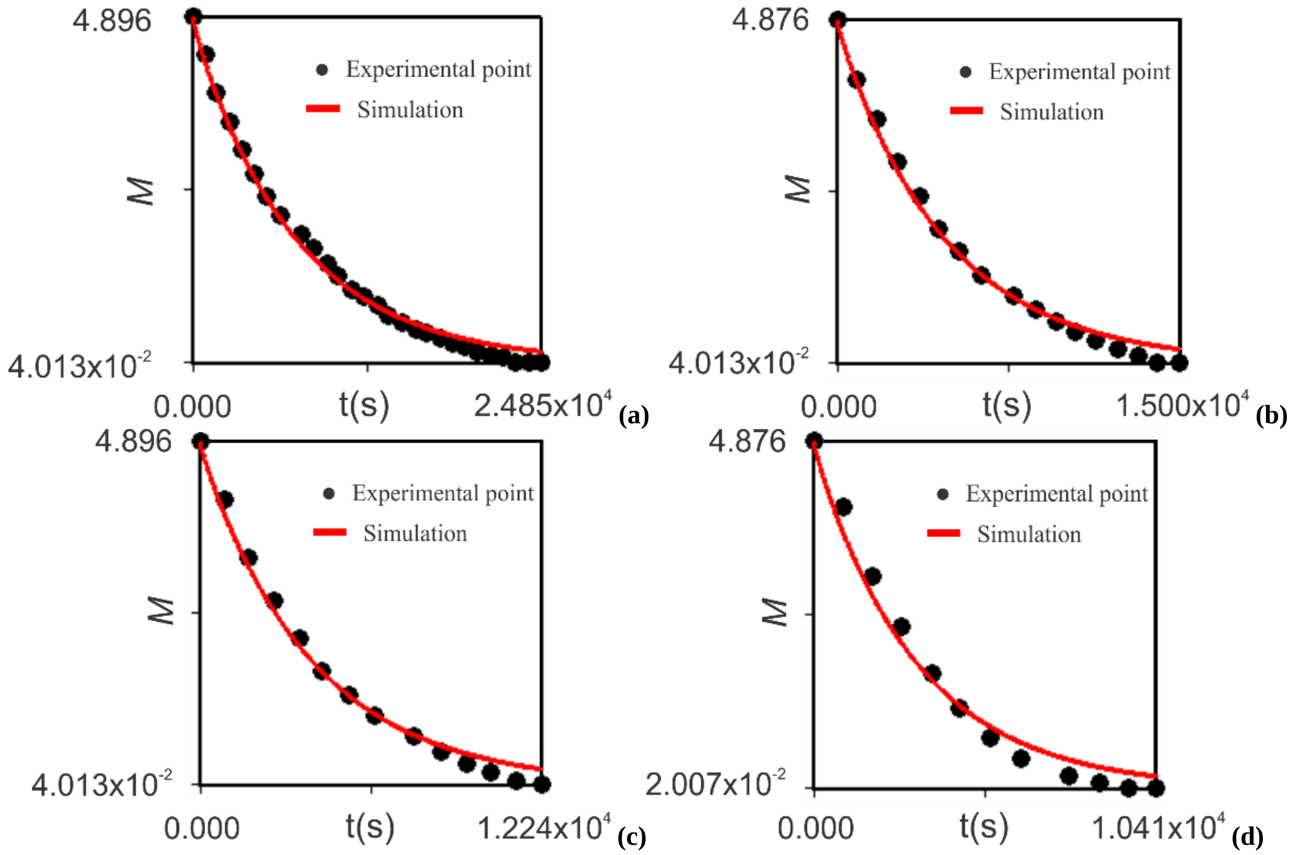

Figure 1: Drying kinetics obtained for the temperatures: (a) 40°C; (b) 50°C; (c) 60°C; (d) 70°C.

In Fig. (2) are the simulations for the four temperatures studied. In this figure it is possible to observe the influence of the temperatures on the drying time. Note that the influence was greater from 40 to 50°C.

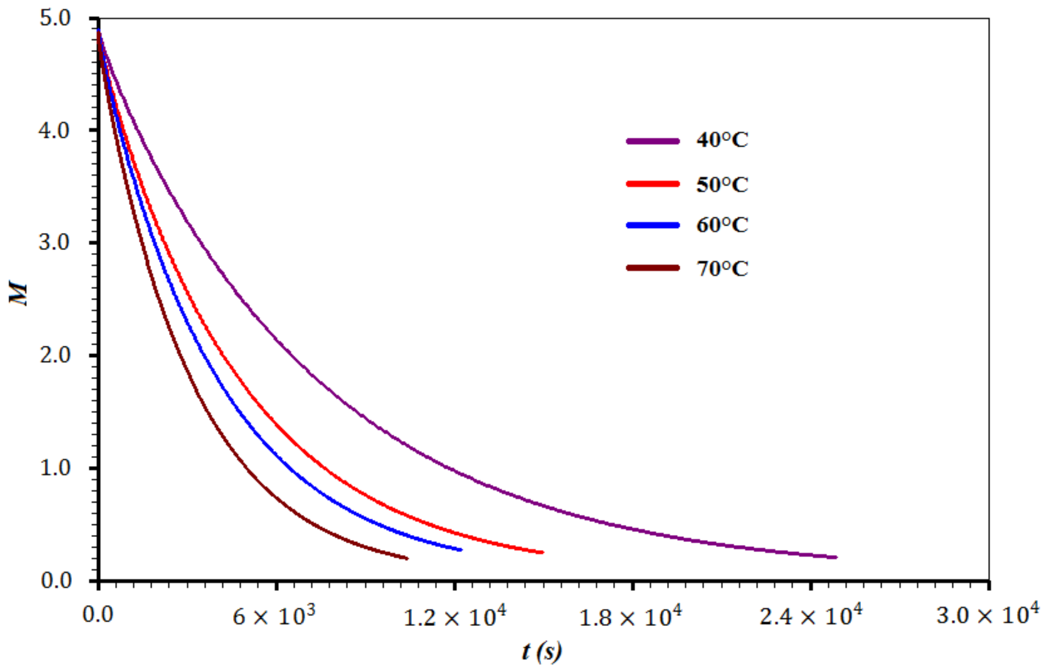

Figure 2: Water loss simulations for the four drying temperatures.

4. CONCLUSIONS

The proposed model satisfactorily described the drying kinetics of ginger. By the values obtained for the Biot number, it is possible to conclude that the boundary condition adequate to describe the drying of ginger is the one of the third kind. Many papers found in the literature consider the boundary condition of the first kind to describe the drying of agricultural products. In this way, the model proposed in the present work can be applied in the description of the drying of other products, in order to study the boundary condition.

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